## Sample Problems from previous Andalafte Competitions

**Problem 1.** Evaluate $\int_0^\infty \frac{\ln x}{1+x^2}dx$ (let u = 1/x)

**Problem 2. Evaluate** $\int_{-1}^{1} \sqrt{(1+x)/(1-x)}dx$

**Problem 3. The Fibonacci sequence** $(f_n)$ **is defined by** $f_1 = 1, f_2 = 1$ **and for n**$\geq 2$**,** $f_n = f_{n-1} + f_{n-2}$**. Find the radius of convergence of the power series** $\sum_{n=1}^{\infty} f_n x^n$

**Problem 4. Evaluate** $\int_0^1 \sqrt[5]{1-x^3} + \sqrt[3]{x^5-1}dx$

**Problem 5. Evaluate** $\int_0^{\pi/2} \cos^3(x)/(\cos^3(x)+\sin^3(x))dx$

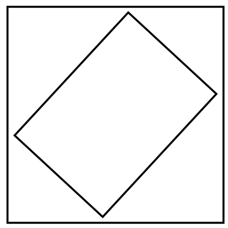
**Problem 6**. Let p and q be distinct primes. Show that $\log_p q$ is always an irrational number.

**Problem 7.** Let a and b be positive numbers. Show that a(1-b) and b(1-a) cannot both be larger than ¼.

**Problem 8.** Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers. Let $\mathbb{N} = \{1,2,3,\ldots\}$ be the set of positive integers and let $h:\mathbb{N} \to \mathbb{N}$ be a one-to-one and onto function. Define a new sequence
$\{b_n\}_{n=1}^{\infty}$ by setting $b_n = a_{h(n)}$. We call the sequence $\{b_n\}_{n=1}^{\infty}$ a "rearrangement" of the original sequence $\{a_n\}_{n=1}^{\infty}$ . For example suppose we take for h, the function h(1)=2, h(2)=1, h(3)=4, h(4)=3, and so on. Then the corresponding rearrangement of $\{a_n\}_{n=1}^{\infty}$ is
$\{a_2, a_1, a_{4,} a_3, a_6, a_5, .....\}$
Show that if the sequence $\{a_n\}_{n=1}^{\infty}$ converges to a limit L then any rearrangement of $\{a_n\}_{n=1}^{\infty}$ also must converge to L.

**Problem 9.** Suppose that *f* and *g* are continuous real-valued functions on [a, b] and are differentiable on (a, b). Prove that if *f*(a) = *g*(a) and $f'(x) < g'(x)$ for all x in (a, b) then *f*(b) < *g*(b)

**Problem 10.** Let R be a rectangle having length L and width W. Find the maximum area of a rectangle that can be circumscribed about R. ("circumscribed" means that the vertices of R must lie on the sides of the larger, circumscribed, rectangle – as in the figure below )



**Problem 11.** A tank contains 20 kg of salt dissolved in 5000L of water. Brine that contains .03 kg of salt per liter of water enters the tank at a rate of 25L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt remains in the tank after ¾ of an hour ?

**Problem 12**. Show that the series $\sum_{n=0}^{\infty} \frac{2^n}{5^{2^n}+1}$ converges and find its sum

**Problem 13**. Evaluate $\int_0^{\pi/2} \frac{\cos^{2008}(x)}{\sin^{2008}(x)+\cos^{2008}(x)}dx$