Wall clocks on a shuttle to Vega

P. Fraundorf[1, a)]
Physics & Astronomy/Center for Nanoscience, U. Missouri-StL (63121), St. Louis, MO, USA

(Dated: 23 January 2016)

Suppose that in the future we find a way to power sustained-acceleration we can live with e.g. for roundtrips to Vega about 25 lightyears away. How would you design a set of shuttle wall-clocks to help crew and passengers keep track of voyage progress in local-time, as well as the "current time" at your launch and destination points? Also, might this design project provide insight into the nature of spacetime, and its effects on the low-speed world in which most of us live?

I. INTRODUCTION

Although coordinate-acceleration, defined as the second-derivative of map-position with respect to map-time, cannot be held constant as one's speeds become relativistic, the frame-invariant proper-acceleration of a rocketship can be held constant as long as you can exert the thrust and carry the fuel needed to power it[1,2]. In spite of modern movie tales about interstellar travel this is unlikely to be practical in the near future, but it is fun to think about nonetheless. We make the case here that thinking about it may be educational as well.

II. THE CHALLENGE

Let's imagine that an outpost in the neighborhood of Vega (about 25 lightyears away) has been established, and that a "shuttle service" for transporting physical objects (like colonists) between Sol and Vega has been established using "sustained 1-gee acceleration" technology. Shuttle crew and passengers might enjoy seeing a display marking progress during their constant proper-acceleration roundtrip (about 6.6 years on ship-clocks each way, with a thrust-direction turn-around half-way between), and your task is to come up with something that will at a glance give them a picture of where things stand.

In addition to aesthetic matters of design, there are technical issues to consider as well. Far-away simultaneity is ambiguous because of the "causality-gap" which e.g. deprives us of a real-time view of unfolding events on the surface of Mars, and in addition prevents us from giving real-time feedback even if we had that real-time view e.g. on how a navigating rover should respond. In other words, Mars surface (and that of any far-away object) is obscured by such a causality-gap.

Is the "current time" on Mars' surface: (i) the last instant (in Mars' past) on which we have incoming data e.g. when the crowd cheers at the incoming video of a rescue in Mars' orbit, (ii) the first instant (in Mars' future) on which earth actions now can have an effect e.g. when the crowd cheers as we push the button just in time to warn Mars colonists to take cover following a recent solar flare, or (iii) something else? Halfway between is the standard answer to this question if Mars is not being accelerated to relativistic speeds, but otherwise the alternatives are more varied.

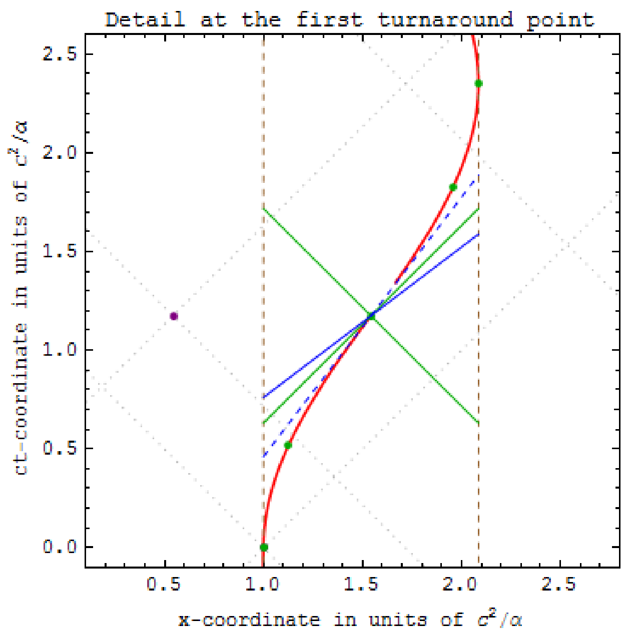

FIG. 1. Closeup of the first turn-around event on an x-ct plot of a generic constant proper-acceleration roundtrip. This is where the Sol (left brown-dashed vertical line) and Vega (right brown-dashed vertical line) "causality-gap" boundaries (green right-angle intersections) share a common symmetry. The tangent free-float-frame position isocontour at this point is the blue dashed-tangent, while its time-isocontour is a blue solid-line.

A. causality limits

Figure 1 here shows closeup of the first half of the standard x-ct diagram for a generic constant proper-acceleration round-trip trajectory (in red), with one-

a) Electronic mail: pfraundorf@umsl.edu

eighth round-trip benchmarks in traveler-time superposed (as green dots). The focus is on the first thrust-reversal point halfway to the destination. The causality-gap at each destination (dashed vertical lines) is defined the by the 45-degree diagonals (in green), and depends only on the shuttle's position as measured by galactic map-frame (e.g. "Milkyway Mean Position") coordinates.

The vertical axis of this plot similarly represents time on (to first order) synchronized galactic map-frame clocks (e.g. "Milkyway Mean Time" or MMT). By sliding that diagonal intersection along the trajectory, of course, you can see that the causality gap will be smaller for the destination to which we are nearest. Most probably agree[3] that the "current time" at a far-away point, from the perspective of our shuttle travelers, should probably lie somewhere within this causality gap.

B. tangent free-float-frame notions

A popular notion of extended-simultaneity that might work for accelerated travelers in flat spacetime imagines a co-moving "free-float-frame" observer (unaccelerated in flat spacetime) who is attached to a co-moving extended network of yardsticks and synchronized clocks separate from the galactic network already discussed. Time and space axes for this "tangent free-float-frame" network are shown in Fig. 1 by blue dashed and solid lines, respectively, obtained as usual with help from special-relativity's Lorentz transform.

The solid blue space-axis is an "isocontour" of constant time in that co-moving frame, which can be extrapolated to either of the roundtrip destination end-points, for an answer to "What time is it at that destination now?". This answer depends on both shuttle position (which determines the intersection location along the trajectory) and shuttle-velocity (which determines the angle between space and time axes).

C. radar-time notions

Radar-time simultaneity[4], like all simultaneity models discussed here, reduces to the halfway in-between convention for "current time on Mars and other non-accelerating objects" discussed at the beginning of this section.

Radar-time's main advantage may be that requires no frames of synchronized clocks at all, and as a result works also in curved spacetime. Its main disadvantages are that it depends on shuttle trajectory throughout the causality-gap, and is messier to calculate[5] as suggested by the "25 zone" plot of radar-time and radar-position isocontours in Fig. 2.

For the application here, its estimate of "current time at the destination points" is closer to the local MMT value. To observers at rest in the galaxy, of course, both

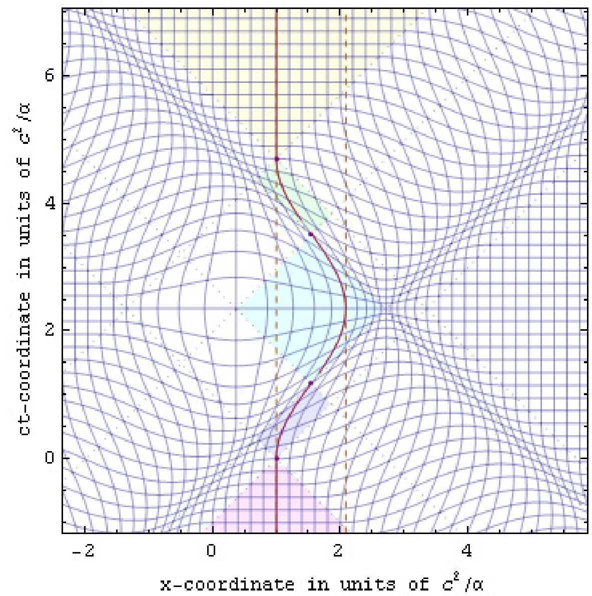

FIG. 2. Radar-separation hypersurfaces for an accelerated traveler on a generic constant proper-acceleration roundtrip.

local MMT clocks and clocks at the destination points are synchronized. Which brings me to the final alternative.

D. local map-time notions

If we can at least assume a local galactic network of synchronized MMT clocks (e.g. by neglecting the effects of gravitational potential differences within this region of the Milkyway), yet another alternative may be to ask what MMT time is on clocks in the current neighborhood of the shuttle. This time has no causality-gap, is centered within the causality-gaps for both end-point destinations, and has the advantage that it fixes the current time at both Sol and Vega to be one and the same.

III. SAMPLE SOLUTION

What will **your** wall clock look like in addressing this challenge?

For instance, since 6.6 years should provide lots of opportunity to think about time and space on a one-way trip, the sample solution illustrated with a collection of snapshots in Fig. 3 (and available with equations in gif-animation form on Wikimedia Commons[6]) is fairly rich in detail. It uses 4 clock faces to show the current causality-gap for each end-point destination, as well as times at all of the locations using each of the definitions discussed above.

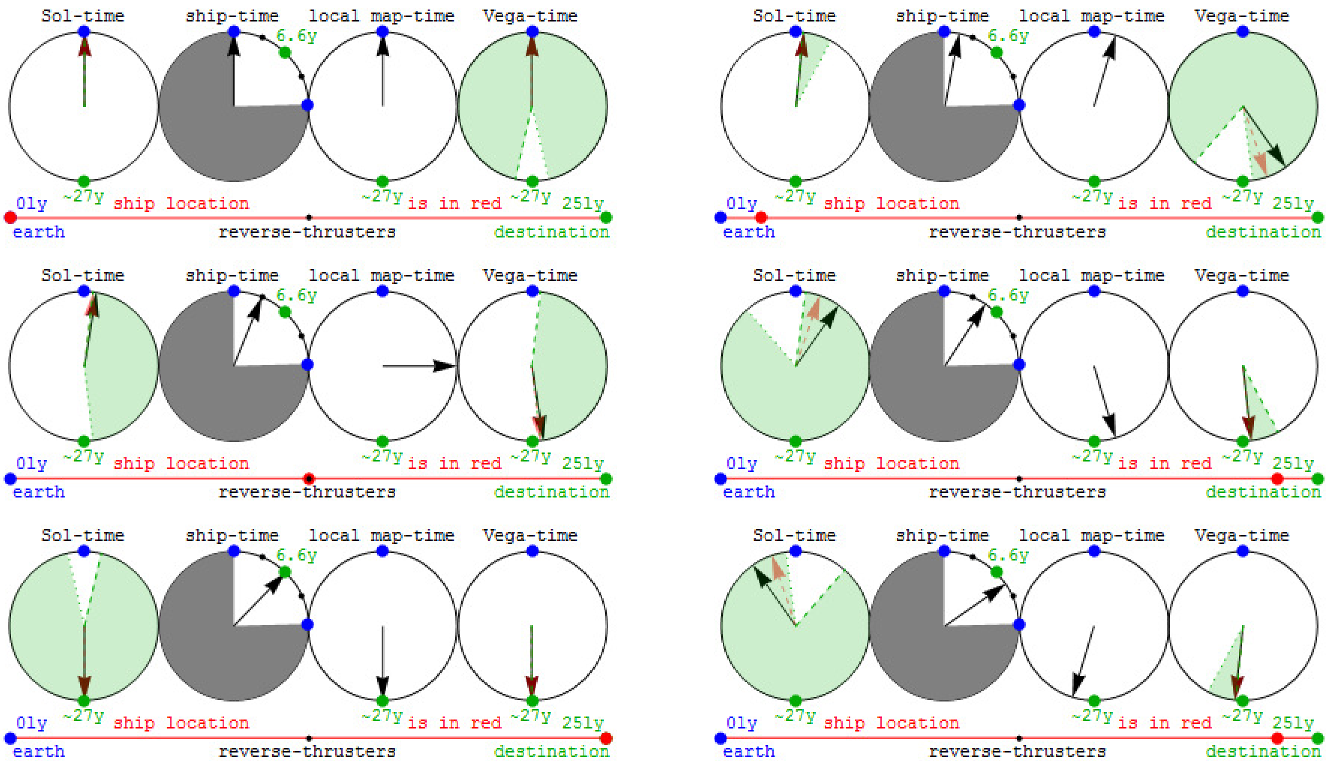

FIG. 3. Selected set of clock snapshots on a 6.58 traveler-year 1-gee constant proper-acceleration roundtrip to Vega about 25 lightyears away, at 1/8th roundtrip time-increments from Sol-departure in row 1 of: 0, 1.645, row 2 of: 3.29, 4.935, and row 3 of: 6.58, and 8.225 traveler-years.

IV. DISCUSSION

In the top-left corner "earth launch" panel of Fig. 3, note the ±25 year causality-gap (shaded) we have today as more or less stationary observers looking out at Vega. It's this causality-gap that is responsible for the 8 to 48 minute communications turn-around time (depending on orbital positions) between earth and Mars, as well as for the much larger gaps associated with the study of objects outside of our galaxy.

Given this causality-gap when traveling interstellar distances, the timing from your perspective of far away events admits to more than one model. Since comoving networks of yardsticks of synchronized clocks in flat spacetime are not usually available, tangent free-float-frame simultaneity has been useful mainly for exploring special-relativistic symmetries.

When a preferred extended map-frame of yardsticks and synchronized clocks is available, the best way to describe the timing of a far away event may be cite the local map-time of that event. When no map-frame of yardsticks and synchronized clocks is available, the metric-equation's bookkeeper-time may have limited physical meaning. Hence the only alternative for the "What's happening there now?" question may then be the "half-way in-between" radar-time of that event.

One message from the causality-gap is therefore this: The timing of far-away events may not be set in stone until light from those events reaches your grasp.

ACKNOWLEDGMENTS

Thanks to Edwin Taylor and the late Bill Shurcliff for interesting notes and discussions on these topics.

[1]E. Taylor and J. A. Wheeler, *Spacetime physics*, 1st ed. (W. H. Freeman, San Francisco, 1963) contains some material not found in the 2nd edition.

[2]C. Lagoute and E. Davoust, "The interstellar traveler," American Journal of Physics **63**, 221 (1995).

[3]T. A. Debs and M. L. G. Redhead, "The twin 'paradox' and the conventionality of simultaneity," American Journal of Physics **64**, 384–392 (1996).

[4]Carl E. Dolby and Stephen F. Gull, "On radar time and the twin paradox," American Journal of Physics **69**, 1257–1261 (2001).

[5]Antony Eagle, "A note on Dolby and Gull on radar time and the twin 'paradox'," American Journal of Physics **73**, 976–979 (2005).

[6]P. Fraundorf, "Round-trip to Vega," (2016), archived by WebCite at http://www.webcitation.org/6ehp04Wui.