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Abstract

The paper shows a key role of human capital in explaining how US postwar growth and welfare could have increased while tax rates declined. As in evidence, we assume that the share of government revenue in output has remained stable and model tax evasion within an endogenous growth model with human capital. A trend upwards in the productivity of the goods or human capital sectors gradually decreases the degree of tax evasion, and causes a trend upwards in time spent in human capital accumulation. These productivity increases also increase the ratio of tax revenue to GDP at any given tax rate such that the tax rate must be reduced in order to be consistent with the stylized fact of a constant share of government revenue in output. Based on estimated US postwar goods and human capital sectoral productivities, the model explains 30% of the actual decline in a weighted average of postwar US top marginal personal and corporate tax rates. The estimated joint sectoral productivity increases are asymmetric with a larger relative increase in the human capital investment sector, a result related to McGrattan and Prescott's (2010) relatively larger increase in the productivity of the sector producing intangible capital relative to the goods sector. We show that in a special case of exogenous growth without human capital investment, the explanatory power of the tax trend drops significantly.

JEL Classification: *E13, E62, H26, O41*

Keywords: Tax evasion, intermediation technology, endogenous growth, human capital productivity, dynamic general equilibrium.

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1 Introduction

It is well-known that postwar US top marginal personal and corporate income tax rates have trended downwards while US government revenue as a proportion of GDP has remained stable. Figure 1 (solid line) shows that a weighted average of the top US marginal personal (x-line) and corporate (■-line) income tax rate fell from 75% to 35% from 1951 to 2012. Figure 1 also shows that federal tax revenue as a percentage of GDP (▲-line) varied little from its average value of 18% over this postwar period. Similarly the average personal income tax rate for the top 0.5% of tax payers (accounting for an estimated 31.67% of federal personal income tax receipts in 2010) fell from 56% to 34% between 1960 and 2004; the average US corporate income tax rate fell from 52% to 27% between 1951 and 2011; and the weighted average decline was from about 52 to 33% from 1960 to 2004 (average rates are shown in Appendix $A.1$, Figure A, and the online Appendix).[1] Such tax trends also are found for the UK.

The incentive effect of top marginal tax rates, or the average rate on the highest income taxpayers, is stressed by many from McGrattan (2012) to Saez et al. (2012).[2] Besides documenting the decline in US postwar tax rates, Saez et al. also document a more than doubling of the share of the top 1% of US income earners from a steady 8% level from 1960 to 1981 to 18% in 2006, with acceleration upwards after the 1981 tax reductions and after the 1986 tax reductions. Saez et al. find this income share rise puzzling as it apparently is not explained by "real" factors such as labor supply response. They conclude instead that the cause is that tax evasion, or avoidance, decreased as the tax rate fell and the "tax base" of reported income rose.[3] As Thornton (2012) puts it, "Higher marginal tax rates also provide a stronger incentive to go 'underground'". With evasion a part of the explanation, the elasticity of reported income to the tax rate would be expected to be higher than if there were no evasion. Saez et al. also present evidence (their Table 1) showing the existence of high estimated elasticities of the income share of the top 1% to the tax rate after the US 1986 tax act, as consistent with high elasticity evidence in Mertens and Ravn (2013a, 2013b). High elasticities appear difficult to explain theoretically in standard models without tax evasion, such as those reviewed by Saez et al. [4]

This paper models tax evasion, explains how reported income elasticities to tax

[1] The historical contribution of the personal income tax to total US taxes was around 7% in 1949 and in 2012 with some variation across time, while the corporate tax share of taxes has fallen some from about 4% to 2% from 1949-2012. Sources for the top marginal tax rates in Figure 1: Tax Foundation, "U.S. Federal Individual Income Tax Rates History, 1913-2013 (Nominal and Inflation-Adjusted Brackets)" and "Federal Corporate Income Tax Rates, Income Years 1909-2012". Weights are calculated using shares in OMB Historical Table 2 "Percentage Composition of Receipts by Source: 1934–2018". Tax receipts as a fraction of GDP are from OMB Historical Table 1.3; 2012 is their estimate. Please see online Appendix Tables A1-A4 for details.

[2] McGrattan (2012) focuses on how higher tax rates on dividend income, paid mainly by those in higher income brackets, can help to explain key facets of the Great Depression, while for example Romer and Romer (2010) and Cloyne (2013) find that tax cuts have a large impact on output.

[3] "While such policy options may have little impact on real responses to tax rates (such as labor supply or saving behavior), they can have a major impact on responses to tax rates along the avoidance or evasion channels" (Saez et al., p.42, 2012). Slemrod and Weber (2012) review the evasion literature.

[4] Inbetween the US 1981 and 1986 tax reforms was the lesser-known US Tax Reform Act of 1984 that broadened the tax base by ending a decade long Congressional deadlock on IRS determination of non-statutory fringe benefits; the act specified exactly how a variety of such benefits should be taxed by the IRS.

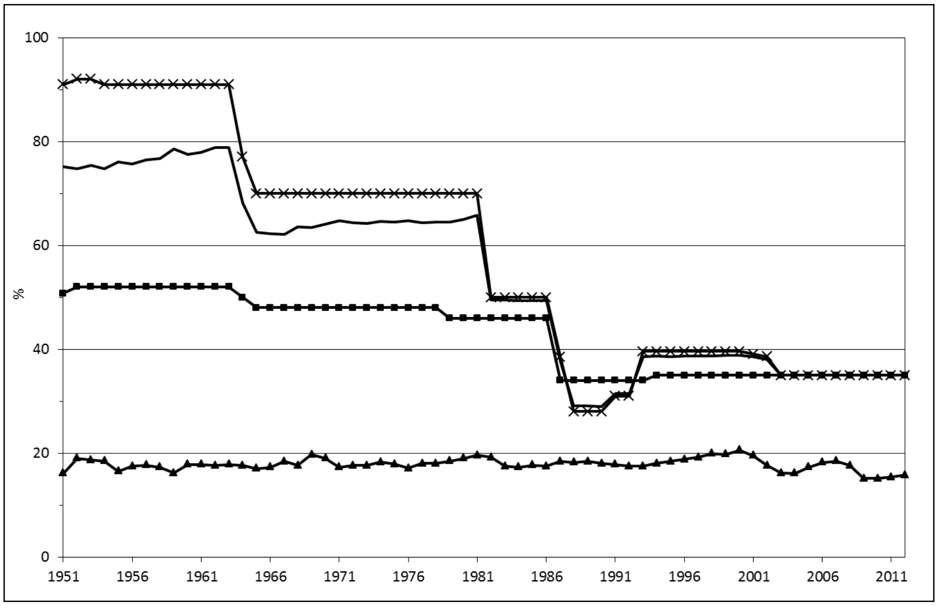

Figure 1: US Postwar Marginal Personal and Corporate Income Tax Rates, 1951-2011.

rates are higher by making the tax rates a function of the degree of evasion, and provides an analysis of how postwar US tax rates may have fallen. Exploiting the role of human capital investment is key to our explanation of the downward US tax rate trend, while also implying rising growth and welfare. Assuming flat rates of tax on capital and labor income, a constant share of government revenue as a percentage of income (Lucas, 2000, Sections 4,5), and modeling tax evasion in a general equilibrium with human capital investment, we show that tax rates decrease as sectoral productivity rises. Tax evasion creates a higher elasticity (magnitude) of the share of taxable income compared to no evasion that depends on the ratio of unreported to reported income. Increases in productivity causes a lesser degree of evasion, a lower tax rate elasticity of taxable income and, given the constant share of tax revenue in output, a decrease in the tax rate. Estimated postwar productivity trends for the goods and human capital investment sectors are based on the Baier et al. (2006) database. With this evidence the US calibrated model can explain 30% of the Figure 1 downward trend in postwar US tax rates. This fraction is more than 30% if we define the empirical tax rate decline more narrowly, such as found in Saez et al. But this fraction and our ability to explain the postwar tax trend downwards drops significantly without the human capital investment sector and endogenous growth.

2 Methodology and Role of Human Capital

The representative agent economy is a human-capital based "second-generation" endogenous growth economy as in Lucas (1988) but without externalities, with flat taxes as in King and Rebelo (1990), and with a wasteful activity as related to the political capital for corruption in Ehrlich and Lui (1999) except that here this activity takes the form of a decentralized tax evasion service.[5] The consumer's degree of tax evasion determines the curvature of the tax revenues per unit of output as graphed against the tax rate, and this curvature in turn translates directly into the elasticity of the reported income share relative to the tax rate.[6] Increases in goods and human capital productivities reduce the degree of evasion and induce a lower tax rate in order to keep the share of government revenue in output constant, while increasing the stationary time spent in human capital investment, and stationary growth and welfare.

The human capital sector is key for four main reasons. First the estimated human capital productivity increase impacts by more than the goods sector productivity the degree of potential tax rate decline. This results in the sense that the human capital productivity increase is found to be five fold larger than that coming from the goods sector, while having half the effect as that of the goods sector productivity in lowering tax rates per unit of productivity increase. Second, taking the goods sector productivity increase by itself, it causes a significantly larger decrease in the tax rate in the endogenous growth baseline model with human capital investment as compared to a similarly calibrated exogenous growth model in which the human capital grows exogenously. Therefore the model with human capital investment provides a fuller explanation of the actual US downward tax trend both because it includes human capital productivity increases along with goods sector productivity increases and because the latter have a stronger effect on tax rates within the human capital- based endogenous growth economy than in the exogenous growth economy. Third, the estimated productivity increases as combined with the implied tax reduction also imply that the time spent in human capital gradually rises over time, consistent with how average time spent in education apparently has trended upwards. Fourth, only within our human capital investment model do we find rising stationary growth and welfare from the productivity increases.

The way the evasion service works is that reported and unreported income are perfect substitutes for buying goods once the unreported income has been "laundered" through the competitive evasion intermediary. This intermediary has a rising marginal cost of evasion so that as the tax rate increases there is a greater waste of resources lost to evasion activity, comparable to the resource cost of the political capital investment time lost to corruption in Ehrlich and Lui (1999). Given perfect substitutability between reported and laundered unreported income, the rising marginal cost of the evasion service conversely is shown to imply an equilibrium outcome similar to a rising tax elasticity of reported income. This gives a bigger tax elasticity than without evasion at any given tax rate, as episodal evidence may suggest. This tax elasticity is also such that the higher it is, the greater the reduction in evasion that results from a given productivity increase and the greater the subsequent decrease in tax rates given a constant share of tax revenue in output. This mechanism when viewed more broadly implies that there is a link between low tax evasion and high productivity. This link is consistent with lower tax evasion being widely found in developed countries and higher evasion in developing countries (Schneider and Enste, 2000), since higher productivity is found in developed countries and lower productivity in developing countries (Klenow and Rodríguez-Clare, 1997; Hall and Jones, 1999). And the link of less evasion with lower tax rates is consistent with the movement towards lower flat taxes as designed

[5] What we call tax evasion in terms of avoiding legal taxes can also be interpreted to include avoidance through various means that lower the effective tax rate.

[6] At a given tax rate, the tax elasticity equals the slope of the output-normalized tax revenue "Laffer curve", as Agell and Persson (2001) call it, divided by the slope of a ray from the origin: or the marginal change divided by the average change.

to broaden the tax base, for example, as seen starting in 1993 in Eastern Europe.[7]

Estimates of upward trends in sectoral productivities are widespread in the Solow-growth-accounting/RBC framework for the goods sector and are emphasized for example in terms of human capital by Guryan (2009) and Baier et al. (2006). Beaudry and Francois (2010) emphasize how standard growth accounting has understated the role of human capital productivity. McGrattan and Prescott (2010) present an alternative growth accounting framework that adds in the productivity of the intangible capital sector which has been interpreted in part as including human capital. We similarly use extended growth accounting to estimate the productivities of the goods and human capital sectors, using the data set of Baier et al. for the growth rate of output, physical capital and human capital (see Appendix $A.5$).

In Section 3, the tax evasion model is first presented with only physical capital, as an "Ak" model. The paper analytically derives the Ak elasticity features and shows how productivity increases tax revenue per GDP at any given tax rate and so induces tax rate reduction. As it quantitatively cannot account for human capital sectoral productivity trends, Section 4 extends the model with a human capital investment sector and Cobb-Douglas production for both the goods and human capital sectors; this is similar to how McGrattan and Prescott (2010) extend their basic model to account for increases in its productivity in the intangible investment sector. Section 5 provides the US calibration; and Section 6 presents the simulation results of how productivity changes affect evasion, tax rate reduction, growth and welfare. Section 7 estimates postwar growth rates in the goods and human capital investment productivities and uses these in turn to estimate what proportion of the observed downward trend in US postwar tax rates can be explained by the extended economy. Section 8 provides discussion and Section 9 concludes.

3 Ak Model with Evasion

In this representative agent economy, the consumer invests in physical capital k_t and rents it to the goods producing firm and to the intermediary that provides tax evasion services. With the share of capital going to the goods sector denoted by s_{Gt} and that going to the evasion sector by s_{Et}, we have that $s_{Gt}+s_{Et}=1$. With r_t the competitively determined rental price of capital goods, the rental income that the consumer receives is $r_t\left(s_{Gt}+s_{Et}\right)k_t = r_t k_t$. The representative agent places deposits d_t, equal to all income $r_t k_t$, into the intermediary:

$$d_t = r_t k_t. \tag{1}$$

By choosing the fraction of income to report to the tax authority, $a_t \in [0,1]$ (similar to Fullerton and Karayannis, 1994) the household pays taxes on $a_t r_t k_t$ and demands tax evading services for the income equal to the remainder, $(1-a_t)\,r_t k_t$. The statutory

[7] Tax evasion here occurs in a similar manner to the inflation tax avoidance in the literature going back to Bailey (1956) and Cagan (1956); in both of these there is a rising interest elasticity of money demand as the inflation rate rises, supported empirically in international money demand evidence by Mark and Sul (2003). Tax evasion takes place in a competitively decentralized market (see also Becker, 1968, Ehrlich, 1973), analogous to inflation tax avoidance through a decentralized competitive exchange credit intermediary in Benk et al. (2010) in which there is an equilibrium relation of a rising interest elasticity of money demand. This rising inflation tax elasticity feature is the basis of Cagan's (1956) explanation of hyperinflation, Gillman and Kejak's (2005) explanation of the negative long run inflation - output growth relation found in evidence, and Eckstein and Leiderman's (1992) explanation of Israeli inflation tax revenue.

tax rate on capital income is τ_k and the competitive market price for the tax evasion service in per unit terms is denoted by p_{Et}. The income that evades tax net of the price of evasion is $(1 - p_{Et})(1 - a_t) r_t k_t$. However, as the agent owns the intermediary the profit produced by the evasion intermediary is paid back to the consumer in the form of a return per unit of deposits, denoted by r_{Et}, and thus total profit returned to the consumer is $r_{Et} d_t$. This makes the actual average cost of evasion less than p_{Et} once the intermediary's dividend payments are accounted for. Using the sum of after-tax reported income, after-evasion unreported income, and dividends from the intermediary, the agent decides how much new investment to make in capital, denoted by i_t, and the level of goods consumption c_t. Assuming a depreciation rate of δ_K on capital, the capital accumulation equation is:

$$\dot{k}_t = i_t - \delta_K k_t. \tag{2}$$

The representative consumer also receives a government transfer, denoted by v_t, hence the representative consumer's budget constraint is:

$$\dot{k}_t = (1 - \tau_k) a_t r_t k_t + (1 - p_{Et})(1 - a_t) r_t k_t + r_{Et} d_t - c_t - \delta_K k_t + v_t. \tag{3}$$

The representative consumer derives utility only from consumption goods, c_t, and maximizes lifetime utility $V(k_0)$ at time 0:

$$V(k_0) = \max_{c_t, a_t, d_t, k_t} \int_0^\infty \ln c_t e^{-\rho t} dt, \tag{4}$$

subject to the deposit (1) and budget (3) constraints given the initial capital stock k_0.

The production of the output of goods, denoted by y_{Gt} and with $A_G > 0$, is a linear function in only the physical capital allocated to the goods sector $(s_{Gt} k_t)$:

$$y_{Gt} = A_G s_{Gt} k_t. \tag{5}$$

In this "*Ak*" model, the representative agent as goods producer takes the price of capital services, r_t, as given and maximizes profit Π_{Gt} by choosing the capital input:

$$\max_{s_{Gt} k_t} \Pi_{Gt} = A_G s_{Gt} k_t - r_t s_{Gt} k_t, \tag{6}$$

so that in equilibrium $r_t = A_G$.

The government receives tax revenue $a_t \tau_k r_t k_t$ from reported capital income; it transfers the lump sum v_t to the consumer; and it consumes an amount Γ_t :

$$a_t \tau_k r_t k_t = v_t + \Gamma_t. \tag{7}$$

The intermediation sector produces the tax evasion service that enables the consumer to report only a fraction of the capital income; it is owned by the representative agent, just as in the goods producer. A Leontief one-to-one "household" production technology is implicitly assumed such that a unit of the tax evasion service and a unit of "laundered" income are combined to yield a unit of untaxed income that the consumer can use for goods purchases. Therefore the quantity of evasion services, denoted by κ_t, equals the quantity of unreported income; $\kappa_t = (1 - a_t) r_t k_t$.[8]

[8] A related Leontief approach is formalized in Gillman and Kejak (2005, eqt. 8-11), as based on Becker's (1965) household production technology.

The intermediary takes as given prices p_{Et} and r_t, and maximizes profit Π_{Et}, which equals total revenue $p_{Et}\kappa_t$ minus the rental costs of capital used in producing the intermediation service, $r_t s_{Et} k_t$, and minus the dividend payouts on the income deposits $r_{Et} d_t$. There is zero profit after paying out the residual dividend income:[9]

$$\max_{s_{Et}k_t, d_t} \Pi_{Et} = p_{Et}\kappa_t - r_t s_{Et} k_t - r_{Et} d_t. \tag{8}$$

Note that the consumer owns the intermediary because, as with a mutual bank, each dollar deposited buys an ownership share where the price per share is fixed at one.[10]

Given $\omega_E \in [0, 1)$, the technology of the intermediary's tax evasion service is assumed to be constant returns to scale (CRS) in its inputs of physical capital and deposited funds (a form of "financial" capital; see Berger and Humphrey, 1997):

$$\kappa_t = A_E \left(s_{Et} k_t\right)^{\omega_E} \left(d_t\right)^{1-\omega_E}. \tag{9}$$

Per unit of deposits, the production function, $\kappa_t / d_t = A_E \left(s_{Et} k_t / d_t\right)^{\omega_E}$, exhibits diminishing returns to the normalized capital input factor and so has an upward sloping marginal cost per unit of deposits and a unique equilibrium.[11]

The first-order conditions imply that the cost of capital equals its marginal product; $r_t = p_{Et}\omega_E A_E \left(s_{Et} k_t / d_t\right)^{(\omega_E - 1)}$, that the residual return on deposits equals its marginal product, $r_{Et} = p_{Et}\left(1 - \omega_E\right) A_E \left(s_{Et} k_t / d_t\right)^{\omega_E}$, and that the unique solution for the normalized input ratio is $s_{Et} k_t / d_t = \left(\omega_E A_E p_{Et} / r_t\right)^{[1/(1-\omega_E)]}$. Substituting this ratio into the production function in equation (9) yields the equilibrium ratio of tax evasion dollars to deposits: $\kappa_t / d_t = A_E \left(\omega_E A_E p_{Et} / r_t\right)^{[\omega_E/(1-\omega_E)]}$. Given $\kappa_t = \left(1 - a_t\right) r_t k_t$ and $d_t = r_t k_t$, this implies an equilibrium fraction of unreported income of $1 - a_t = A_E \left(\omega_E A_E p_{Et} / r_t\right)^{[\omega_E/(1-\omega_E)]}$. One can rewrite these equilibrium conditions to show that the marginal cost of the evasion service (MC) is equated to the price p_{Et}, with MC defined as the marginal factor price divided by the marginal factor product:

$$p_{Et} = r_t / \left[\omega_E A_E \left(s_{Et} k_t / d_t\right)^{(\omega_E - 1)}\right] \equiv MC. \tag{10}$$

[9] The evasion intermediation activity, or avoidance more broadly, can be viewed as taking place in a branch of the firm, in a small segment of the banking sector, or in other ways whereby the income is reprocessed into non-taxable income through an intermediary (see also Gillman and Kejak, 2011). This income can be from legal enterprises or criminal industries such as drugs, trafficking and illegal arms trade; presumably most large sums of both legal and illegal income are deposited in banks. Tax evasion through banks is the focus of ongoing US Congress (2001) hearings and continuous news reports; eg. the Wall Street Journal (2013, June 3) reports a "detailed account of a system the bank allegedly helped put in place to allow some wealthy French people to evade taxes".

[10] We assume that dividends are not taxed since the total value-added of the intermediary equals the factor income $s_{Et} r_t k_t$ that is used up in production and this is already subject to full taxation (although some is evaded); taxation of dividends $r_{Et} d_t$ would amount to a type of double taxation that we prefer to avoid in this context.

[11] With $\omega_E = 1$, it can be shown that no unique equilibrium exists. See also Sealey and Lindley (1977), Clark (1984), Hancock (1985), and Gillman and Kejak (2005, 2011) for the intermediation approach. The "productivity" parameter is shorthand for the myriad of factors affecting the evasion that Becker (1968) and Ehrlich (1973, 1996) detail more generally for participation in an illegal activity. Our service production is for the evasion industry itself, while Ehrlich's (1973) production function is for a certain probability of the good state (apprehension of criminal activity).

3.1 Equilibrium

A competitive equilibrium for this economy consists of a set of allocations $\{c_t, a_t, k_t, s_{Gt}, s_{Et}, d_t\}$, a set of prices $\{r_t, p_{Et}, r_{Et}\}$, the government's policy $\{\tau_k, v_t, \Gamma_t\}$, and the initial condition k_0 such that i) given r_t, p_{Et}, and r_{Et}, the consumer maximizes utility $V(k_0)$ in equation (4) with respect to $\mathbf{u_t} \equiv (c_t, a_t, d_t, k_t)$ subject to the deposit constraint (1) and the budget constraint (3); ii) given r_t, the goods producing firm maximizes profit Π_{Gt} in (6), with respect to $s_{Gt}k_t$; iii) given r_t, r_{Et}, and p_{Et}, the intermediary maximizes its profit Π_{Et} in (8) subject to (9) with respect to $s_{Et}k_t$ and d_t; iv) the government budget (7) is always satisfied; v) all markets clear at given prices.

3.2 BGP Growth and Welfare

Along the balanced growth path (BGP) the variables, $k_t, c_t, y_{Gt}, \kappa_t, d_t$, grow at the constant rate g and remain time indexed while the other shares and factor prices, s_G, s_E, a, r, and r_E, are constant.

Proposition 1 *A necessary condition for an interior solution for the fraction of reported income $a_t \in (0, 1)$, is that the competitive equilibrium price of tax evasion services for capital income tax evasion equals the tax rate,*

$$p_{Et} = \tau_k. \tag{11}$$

Proof. This follows directly from the consumer's first-order condition with respect to the fraction of reported income, a_t.[12] ■

A competitive equilibrium market price for illegal evasion services that is equal to the tax rate relates to the literature of Becker (1968) and Ehrlich (1973, 1996), with an analogous result found in inflation tax theory (Gillman and Kejak, 2005).[13]

Solving for $s_{Et}k_t/d_t$ from equation (9) and using that $(\kappa_t/d_t) = 1-a$, it results that $(s_{Et}k_t/d_t) = [(1-a_t)/A_E]^{(1/\omega_E)}$. Substituting this input ratio back into (10), using that $p_{Et} = \tau_k$ and that $r = A_G$, gives an upward-sloping MC in price-quantity space $(\tau_k, 1-a_t)$[14]:

$$\tau_k = A_G \left(\omega_E A_E^{(1/\omega_E)}\right) (1-a_t)^{[(1-\omega_E)/\omega_E]} \equiv MC. \tag{12}$$

[12] The first-order condition with respect a_t implies that if $p_{Et} > \tau_k$ then $a_{kt} = 1$ and the consumer will report the whole income and not use any tax evasion services; excluding the case $a_{kt} = 0$ rules out having no taxes paid.

[13] Ehrlich (1996) notes that he does "not necessarily mean a physical setting where such illegitimate transactions are contracted", but in general where "A person's decision to engage in an illegal activity i can be viewed as motivated by the costs and gains from such activity." He focuses not on the implicit tax caused by the law itself, but rather on expenditure to reduce the benefit of evading the law. This approach also gives a "tax equals market price" result, but it is with respect to the margin of the protection/enforcement activity: "Private self-protection and public law enforcement set a 'price', or 'tax', on criminal activity by reducing the marginal net return to the offender." We take a more primitive, related, approach by explicitly modeling the market for an illegal activity, but abstract from more detailed modeling of the protection/enforcement activity by reflecting the outcome of all such activity in the productivity parameter of the evasion intermediary sector: the statutory tax rate reduces the marginal return to reporting income and induces "offending" in the form of evading the tax up until the marginal cost of the share of unreported income equals the tax rate itself.

[14] See for example Ehrlich (1973) equation 2.2, the "aggregate supply curve of offenses", with equation 3.1 being normalized to the rate of offense; we use a similar normalization in that $1-a$ is the percent of income not reported.

When plotted, the area under the MC curve represents the total resource cost of tax evasion, $r_t s_{Et} k_t$, while the producer surplus that is returned to the consumer is the dividend, $r_{Et} d_t$. At the margin the cost of using reported income to purchase goods is equal to the cost of using unreported income for the same purpose, with the key incentive being to optimally evade the income tax. Solving for a_t from equation (12),

$$a_t = 1 - A_E \left(\omega_E A_E \tau_k / r_t\right)^{[\omega_E/(1-\omega_E)]}. \tag{13}$$

The higher the tax rate the lower is the equilibrium fraction of income that is reported. While the consumer is a competitive price taker with an infinitely elastic demand for evasion at the price $p_{Et} = \tau_k$ as in Proposition 1, the equilibrium outcome for $a(\tau_k)$ is a steady state relation that is a "downward sloping" function of the price.

The parameters of the evasion intermediary technology tie down what we will call the BGP "equilibrium tax rate elasticity for the reported income", or just tax elasticity for short, in a precise fashion. To see this, define the economy's total income as the value added from both goods and evasion intermediary sectors, so that $y_t \equiv s_{Gt} r k_t + s_{Et} r k_t = r k_t$, and derive the elasticity with it denoted by $\eta^a_{\tau_k}$.

Proposition 2 *The elasticity of the taxable income as a share of total income relative to the tax rate equals* $\eta^a_{\tau_k} \equiv (\partial a/\partial \tau_k)(\tau_k/a) = -[(1-a_t)/a][\omega_E/(1-\omega_E)] \leq 0$; *it approaches* 0 *as* τ_k *approaches* 0 *and* a *approaches* 1.

Proof. Given that taxable income equals the reported income of $a y_t$, and that as a share of total income the ratio of taxable to total income is $a y_t / y_t = a_t$, take the derivative with respect to the tax rate using equation (13) and the proof follows. ■

Corollary 3 $\partial \eta^a_{\tau_k} / \partial \tau_k = -[\omega_E/(1-\omega_E)]^2 (1-a)/(\tau_k a^2) < 0.$

The absolute value of the tax elasticity rises (becomes more negative) as τ_k increases and a falls, with marginally more substitution towards unreported income. The consumer in equilibrium becomes increasingly sensitive to the tax and substitutes away from it through greater use of the evasion service. [15] This means that the elasticity rises at an increasing rate as the tax rate rises, as related to Cagan (1956) and Gillman and Kejak (2005) inflation tax elasticity features.[16]

[15] An increase in the statutory tax rate also increases the elasticity of substitution between reported income and unreported income; defined as $\varepsilon \equiv \left\{\partial[a/(1-a)] \Big/ \partial\left[\omega_E \tau_k (A_E)^{1/\omega_E} / A_G\right]\right\}\left\{[a/(1-a)] \Big/ \left[\omega_E \tau_k (A_E)^{1/\omega_E} / A_G\right]\right\}$, with the relative price being $\left[\omega_E \tau_k (A_E)^{1/\omega_E} / A_G\right]$, where $\varepsilon = -[\omega_E/(1-\omega_E)]/a = \eta^a_{\tau_k}/(1-a)$. A price-theoretic form of Alfred Marshall's factor input laws results: $\eta^a_{\tau_k} = \varepsilon(1-a)$ (see Layard and Walters, 1978; Gillman and Kejak, 2005).

[16] This is analogous to the consumer in monetary theory taking the nominal interest rate (the inflation tax) as given while facing a downward sloping money demand per unit of consumption (eg. Lucas, 2000 or Gillman and Kejak, 2005). In Lucas, with m denoting the money income ratio and r the interest rate, he writes "Let m(r) denote the m value that satisfies (3.7), expressed as a function of the interest rate. Throughout the paper, it is this kind of steady state equilibrium relation $m(r)$ that I call a "money demand function," and that I identify with the curves shown in Figures 2 and 3". In Gillman and Kejak (2005, 2011), equation (12) is analytically synonymous with a "Baumol" (1952) type condition that equalizes the marginal cost of the different exchange means of money and interest-bearing bank deposits while optimally avoiding the inflation tax, and from which the money demand equilibrium relation is derived.

The tax elasticity also affects how the BGP growth rate g responds to the tax. To see this consider that the "after-evasion effective tax rate" is less than the actual tax rate because the intermediary returns $r_E d_t$ to the consumer as dividends. Defined here as the statutory rate τ_k minus r_E, this effective rate in the BGP equilibrium is given by $\tau_k - r_E = \tau_k - [\tau_k (1-\omega_E)(1-a)] < \tau_k$, where $1-a$ is given by equation (13). It can be shown that the effective rate rises as τ_k rises, falls as evasion productivity A_E rises, and falls as goods sector productivity A_G rises. Also, it can be rewritten as a weighted average of the unit cost of reported and unreported income, with weights a and $1-a$, and with the average cost when reporting income equal to τ_k, and when not reporting income equal to $\tau_k \omega_E$; i.e., $\tau_k a + \tau_k \omega_E (1-a)$.[17] Tax evasion lowers this "effective tax rate" because of the lower average cost of unreported income, which in turn allows for a higher BGP rate of growth g.

The BGP growth rate g depends on the effective tax rate, being given by

$$g = r(1-\tau_k + r_E) - \delta_K - \rho. \tag{14}$$

It can be shown that $\partial g/\partial \tau_k = -ra$, with $\partial^2 g/\partial \tau_k^2 = r[(1-a)/\tau_k][\omega_E/(1-\omega_E)] > 0$. The growth rate falls at a decreasing rate as τ_k rises except when $a = 1$.

The welfare effect of including evasion is shown by deriving the BGP welfare W

$$W \equiv \int_0^\infty \ln c_t e^{-\rho t} dt = \frac{1}{\rho}\left(\ln k_0 + [\ln(c_t/k_t) + (g/\rho)]\right). \tag{15}$$

From equations (3) and (7), $c_t/k_t = \rho + \tau_k ra$. Since $\partial(c_t/k_t)/\partial\tau_k = ra\left(1+\eta^a_{\tau_k}\right)$ and $\partial g/\partial\tau_k = -ra$, the effect of τ_k on welfare is simply $\partial W/\partial\tau_k = -ra\left[1-\rho\left(1+\eta^a_{\tau_k}\right)\right]/\rho$. An increase in A_E lowers the effective tax rate and so increases g and W.

Imposing the assumption that government tax revenue is a constant share of output effectively endogenizes the tax rate and changes the welfare effects of increasing evasion productivity. Given the consistency of this assumption with the US empirical trend, for the rest of the paper we assume a constant share of revenue and denote this share by $\gamma \in (0,1)$. With $y_t \equiv s_{Gt} r k_t + s_{Et} r k_t = r k_t$, and with government revenue v_t transferred back to the consumer in lump sum fashion, the government budget constraint becomes

$$v_t + \Gamma_t = \tau_k a r k_t = \gamma y_t. \tag{16}$$

Given $y_t = rk_t$, this implies that $\tau_k = \gamma/a$, where $a \leq 1$, and $\tau_k \geq \gamma$; with $A_E = 0$, $a = 1$, and $\tau_k = \gamma$.

Proposition 4 *Given equation (16) and $\left|\eta^a_{\tau_k}\right| < 1$, a marginal increase in A_E decreases welfare W.*

See Appendix A.2 for proof. A more productive evasion sector requires a higher tax rate in order to keep revenue the same fraction of output, causing growth and welfare to fall as resources are increasingly used up in tax evasion. This result is analogous to the BGP loss of resources, growth and welfare in Ehrlich and Lui (1999) when their productivity of producing political capital increases.[18]

[17] To compute the average cost of unreported income, divide the capital rental cost for evasion production by the quantity of evasion services produced, or $rs_E k_t/\kappa_t$. Since the share of capital in evasion sector output is the factor cost divided by the value of evasion output, or $rs_E k_t/(\tau_k \kappa_t) = \omega_E$, then $(rs_E k_t/\kappa_t) = \tau_k \omega_E < \tau_k$.

[18] Ehrlich and Lui (1999) have a related parameter to our γ in their θ, which is the "proportion of all transactions subject to government intervention".

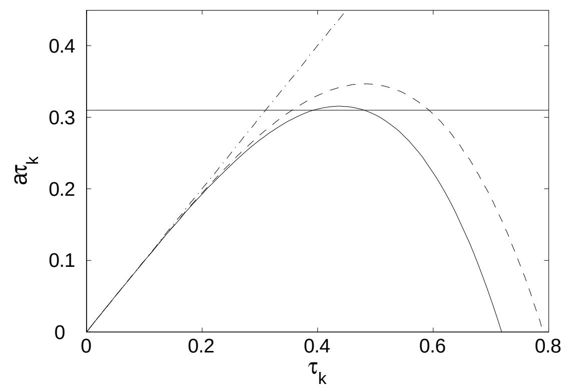

Figure 2: Tax Revenue Normalized by Output and τ_k.

3.3 Revenue Curve, Tax Rate and Productivity Change

Following Agell and Persson (2001), we now derive the relation of the tax rate to the total tax revenue per unit of output. In the BGP equilibrium revenue per output is simply $a\tau_k (A_G k_t)/y_t = a\tau_k$. Graphing $a\tau_k$ against τ_k, the peak occurs at $\eta^a_{\tau_k} = -1$.

Assuming $\omega_E = 0.72$, $\delta_k = 0.07$, $A_E = 0.46$, $A_G = r = 0.176$, $\rho = 0.02$, and $\gamma = 0.31$ as is similar to our calibration for the extended model detailed in Section 5 below, Figure 2 graphs $a\tau_k$ as the solid line; with $A_E = 0$, the straight 45 degree dashed ray results. A 10% increase in A_G causes an increase in the ratio of tax revenue to output at any given tax rate as seen in the dashed curve. As long as $\gamma = 0.31$ intercepts the baseline curve to the left of its peak, then when the curve pivots upwards because A_G increases the rate τ_k needs to be reduced to keep $\gamma = 0.31$. This result would not follow without tax evasion as on the 45% line. The possible tax reduction becomes smaller for a given A_G increase as the tax elasticity falls (in magnitude).

Proposition 5 *Under the condition of the fixed share of tax revenues in output, γ, a marginal increase in A_G causes a decrease in the statutory tax rate τ_k as given by $d\tau_k = -\left\{A_G\left[\left(1/\left|\eta^a_{\tau_k}\right|\right) - 1\right]\right\}^{-1} dA_G$. For $\left|\eta^a_{\tau_k}\right| < 1$, i.e. being on the upward sloping part of the normalized tax revenue curve, the size of the decrease in the statutory tax rate is smaller, the smaller is the elasticity of reported income $\left|\eta^a_{\tau_k}\right|$.*

Proof. See Appendix A.3. ■

4 Extension with Human Capital Investment

Productivity increases are empirically documented to be significant in both goods and human capital investment sectors. Therefore the qualitative result that a goods productivity increase allows for a lower tax rate can be better quantified by extending the economy to include human capital investment. Then productivities of both sectors can be included in simulation results of an economy calibrated for postwar US data.

The extended economy consists of three sectors. The goods sector produces output; the human capital sector produces gross investment in human capital; and both sectors use CRS production with inputs of physical capital and human capital with the human capital sector more human capital intensive than the goods sector. The third sector is evasion intermediation which uses CRS production with inputs of physical capital k_t, human capital h_t and deposited income d_t.

The representative consumer allocates one unit of time across work in goods production, l_{Gt}, in human capital investment, l_{Ht}, and in evasion, l_{Et}, with leisure time, x_t, the residual; $l_{Gt} + l_{Ht} + l_{Et} = 1 - x_t$. The quantity of human capital input in three sectors is $l_{Gt}h_t$, $l_{Ht}h_t$, and $l_{Et}h_t$. Similarly, the share of physical capital allocated to goods production is denoted by s_{Gt}, the share to human capital production by s_{Ht}, and the share to the evasion intermediary sector by s_{Et}; $s_{Gt} + s_{Et} + s_{Ht} = 1$. The quantity of physical capital input in each sector is $s_{Gt}k_t$, $s_{Ht}k_t$, and $s_{Et}k_t$.

With productivity parameters $A_G > 0$ and $A_H > 0$, labor shares $\beta \in (0,1)$ and $\varepsilon \in (0,1)$ where $\beta > \varepsilon$, and the depreciation rate of human capital given by $\delta_H \in [0,1]$, let goods output be denoted by y_{Gt} and its production function be given by:

$$y_{Gt} = A_G \left(l_{Gt}h_t\right)^{\beta} \left(s_{Gt}k_t\right)^{1-\beta}. \tag{17}$$

Let the gross production of human capital investment be given by:

$$\dot{h}_t + \delta_H h_t = A_H \left[\left(1 - x_t - l_{Gt} - l_{Et}\right) h_t\right]^{\varepsilon} \left[\left(1 - s_{Gt} - s_{Et}\right) k_t\right]^{1-\varepsilon}. \tag{18}$$

While the human capital investment sector is a "home production" sector in this representative agent framework, the goods and evasion sectors are decentralized such that the consumer rents human capital to them at the wage rate w_t and physical capital at the rate r_t. Human capital income is thereby $w_t\left(l_{Gt} + l_{Et}\right)h_t$, and capital income is $r_t\left(s_{Gt} + s_{Et}\right)k_t$. In order to avoid taxes, the consumer reports again only a fraction a_t of the human and physical capital income, where we denote this income by $y_t \equiv w_t\left(l_{Gt} + l_{Et}\right)h_t + r_t\left(s_{Gt} + s_{Et}\right)k_t$. The consumer also pays the fee p_{Et} per unit of evasion service, κ_t, which in turn by the implicit Leontief technology is equal to the quantity of income that evades taxes, $\kappa_t = \left(1 - a_t\right)y_t$. The taxes in the economy are the proportional tax rates on capital income, τ_k, and on labor income, τ_l. The consumer also receives dividends from the evasion intermediary at the rate of r_{Et} per unit of deposits d_t, and a government transfer of v_t. Income is used for gross physical capital investment, $\dot{k}_t + \delta_K k_t$, where the depreciation rate for physical capital is given as $\delta_K \in [0,1]$, and for consumption goods purchases c_t. The budget constraint is

$$\begin{aligned} \dot{k}_t &= a_t\left[\left(1-\tau_l\right)w_t\left(l_{Gt}+l_{Et}\right)h_t + \left(1-\tau_k\right)r_t\left(s_{Gt}+s_{Et}\right)k_t\right] \\ &\quad + \left(1-a_t\right)\left(1-p_{Et}\right)\left[w_t\left(l_{Gt}+l_{Et}\right)h_t + r_t\left(s_{Gt}+s_{Et}\right)k_t\right] \\ &\quad + r_{Et}d_t + v_t - \delta_K k_t - c_t. \end{aligned} \tag{19}$$

The first term on the right-hand side shows the reported income upon which taxes are paid and the next term the usable unreported income after paying the fee p_{Et} to the evasion intermediary. The household deposits in the evasion intermediary are equal to its total income, as given by:

$$d_t = w_t\left(l_{Gt} + l_{Et}\right)h_t + r_t\left(s_{Gt} + s_{Et}\right)k_t. \tag{20}$$

Given (k_0, h_0), $\alpha > 0$, and $\rho \in (0,1)$, the representative consumer maximizes lifetime welfare $V(k_0, h_0)$:

$$V(k_0, h_0) = \max_{\{c_t, x_t, d_t, k_t, h_t, l_{Gt}, l_{Et}, s_{Gt}, s_{Et}, a_t,\}_{t=0}^{\infty}} \int_0^{\infty} \left(\ln c_t + \alpha \ln x_t\right) e^{-\rho t} dt,$$

subject to the human capital accumulation constraint (18), budget constraint (19), and deposit constraint (20); see Appendix $A.4$ for the first-order conditions.

The government receives taxes, spends (unproductively) on government consumption Γ_t and returns the rest as a transfer, v_t, such that

$$a_t \left[\tau_l w_t \left(l_{Gt} + l_{Et}\right) h_t + \tau_k r_t \left(s_{Gt} + s_{Et}\right) k_t\right] = \Gamma_t + v_t.$$

Additionally assume that the size of government consumption is a fixed fraction $\gamma \in [0,1]$ of the value of market output such that:

$$\Gamma_t + v_t = \gamma y_t. \tag{21}$$

The goods producing firm takes r_t and w_t as given, maximizes revenue minus cost, and has the first-order conditions:

$$\begin{aligned} w_t &= \beta A_G \left(s_{Gt} k_t\right)^{1-\beta} \left(l_{Gt} h_t\right)^{\beta-1}, \\ r_t &= \left(1-\beta\right) A_G \left(s_{Gt} k_t\right)^{-\beta} \left(l_{Gt} h_t\right)^{\beta}. \end{aligned}$$

The competitive intermediary is owned by the consumer and maximizes profit Π_{Et} subject to its production function. With $A_E > 0$, $\omega_l \in (0,1)$, $\omega_k \in (0,1)$, and κ_t denoting the amount of evasion services provided to the consumer by the evasion intermediary, the production function for these services is given by:

$$\kappa_t = A_E \left(l_{Et} h_t\right)^{\omega_l} \left(s_{Et} k_t\right)^{\omega_k} d_t^{1-\omega_l-\omega_k}. \tag{22}$$

The quantity of evasion services corresponds to the quantity of unreported income "laundered" into income that can subsequently be used to purchase goods. Implicitly assuming a Leontief production function, combining one unit of the evasion service with one unit of laundered income yields that $\kappa_t = \left(1 - a_t\right)\left[\left(s_{Gt} + s_{Et}\right) r_t k_t + \left(l_{Gt} + l_{Et}\right) w_t h_t\right]$.

The intermediary problem is:

$$\max_{\{l_{Et} h_t, s_{Et} h_t, d_t\}} \Pi_{Et} = p_{Et} \kappa_t - w_t l_{Et} h_t - r_t s_{Et} k_t - r_{Et} d_t, \tag{23}$$

subject to (22). The first-order conditions are:

$$w_t = \omega_l p_{Et} A_E \left(l_{Et} h_t / d_t\right)^{(\omega_l - 1)} \left(s_{Et} k_t / d_t\right)^{\omega_k}, \tag{24}$$

$$r_t = \omega_k p_{Et} A_E^{\omega_l} \left(l_{Et} h_t / d_t\right) \left(s_{Et} k_t / d_t\right)^{(\omega_k - 1)}, \tag{25}$$

$$r_{Et} = \left(1 - \omega_k - \omega_l\right) p_{Et} A_E^{\omega_l} \left(l_{Et} h_t / d_t\right) \left(s_{Et} k_t / d_t\right)^{\omega_k}. \tag{26}$$

The solution for the degree of tax evasion follows as:

$$1 - a_t = A_E^{[1/(1-\omega_l-\omega_k)]} \left(p_{Et} \omega_l / w_t\right)^{[\omega_l/(1-\omega_l-\omega_k)]} \left(p_{Et} \omega_k / r_t\right)^{[\omega_k/(1-\omega_l-\omega_k)]}; \tag{27}$$

in addition, the CRS property implies $r_{Et} = \left(1 - \omega_l - \omega_k\right) p_{Et} \left(1 - a_t\right)$.

From the consumer problem, the price of tax evasion services p_{Et}, is a weighted average of the capital and labor tax rates; $p_{Et} = \left[\tau_l w_t \left(l_{Gt} + l_{Et}\right) h_t / d_t\right] + \left[\tau_k r_t \left(s_{Gt} + s_{Et}\right) k_t / d_t\right]$. In the case of a uniform tax rate for both capital and labor income, $\tau_l = \tau_k \equiv \tau$, this reduces to $p_{Et} = \tau$, as in Proposition 1; and then $r_{Et} = \left(1 - \omega_l - \omega_k\right) \tau \left(1 - a_t\right) < \tau$. In this case, the BGP equilibrium solution for the growth rate is $g = r\left[1 - \tau + r_E\right] - \delta_K - \rho$. As the tax rate τ rises, the consumer is increasingly less willing to substitute

Calibration targets	Target Variable	Target Value	Achieved Value	Target Source
Output growth rate	g	0.02	0.02	Trabandt and Uhlig (2011)
Inverse of Intertemp. Elast.Subst.	θ	$1-2$	1	Trabandt and Uhlig (2011)
Govt. Rev to GDP	γ	0.26	0.31	Trabandt and Uhlig (2011)
Average income tax rate	τ	0.31	0.4	Trabandt and Uhlig (2011)
Govt. consumption to GDP	Γ/y	0.18	0.20	Trabandt and Uhlig (2011)
Government transfers to GDP	v/y	0.08	0.11	Trabandt and Uhlig (2011)
After-tax net real interest rate.	r'	0.04	0.04	Trabandt and Uhlig (2011)
Capital to Output ratio	k/y	2.38	2.38	Trabandt and Uhlig (2011)
Leisure time	x	0.5	0.5	Jones et al. (2005)
Labor time	l_G	0.17	0.16	Jones et al. (2005)
Fraction of income reported	a	0.78	0.78	Waud (1988)
Tax Elasticity ETI	$\eta^a_{1-\tau}$	$0.4-3.0$	1.08	Saez et al. (2012), Feldstein (1995)

Table 1: Target values of the baseline calibration for the US benchmark model.

from goods consumption to leisure and more willing to evade income tax. This causes the BGP growth rate to decline at an increasingly lower rate as the tax rate rises compared to the economy without evasion, as in the Ak economy.

In the BGP equilibrium all growing variables evolve at the same rate g, with k_t/h_t constant and the BGP welfare W is equal to:

$$W = \int_0^\infty (\ln c_t + \alpha \ln x_t)\, e^{-\rho t} dt = \frac{1}{\rho}\left(\ln k_0 + \ln\left[\left(\frac{c_t}{k_t}\right) x^\alpha\right] + \frac{1}{\rho} g\right). \tag{28}$$

With both human and physical capital in the extended economy, and an assumed common tax rate τ, the output normalized tax revenue curve is $a_t\tau$ as in the Ak economy. But now increased human capital productivity also increases the ratio of tax revenue to output for any given tax rate, and forces a reduction in τ given equation (21); it also increases growth and welfare.

The revenue per output for any given tax rate depends on the degree of evasion a and tax elasticity. Denoting the tax elasticity of reported income by η^a_τ, and of unreported income by η^{1-a}_τ, and using equation (27), it can be shown that $\eta^a_\tau = -\left[(1-a)/a\right]\eta^{1-a}_\tau \simeq \eta^a_{\tau_k}$ of the Ak model in the previous section.[19] The higher the tax elasticity magnitude, the stronger the revenue increase from improved productivity, and the larger the tax rate decrease required to keep a constant γ.

5 Calibration

The BGP equilibrium of the model is calibrated annually based on Trabandt and Uhlig's (2011) US averaged data from 1995 to 2007; we get targets from this data

[19] $-\left[(1-a)/a\right]\eta^{1-a}_\tau = -\left[(1-a)/a\right]\left[(\omega_l+\omega_k)/(1-\omega_l-\omega_k)\right]\left[1-0.5\left(\eta^w_\tau+\eta^r_\tau\right)\right]$, where $\eta^w_\tau+\eta^r_\tau$ denotes the sum of the wage and interest rate elasticities to τ. Quantitatively in our calibration below $\eta^w_\tau+\eta^r_\tau$ is negligible, so the approximation results.

Parameters		Baseline value
Time preference rate	ρ	0.02
Labor share in goods sector	β	0.62
Inverse of IES	θ	1
Depreciation rate of physical capital	δ_K	0.07
Depreciation rate of human capital	δ_H	0.07
Productivity in goods sector	A_G	1
Productivity in human cap sector	A_H	0.29
Productivity in evasion sector	A_E	1.44
Weight of labor in preferences	α	2.5
Labor share in education sector	ε	0.8
Capital share in evasion sector	ω_k	0.36
Labor shares in evasion sector	ω_l	0.36

Table 2: Parameter values of the baseline calibration for the US benchmark model.

and then make adjustments to these targets so that we can better capture the entire postwar period rather than just 1995-2007 which is the end of the period when tax rates have already fallen. We also follow Gomme and Rupert (2007), who refine the calibration methodology in general and in particular for a two sector market and non-market household economy; our human capital investment sector is the non-market "household" sector.[20] As such we present in Table 1 twelve independent pieces of information on our variables from different sources, eight from Trabandt and Uhlig, two on leisure and labor time from Jones et al. (2005), one on fraction of reported income from Waud (1988), and one on tax elasticity from Saez et al. (2012). These form our calibration targets that in turn enable us to uniquely pin down the model's parameter values. Table 2 presents twelve parameters for which values are assigned in order to get the "achieved" calibration targets of Table 1. Note that parameter calibration depends often on the solution to BGP variables from implicit equations; for example x, a, and $s_G k/l_G h$ are solved only implicitly within a system of three nonlinear equations.

As in Trabandt and Uhlig (2011), the target value of the real growth of output, g, is set to 2%. Denoting by r' what Trabandt and Uhlig call the annual real interest rate, we define this as consistent with their usage as $r' \equiv (1 - \tau_k + r_E)\, r - \delta_K$ and set it at $r' = 0.04$ as in their calibration. Given our assumption of log utility, a special case of one given in Trabandt in Uhlig, together g and r' imply a time preference rate of $\rho = 0.02$. As in Trabandt and Uhlig's Table 2, we set the share of labor income in the goods sector such that $\beta = 0.62$. The labor share of the human capital sector, ε, is equal to 0.80, similar to e.g. Pecorino (1995) and Bowen (1987). Also following Trabandt and Uhlig's Table 1, we target $\gamma = 0.26$ as the sum of their government consumption plus investment ($\Gamma/y = 0.18$), plus their government transfer of ($v/y = 0.8$). Then using

[20] We do not independently estimate time in human capital investment, although data is becoming more available for this as a task for future research; instead we use their concept of a much lower leisure time share around 0.5 relative to one sector exogenous growth economies that typically set leisure above 0.7; and we set a similar time share for that spent in household production as do they, although their household output is not specified to be human capital investment.

Trabandt and Uhlig's β and their τ_l and τ_k, we assume an average tax rate τ from the labor and capital tax equal to $\tau = \tau_l \beta + \tau_k (1-\beta) = (0.28)(0.62) + (0.36)(0.38) = 0.31$. Accounting for a depreciation tax element found in Trabandt and Uhlig we would revise this τ upwards somewhat; we impute the depreciation rate according to our equilibrium conditions such that $\delta_K = 0.07$.[21]

In our model, it is the share of reported tax revenue per output that equals the spending share γ. Therefore we have $a\tau = \gamma$, while in Trabandt and Uhlig (2011) it is implicit that $\tau = \gamma$ in our notation. For targeting a we use 0.78 since Waud (1988) reports that 22% of federal income tax was lost in 1981 due to unreported income (total federal corporate and personal income), implying $1 - a = 0.22$; Fullerton and Karayannis (1994) report that 20% of non-corporate income evades taxation in the US; other estimates abound, e.g. Schneider and Enste (2000). Given $a = 0.78$, we now have $a\tau = \gamma = 0.26$, or $\tau = (0.26)/(0.78) = 0.33$.

We then make adjustments in order to try to better capture the entire postwar period. We chose a higher tax rate than that which existed in the Trabandt and Uhlig data set from 1995-2007, or as implied at $\tau = 0.33$. We set the baseline τ at $\tau = 0.40$ as an approximation of the midpoint of the postwar economy tax rates, in that this increase puts us approximately in the midrange of the twenty points between lowest and highest weighted average tax rates, of 30% and 50%, on the top 1% of income that we find for postwar US in Figure A in Appendix A.1. This implies $a\tau = (0.4)(0.78) = 0.31$, and so $\gamma = 0.31$, instead of the $\gamma = 0.26$ target. This increase is distributed between government spending of Γ/y going to 0.20 and v/y to 0.11, our achieved values, instead of the Trabandt and Uhlig values of 0.18 and 0.08.

In calibrating the evasion intermediary technology factors, Slemrod and Weber (2012) make clear that there is a great deal of uncertainty over reliable micro-based or macro-based evidence that can be used for such a purpose. In order to calibrate the labor and capital shares in the evasion sector we first assume they are equal, so that $\omega_l = \omega_k = \omega$, as seen in Benk et al. (2010). Then in order to pin down ω precisely, while being given $a = 0.78$, we make use of the large literature reviewed by Saez et al. (2012) on the tax elasticity of reported income. Here Saez et al. focus on estimation of the elasticity of reported income with respect to the net-of-tax rate, or $1-\tau$; this uses the acronym "ETI," and in our notation this is $\eta^a_{1-\tau}$.

Saez et al. (2012) report that substantial variance in the ETI is found in the literature, depending on the year, the percentile income share and on econometric methodology. For the period of the 1986 tax act, Feldstein (1995) finds an ETI between 1 and 3 while Moffett and Whilhelm (2000) obtain a range of 0.35 to 0.97. Saez et al. report how the ETI is found to be significantly lower over the long run period, even though for the long run they find "no truly convincing estimates" of the ETI. Still, they put the upper end of this long run range at 0.4 for the top 1% percentile. Our baseline calibration is designed to get an average effect for the postwar period, including the high responses during tax reform periods. And so we chose an intermediate value equal to 1.08 that is between certain reform period point estimates and the long run estimates of the ETI. Then using the fact that $\eta^a_\tau = -[\tau/(1-\tau)]\,\eta^a_{1-\tau}$, and given our baseline τ, the implied tax elasticity of reported income is $\eta^a_\tau = 0.76$. In turn this implies an

[21] Trabandt and Uhlig assume capital income taxes are levied on dividends net-of-depreciation, i.e. $\tau_k\left(1-\beta-\delta_K \frac{k}{y}\right)$, so that the weighted average is $\tau = \tau_l\beta + \tau_k\left(1-\beta-\delta_K\frac{k}{y}\right) = 0.33$ given $\delta_K = 0.07$, instead of 0.31. We have also ignored their consumption tax of 0.05. Our BGP equilibrium implies that we impute δ_K as $\delta_K = (y/k)\left[1-\tau+(1-\omega_l-\omega_k)\tau(1-a)\right]\left[\omega_k\tau(1-a)\right]\left\{1+\left[\omega_k/(1-\beta)\right]\tau(1-a)/\left[1-\tau(1-a)(\omega_l+\omega_k)\right]\right\}$.

approximation for the input share in the intermediary sector that gives $\omega = 0.36$.[22] The other part of the evasion technology is A_E; this is set at 1.44 to achieve the elasticity target in conjunction with $\omega = 0.36$, while giving a share of labor time in evasion equal to less than 1% of total time and so achieving the other targeted time allocations.

Leisure time is targeted at $x = 0.5$, based on Jones et al. (2005), Ramey and Francis (2009) and Gomme and Rupert (2007); Ragan (2013) also argues that leisure is 51% of 14 hours a day. Labor time l_G is targeted at the Jones et al. (2005) value of 0.17. We assume $\delta_K = \delta_H$, although some estimates place human capital depreciation at a lower rate than physical capital. Given that $l_H = \varepsilon(1-x)(g+\delta_k)/(r'+\delta_k)$, using the target values for g, r', and x, $\theta = 1$, the imputed value of $\delta_K = \delta_H = 0.07$ and the standard value for ε of 0.2, we then impute a standard value for leisure preference of $\alpha = 2.5$ so that $x = 0.5$, $l_G = 0.16$ and $l_H = 0.33$.

6 Simulation Results

Tables 3 and 4 present the baseline calibration results, with $g = 0.02$, $\tau = 0.40$, $1-a = 0.22$, $\gamma = 0.31$ and $\omega = 0.72$, of simulations of the effects of a 10% productivity increase in each of the goods and human capital sectors, in terms of A_H and A_G rising, and of a 10% decrease in the productivity of the evasion sector, in terms of A_E falling. This is done for when there is a single common tax rate τ on both labor and capital income in Table 3, and for the cases of when there are separate taxes on labor income τ_l and on capital income τ_k in Table 4. The difference is that in Table 3 the common tax rate responds to changes in productivity while in Table 4, first the capital tax τ_k is held constant at the baseline and only the labor tax τ_l is allowed to fall, and then the labor tax τ_l is held constant at the baseline and only the capital tax is allowed to fall. The two tables present the new levels of τ, g and $1-a$, plus the percentage change in l_H, l_G, s_H, s_G, (k/h), $s_G k/(l_G h)$, $s_H k/(l_H h)$, and x, as induced by the productivity changes. And each table includes an exogenous growth special case for comparison; here human capital is specified to grow exogenously at the baseline rate $g = 0.02$, while assuming that $A_H = 0$ and $\delta_h = 0$.[23] Tables 3 also show the results for both the baseline $\omega = 0.36$ and when it is increased to 0.39.

Increases in goods and human capital investment sector productivities induce a lesser degree of tax evasion $1-a$ and this in turn allows a lower tax rate in all cases for all models. In Table 3, the increase in A_H allows τ to fall by 2 points, with g rising from 2 to 3.57 percent, while the A_G increase causes a 4 point fall in the tax rate but a smaller growth rate increase. The growth and welfare (not shown) gain is highest from the A_H increase in all cases.

Table 3 shows that the factor reallocation from both goods and human capital sector productivity increases is away from leisure and more towards the human capital sector. The input ratios are given in the penultimate column. The capital to effective labor ratio decreases with an A_H increase and increases with an A_G increase; the input factor ratio w/r falls with an A_H increase and rises with an A_G increase; and in the third to

[22] $\omega \simeq 0.5\left\{1 + \left[(1-a)/a\right]\left[\tau(1-\tau)\right]/\eta^{a}_{1-\tau}\right\}$.

[23] This is similar to an exogenously increasing productivity parameter defined as $A_{Gt} \equiv A_G (h_t)^{\beta}$; the exogenous growth case with the same baseline calibration gives a larger leisure time allocation, such that $x = 0.8$ and $l_G = 0.19$, close to values used in standard exogenous growth models; the same baseline but with human capital investment gives $x = 0.5$, close to the two-sector household economy of Gomme and Ruppert (2007).

last column, k/h decreases with an A_H increase, and increases with an A_G increase.[24] With ω higher, the tax elasticity is higher and so the degree of lesser evasion and the tax rate reduction are greater from the goods and human capital sector productivity increases.

The time spent in human capital investment l_H increases by more than does l_G for all productivity changes in Tables 3 and 4. This is consistent with the human capital sector being relatively more labor intensive than the goods sector and so it is the sector to expand more as leisure is reduced. The increase in s_H is larger than in s_G for all productivity changes in Table 3, but not in Table 4 when s_H falls with a capital tax decrease alone.

For exogenous growth in Table 3, there is no human capital sector and the good sector productivity increase shows similar patterns as with endogenous growth. The exception is that for exogenous growth the decline in leisure in the last column is much smaller and the increase in the capital to labor ratio in the second to last column more than three fold bigger; similarly the k/h increase is double what it is in endogenous growth when A_G increases. This leads to what we interpret as greater diminishing returns being experienced in the goods sector during the sectoral reallocation so that it becomes more productive relative to the evasion sector but by a lesser degree than in the case with endogenous growth. This can explain why the tax evasion decrease is less in the exogenous growth case and the tax rate decrease also significantly less.

A 10% reduction in the evasion sector productivity A_E in Table 3 shows that the evasion degree falls by more, and the growth rate increases by less as compared to the goods and human capital sector productivity increases; welfare also rises but by the least amount as compared to A_H and A_G increases (not shown).

Table 4 shows how allowing for a reduction in either the labor tax or the capital tax alone, compares to instead decreasing a common tax rate as in Table 3. Increasing either A_H or A_G while lowering the labor tax τ_l causes a larger decline in the degree of evasion and a bigger growth rate increase than does lowering the capital tax τ_k. This happens even as there is a smaller decrease in the labor tax as compared to the capital tax. For exogenous growth, as compared to the endogenous growth baseline, the A_G increase again causes a smaller fall in the degree of evasion and in the tax rate, for both taxes.

Table 4 also shows that the large leisure decrease again induces the l_H time to increase by more than l_G for all the productivity changes reported and for either the labor or capital tax rate reduction, due in part to the large leisure decrease in endogenous growth. For an A_H increase, the share s_H rises absolutely and relatively by more than s_G for either tax rate reduction, and $s_H k/(l_H h)$ falls. However, for an A_G increase or an A_E decrease, s_H expands or contracts depending on whether it is a labor tax reduction or a capital tax reduction, and $s_H k/(l_H h)$ rises for either tax reduction.

Table 4 may be interpreted as indicating an advantage of a labor tax versus a capital tax. As a result of an increase in either A_H or A_G, the associated per unit decrease in the labor tax allows for less evasion and higher growth, compared to the capital tax per unit decrease, even though the total tax reduction per unit of productivity increase is greater with a capital tax reduction.

The 10% A_E decrease in Table 4, with a labor tax decrease, comes with a decrease in the capital to labor ratio in the goods sector, in the third to last column; a capital tax decrease in contrast comes with an increase in the capital to labor ratio in the goods sector. For exogenous growth with a A_E decrease, and either a labor or capital tax decrease, the results are similar to those for endogenous growth for a common tax τ. The exception is a rise in k/h for the labor tax in exogenous growth and a fall in

[24] It can be shown that w/r depends linearly and positively on the capital to labor ratio.

	ω	After-reform rates τ	g	$1-a$	After-reform percentage changes l_H	l_G	s_H	s_G	$\frac{k}{h}$	$\frac{s_G k}{l_G h}, \frac{s_H k}{l_H h}$	x
10% ↑ in A_H											
Endog. Gr.	0.36	38.2	3.57	18.9	8.4	2.9	3.8	−1.5	−11.8	−15.6	−6.2
	0.39	36.6	3.97	15.2	12.7	6.7	5.0	−0.6	−10.7	−16.8	−9.7
10% ↑ in A_G											
Endog. Gr.	0.36	35.8	3.25	13.5	10.8	7.9	3.7	1.0	14.1	6.7	−8.9
	0.39	34.3	3.63	9.5	14.9	11.8	4.9	2.0	15.6	5.5	−12.3
Exog. Gr.	0.36	36.3	2.00	14.6	–	8.0	–	3.5	29.6	24.1	−1.4
10% ↓ in A_E											
Endog. Gr.	0.36	34.6	3.11	10.4	12.1	10.5	3.6	2.2	2.8	−5.0	−10.3
	0.39	33.4	3.42	7.2	15.4	13.7	4.6	3.1	4.0	−5.7	−13.0
Exog. Gr.	0.36	34.9	2.00	11.1	–	11.5	–	4.9	16.0	9.1	−2.0

Table 3: Productivity Effect on Tax Rate, Growth, Evasion

k/h for the labor tax in endogenous growth, related to the lower leisure reduction and larger capital to labor ratios that are induced in exogenous growth.

Figure 3 shows the simulated tax revenue curve as normalized by output in the baseline case as the solid curve, along with the 45 degree line which would apply with no tax evasion. The dashed line shows how the ratio of tax revenue to output increases at any given tax rate following a 10% increase in A_H (labelled $A_H * 1.1$); the dash-dot line shows this for an A_G increase (labelled $A_G * 1.1$). The dotted line shows the largest increase of the ratio per τ following a 10% decrease in A_E (labelled $A_E * 0.9$).

Figure 4 shows the annual effect over time of permanent 10% productivity increases in A_H and A_G, plus a 10% decrease in A_E and in γ (labeled "gamma"): on evasion, $1-a$, the growth rate of y_G (denoted by g_y here), and the $\ln y_G$ (denoted $\ln y$). The dashed lines are the original BGP equilibria and the solid lines show the new equilibria over time after the permanent parameter change. The decrease in A_E causes the largest decrease in $1-a$; while the increases in A_H and A_G lead to larger increases in the growth rate. A decrease in the size of government, as summarized by the parameter γ, causes large evasion and growth effects that would involve a movement down a given normalized tax revenue curve. In practice, such a movement could follow from privatization, deregulation or greater government program efficacy, for example. The A_H increase causes the largest long run increase in both growth and welfare (not shown). The A_H increase, A_G increase, and A_E decrease cause a 22%, 11%, and 6% increase in welfare respectively; in exogenous growth (not shown) the welfare increases for A_G and A_E fall to 3% and 5%.

7 Estimate of Tax Rate Reduction

We now calculate what proportion of the observed decline in US postwar top marginal tax rates can be explained by increases in goods sector and human capital sector productivity. We assume a single tax rate on labor and capital income and compare our estimate of the postwar decline in this single rate to an observed composite top marginal rate over the same period. Using data graphed in Figure 1 (see also the online

ω	After-reform rates τ_K	τ_L	g	$1-a$	After-reform percentage changes l_H	l_G	s_H	s_G	$\frac{k}{h}$	$\frac{s_G k}{l_G h}$	$\frac{s_H k}{l_H h}$	x
10% ↑ in A_H : Endog. Growth												
0.36	40	37.9	3.68	18.3	9.8	3.9	6.2	−3.2	−13.6	−19.5	−16.4	−7.4
	36.8	40	3.41	19.7	6.9	1.7	1.3	0.1	−10.1	−11.6	−14.8	−4.9
10% ↑ in A_G : Endog. Growth												
0.36	40	32.9	3.43	12.8	13.0	9.5	9.3	−3.4	8.4	−4.4	4.8	−10.8
	30.7	40	2.97	14.5	7.5	5.6	−3.6	6.4	21.8	22.8	9.2	−6.1
Exog. Growth												
0.36	40	33.7	2.00	14.0	–	10.0	–	3.8	21.9	15.0	–	−1.8
	31.6	40	2.00	15.4	–	5.8	–	3.2	40.4	36.7	–	−0.9
10% ↓ in A_E : Endog. Growth												
0.36	40	31.1	3.31	9.8	14.5	12.2	10.8	−3.6	−4.0	−17.5	−7.0	−12.4
	27.3	40	2.78	11.4	8.1	7.7	−6.3	10.0	12.9	15.2	−2.1	−6.9
Exog. Growth												
0.36	40	31.5	2.00	11.0	–	13.8	–	5.2	5.7	−2.3	–	−2.5
	27.8	40	2.00	12.0	–	8.5	–	4.7	31.4	26.7	–	−1.3

Table 4: Separate Capital and Labor Tax Effects from Productivity Changes

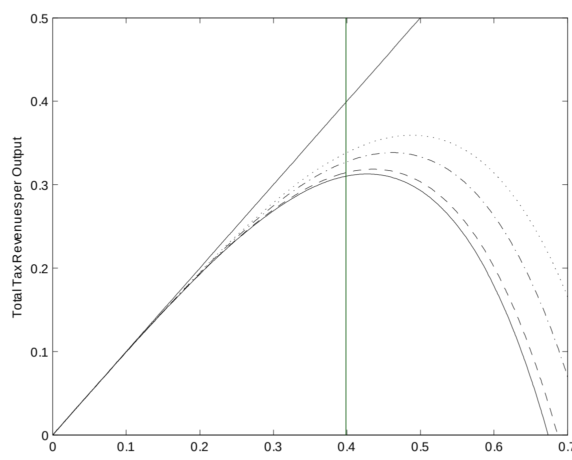

Figure 3: Tax Revenue Curve, 10% Changes in A_H (dash line), A_G (dashdot line), and A_E (dot line); 45% line ($A_E = 0$).

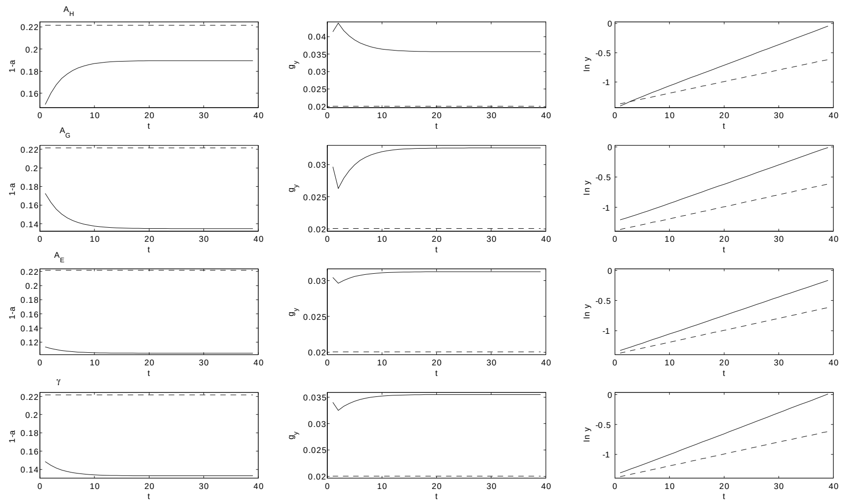

Figure 4: 10% Increase in A_H and A_G; 10% Decrease in A_E and γ; and $1-a$, g_y, $\ln y$.

Appendix Tables A1-A4), a weighted average of the top marginal rates on personal and corporate income falls from 75% in 1951 to 35% in 2012, a 40 point drop. For the average weighted rate on the top 0.5% of the income distribution, Figure A in Appendix A.1 shows that the postwar rate drops from around 50 to 30.

Using the database of Baier et al. (2006), we estimate that human capital productivity has risen at an average rate of 3.69% per decade, from 1950 to 2009, while goods sector productivity has risen at an average rate of 0.756% per decade over the same period (see Table 5 of Appendix A.5). These estimates suggest a 4.9 fold asymmetry between productivity gains in the human capital sector and productivity gains in the goods sector over this period. The size of this asymmetry is comparable to empirical findings in the related intangible capital investment model of McGrattan and Prescott (2010). For 1993 to 2000, they report US goods sector productivity growth of 0.7% per year compared to intangible goods sector productivity growth of 2.7% per year, amounting to a 3.9 fold asymmetry.

Next, we use postwar US data to estimate by how much the hypothetical common tax rate (τ) falls while keeping the share of government revenue in output unchanged. We simultaneously use the estimated changes in A_H and A_G for the 1950-2009 period. We choose the year 2000 as the benchmark period: i.e. $A_H(2000) = A_H = 0.2935$, $A_G(2000) = A_G = 1$ and $\tau(2000) = \tau = 0.3974$. Given the estimated average growth rate of A_H equal to 3.69% per decade, over the six decades, we let $[A_H(2009)/A_H(1950)] - 1 = (1.0369)^6 - 1 = 0.2429$ or 24.3%. It follows that $A_H(1950) = A_H(2000)/(1.0369)^5 = 0.24486$, and $A_H(2009) = A_H(2000)(1.0369)$ $= 0.030433$. Similarly, given the average growth rate of A_G at 0.756% per decade, we let $[A_G(2009)/A_G(1950)] - 1 = (1.00756)^6 - 1 = 0.0462$ or 4.62%. And it follows that $A_G(1950) = A_G(2000)/(1.00756)^5 = 0.96307$, and $A_G(2009) = A_G(2000)(1.00756)$ $= 1.00756$. Ideally, we would use the boundary values for A_H and A_G and compute the implied changes in the tax rate under the condition that tax revenue as a fraction of output remains constant. However, given computational boundaries for the simulation of the baseline BGP equilibrium of $A_H \geq 0.287$ and $A_G \geq 0.99457$, we simulate the increase of A_H in the range 0.2870 to $A_H\,(2009) = 0.30433$ (i.e. only 6.04% of the total 24.3%); and the increase of A_G in the range 0.99457 to $A_G\,(2009) = 1.00756$ (i.e. only 1.31% of the total 4.62%).

The implied decrease in τ from increasing both A_H and A_G simultaneously within the simulation range along the BGP is 0.0317, starting at $\tau = 0.4176$ and going down to $\tau = 0.3859$. Since the simulated changes in A_H forms only $6.04/24.3 = 0.2486$ of the total change, and in A_G it forms $1.31/4.62 = 0.2835$ of the total change, magnitudes which are close to each other, we take their simple average, i.e. $(0.24877 + 0.2827)/2 = 0.2657$, and extrapolate the 0.0317 tax rate decrease for the six decades to $3.17/0.2657 = 11.93$ points. The estimated reduction in the tax rate of 12 points accounts for 30% of the 40 point decline in the weighted top marginal tax rate observed in postwar US data. The estimated decline in the tax rate would double for the weighted average tax rate decline in Figure A in Appendix A.1. However the estimates would be smaller if we built a lower tax elasticity magnitude into the baseline calibration. Note that in this simulation we extrapolate the total six decade change in h/k to be a 17% increase.

8 Discussion

Our approach is driven by the well-articulated goal of satisfying the "input justification criterion" that McGrattan and Prescott (2010) put forth: "requiring our exogenous inputs to be consistent with micro and macro empirical evidence." The use of the rise in goods and human capital sector productivities in our extended model are closely related to McGrattan and Prescott's extended two-sector model which uses unbalanced productivity growth between the goods and intangible capital investment sectors to explain the 1990s expansion in the US. Our human capital investment sector might be viewed as partially encompassing the intangible capital investment sector that McGrattan and Prescott highlight. Similar to their intangible capital, all of our human capital stock is used as an input for goods production. As in their work, the inclusion of a separate investment sector enables a better empirical explanation of growth episodes, in our case the postwar decline in US tax rates, due to a broader consideration of US postwar productivity increases. And as in McGrattan and Prescott, our evidence implies unbalanced productivity growth favoring the investment sector.

The main qualification not yet addressed is that in explaining 30% of the decline in postwar US top marginal tax rates, our model also implies an increase in the BGP growth rate of 5.42 percentage points. This is high compared to the relatively stable 2% measured per-capita postwar US growth rate which some have argued has even indicated a slight decline in US living standards. But the higher growth rate within our economy can be viewed as being consistent with the McGrattan and Prescott (2010) 0.7 percent per year increase in goods sector TFP and their 0.8% higher labor productivity per year. While McGrattan and Prescott (M-P) base their productivity estimates on transition dynamics in an exogenous growth setting of a high growth episode, we abstract from transition dynamics, use comparative static changes in the balanced growth path (BGP) equilibrium within an endogenous growth setting, and consider a broader postwar expansion period. Consequently in our model, output growth rates and labor productivity coincide along the BGP. If the 1990s M-P average growth rate was normalized and extended across the entire US postwar period, then presumably there could result a growth rate more comparable to the 5% that we find.

The results in McGrattan and Prescott (2010) are driven by a shift of resources to the intangible capital sector with a subsequent rise in per capita hours worked and a decrease in leisure time which they emphasize as being plausible during an expansion.

Similarly, from our productivity increases we find a shift in resources towards the human capital sector, more labor time and less leisure (see also Beaudry and Francois, 2010, and Beaudry et al., 2010). And also comparable to the M-P results, our productivity changes cause significant leisure time decreases in the endogenous growth baseline model but leisure decreases only slightly in the exogenous growth version without human capital investment. Such significant declines in leisure time are key to the M-P result of more labor time and in our model are key to increasing the size of the human capital sector and the BGP growth rate.

Buera and Shin (2013) explain long historical periods of accelerated growth resulting from productivity TFP increases, which is related to how we view the postwar US experience when including the human capital sector. Although Buera and Shin emphasize the role of financing costs, which we abstract from, they attribute such increases in TFP to periods of tax reduction and regulation reform, as is related to our focus. While within an exogenous growth framework, tax rate reform may not influence long run economic growth (e.g. Uhlig and Trabandt, 2012), we allow for tax reform's positive affect on growth.

The estimated postwar tax rate decline depends upon our simplified assumption of unchanged productivity in the evasion sector. If this productivity were to have fallen/risen, say because of greater ease of enforcement/evasion through IT advances, then our estimate of the possible tax rate decrease would be larger/smaller. As evidence on evasion productivity change is lacking, instead we use macro evidence to target estimates of the US degree of tax evasion and the tax elasticity of reported income.

The tax evasion literature does not generally use micro evidence. For example Chen (2003) extends a Becker (1968) and Ehrlich (1973) style of illegal tax evasion within an Ak endogenous growth economy in which a transactions cost for evasion enables a lower effective tax rate and higher growth; we have such effects with the intermediation sector instead.[25] A related theoretical treatment of tax evasion by Dhami and al-Nowaihi (2010) improves upon the seminal Yitzhaki (1974) approach by modifying expected utility to provide a theory consistent with broad micro evidence which suggests that tax evasion rises with tax rates and that evasion is sizable relative to the size of the economy; we capture these features with log utility and the evasion intermediation.[26] Micro evidence on evasion sector technology parameters is difficult to obtain by its nature but this sets out a well-identified task for future research, as emphasized by Slemrod and Weber (2012). A promising direction is experimental inference of evidence on the evasion sector (Saez and Kopczuk, 2013).

9 Conclusion

Using a model calibrated to US postwar data, the paper shows how growth, welfare and time in human capital investment can trend upwards due to productivity increases while tax rates trend downwards. The paper employs an endogenous growth economy as extended with taxes and a decentralized competitive intermediation sector that provides tax evasion services. It shows how an increase in productivity induces less tax evasion which causes the ratio of tax revenue to GDP to increase at any given tax rate. In turn, this implies that the tax rate must fall if government revenue as a proportion of GDP is to remain constant in a manner consistent with the data. Upward productivity

[25] Chen (2003) transaction cost is "hiring CPAs and lawyers to dodge taxes, and particularly bribing tax officials and law administrators, along with other concealing activities."

[26] See also Allingham and Sandmo (1972), Roubini and Sala-i-Martin (1995), Slemrod (2001).

trends imply a downward tax rate trend.

Using the estimated upward US postwar productivity trends in the goods and human capital sectors, and assuming a uniform tax rate on labor and capital income, the model explains 30% of the reduction in a weighted average of US postwar tax rates. In simulations, the paper relaxes the assumption of a uniform tax to examine a labor tax reduction versus a capital tax reduction, showing that in either case the time in human capital investment increases. The human capital to physical capital ratio rises with human capital investment productivity increases and falls with goods sector productivity increases; as simulated with our estimated productivities the postwar human to physical capital ratio rises by about 17%.

Without the human capital sector, our explanation of the tax trend would be significantly less strong. Increasing only the goods sector productivity, a lower tax reduction is found in the exogenous growth special case, as compared to the endogenous growth human capital economy. And our estimate of the effect of US postwar productivity growth would be several fold smaller if we took only goods sector data from Baier et al. (2006) and ignored human capital productivity data. Without human capital we would also be inconsistent with the direction of McGrattan and Prescott in using unbalanced goods and intangible capital sector productivity to explain a US growth period, and with the many views and estimates of the impact of rising education and human capital levels in the postwar US experience, e.g. Guryan (2009).

We calibrate tax evasion to fit the US, a developed country, and find that evasion falls as goods and human capital productivity increases. This is consistent with the idea that developing countries experience less evasion and more growth and welfare as they become more developed through rising human capital accumulation. Tax rate reduction then becomes a natural consequence of a relatively constant share of government revenue in output. Even though there may be many other reasons for the observed decline in top marginal and average tax rates, few studies model how this might occur. We provide a potential explanation of this stylized trend based on productivity gains in the goods and human capital sectors, while ignoring other political economy factors. At the same time the analysis is consistent with a rising time allocation in human capital investment, in accordance with claims of such a trend. A potent policy dimension arises that we leave for future research: Education policy that efficaciously continues to boost human capital productivity may interact with public finance considerations by allowing for a gradual reduction in tax rates that further enhances growth and human capital investment in a virtuous cycle. A research issue left untouched here is whether our simulated postwar rise in the human to physical capital ratio could contribute to explaining structural transformation of economies towards more human capital intensive sectors, as related for example to Kabonski (2009) or Buera and Kabonski (2012).

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A APPENDICES

A.1 Average Tax Rates: Figure A

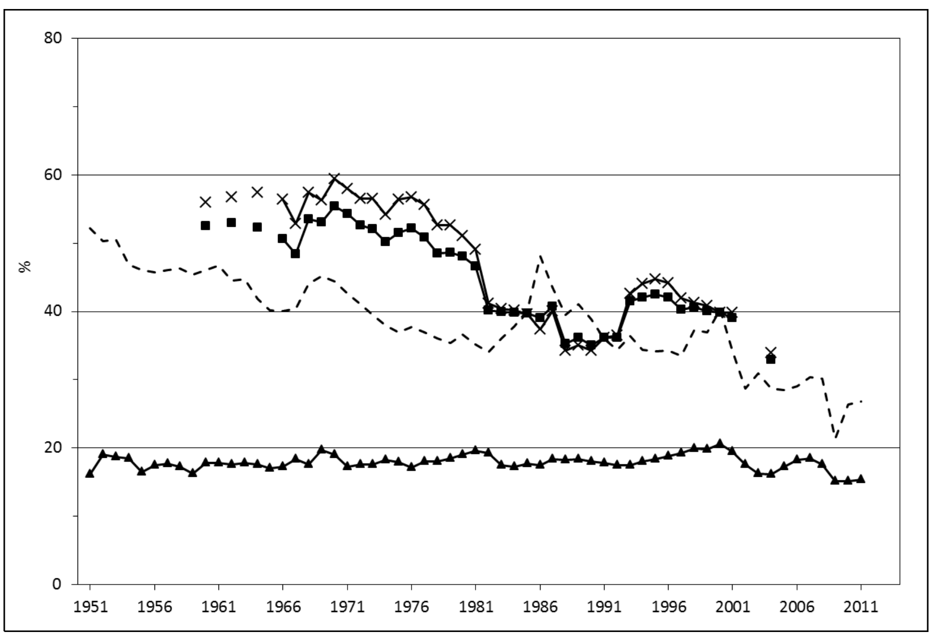

Figure A. Average US Tax Rates, 1951-2011.

The Online Appendix provides details on construction of the data series graphed in Figure A; data sourced from US Bureau of Economics Analysis, OMB, and Piketty and Saez (2006,2007). The lines in Figure A are the ■-line: weighted average of personal and corporate tax rates; x-line: average personal income tax rate on top 0.5%; dashed line: average corporate income tax rate; ▲-line: federal government receipts as % of GDP.

A.2 Proof of Proposition 4

Welfare W consists of terms involving $\frac{c}{k}$ and g. Equation (16) and $y = rk$ imply $\tau_k a = \gamma$ and so $\left[d\left(\tau_k a\right)/dA_E\right] = \left(d\gamma/dA_E\right) = 0$. For $\frac{c}{k}$, it then holds that $\left[\, d\left(c/k\right)/dA_E\right] = A_G\left[d\left(\tau_k a\right)/dA_E\right] = 0$. Since $g = A_G\left[1 - \omega_E \tau_k - \left(1-\omega_E\right)\tau_k a\right] - \delta_K - \rho$ it follows that $dg/dA_E = -A_G \omega_E \left(\partial \tau_k / \partial A_E\right)$. The derivative $\left(d\tau_k/dA_E\right)$ is found from the fact that $\left[d\left(\tau_k a\right)/dA_E\right] = 0$, which implies that $\left(a + \tau_k\left(\partial a/\partial \tau_k\right)\right)d\tau_k + \tau_k\left(\partial a/\partial A_E\right)dA_E = 0$, and so $\left(d\tau_k/dA_E\right) = -\tau_k\left(\partial a/\partial A_E\right)/\left[a\left(\eta^a_{\tau_k}+1\right)\right]$ where $\left(\partial a/\partial A_E\right) \leq 0$. Thus from equation (15), $\left(dW/dA_E\right) = \left(dg/dA_E\right)$ $= A_G \omega_E \tau_k \left(\partial a/\partial A_E\right)/\left[a\left(1-\left|\eta^a_{\tau_k}\right|\right)\right] < 0$ for $\left|\eta^a_{\tau_k}\right| < 1$ and $a \in (0,1)$.

A.3 Proof of Proposition 5

By taking the total differential of the fraction of tax revenue in output given by $a\tau_k$ with respect to changes in the statutory tax rate and the goods sector productivity, we get $d\left(a\tau_k\right) = \left[a + \left(\partial a/\partial \tau_k\right)\tau_k\right]d\tau_k + \left(\partial a/\partial A_G\right)dA_G$. With this share of tax revenue fixed, $d\left(a\tau_k\right) = d\gamma = 0$, and
$d\tau_k = -\left(\partial a/\partial A_G\right)/\left[a + \left(\partial a/\partial \tau_k\right)\tau_k\right]dA_G = \left[\partial\left(1-a\right)/\partial A_G\right]/\left(a - \left[\partial\left(1-a\right)/\partial \tau_k\right]\tau_k\right)dA_G$
$= -\left\{\left[\omega_E/\left(1-\omega_E\right)\right]\left(1-a\right)/A_G\right\}/\left\{a - \left[\omega_E/\left(1-\omega_E\right)\right]\left(1-a\right)\right\}dA_G$
$= -\left(\left|\eta^a_{\tau_k}\right|/A_G\right)/\left(1-\left|\eta^a_{\tau_k}\right|\right)dA_G$ and therefore $d\tau_k = -\left\{A_G\left[\left(1/\left|\eta^a_{\tau_k}\right|\right)-1\right]\right\}^{-1}dA_G$.
Since $\partial\left[\left[\left(1/\left|\eta^a_{\tau_k}\right|\right)-1\right]^{-1}\right]\Big/\partial\left|\eta^a_{\tau_k}\right| = \left[\left(1/\left|\eta^a_{\tau_k}\right|\right)-1\right]^{-2}\Big/\left|\eta^a_{\tau_k}\right|^2$ and $\partial\left|\eta^a_{\tau_k}\right|/\partial\tau_k =$
$\partial\left\{\left[\omega_E/\left(1-\omega_E\right)\right]\left[\left(1/a\right)-1\right]\right\}/\partial\tau_k = \left[\omega_E/\left(1-\omega_E\right)\right]\left[\partial\left(1/a\right)/\partial\tau_k\right]$
$= \left[\omega_E/\left(1-\omega_E\right)\right]\left(1/a^2\right)\left[\partial\left(1-a\right)/\partial\tau_k\right] = \left[\omega_E/\left(1-\omega_E\right)\right]^2\frac{1-a}{\tau_k a^2}$, being on the upward sloping part of the normalized tax revenue curve, i.e. $\left|\eta^a_{\tau_k}\right| < 1$, implies that both $\partial\left[\left[\left(1/\left|\eta^a_{\tau_k}\right|\right)-1\right]^{-1}\right]\Big/\partial\left|\eta^a_{\tau_k}\right|$ and $\partial\left|\eta^a_{\tau_k}\right|/\partial\tau_k$ are positive. So the size of the tax elasticity of reported income $\left|\eta^a_{\tau_k}\right|$ decreases with decreases in the tax rate τ_k and the size of the effect of the goods sector productivity on the magnitude of the tax rate decrease, as captured by the term $\left[A_G\left[\left(1/\left|\eta^a_{\tau_k}\right|\right)-1\right]\right]^{-1}$, likewise decreases with decreases in the tax rate τ_k.

A.4 Extended Model Equilibrium Conditions

Given $\left(k_0, h_0\right)$ and subject to the following three constraints, the representative consumer problem is

$$\begin{aligned}
V\left(k_0, h_0\right) &= \max_{\{c_t, x_t, d_t, k_t, h_t, l_{Gt}, l_{Et}, s_{Gt}, s_{Et}, a_t,\}_{t=0}^{\infty}} \int_0^{\infty}\left(\ln c_t + \alpha \ln x_t\right)e^{-\rho t}dt, \\
\dot{k}_t + \delta_K k_t &= a_t\left[\left(1-\tau_k\right)\left(s_{Gt}+s_{Et}\right)r_t k_t + \left(1-\tau_l\right)\left(l_{Gt}+l_{Et}\right)w_t h_t\right] \\
&\quad + \left(1-a_t\right)\left(1-p_{Et}\right)\left[\left(s_{Gt}+s_{Et}\right)r_t k_t + \left(l_{Gt}+l_{Et}\right)w_t h_t\right] + r_{Et}d_t - c_t + v_t, \\
d_t &= \left(l_{Gt}+l_{Et}\right)w_t h_t + \left(s_{Gt}+s_{Et}\right)r_t k_t, \\
\dot{h}_t + \delta_H h_t &= A_H\left(l_{Ht}h_t\right)^{\varepsilon}\left(s_{\mathfrak{H}t}k_t\right)^{1-\varepsilon}.
\end{aligned}$$

Dropping time subscripts, the first-order conditions with respective multipliers of λ, χ and μ are :

$$
\begin{aligned}
0 &= (1/c)\, e^{-\rho t} - \lambda && (29)\\
0 &= (\alpha/x)\, e^{-\rho t} - \mu\varepsilon A_H \left(l_H h\right)^{\varepsilon-1} \left(s_H k\right)^{1-\varepsilon} \\
0 &= \lambda r_E - \chi && (30)\\
\dot{\lambda} &= -\lambda \left[a\left(1-\tau_k\right) r \left(s_G + s_E\right) + (1-a)\left(1-p_E\right) r \left(s_G + s_E\right) - \delta_K\right] && (31)\\
&\quad -\mu\left(1-\varepsilon\right) A_H \left(l_H h\right)^{\varepsilon} \left(s_H k\right)^{-\varepsilon} \left(1 - s_G - s_E\right) - \chi r \left(s_G + s_E\right) \\
\dot{\mu} &= -\lambda\left[a\left(1-\tau_l\right) + (1-a)\left(1-p_E\right)\right] w \left(l_G + l_E\right) && (32)\\
&\quad -\mu\left[\varepsilon A_H \left(l_H h\right)^{\varepsilon-1} \left(s_H k\right)^{1-\varepsilon} l_H - \delta_H\right] - \chi w \left(l_G + l_E\right) \\
0 &= \lambda\left[a\left(1-\tau_l\right) + (1-a)\left(1-p_E\right)\right] wh - \mu\varepsilon A_H^{\varepsilon-1} \left(l_H h\right)\left(s_H k\right)^{1-\varepsilon} h + \chi wh && (33)\\
0 &= \lambda\left[a\left(1-\tau_k\right) + (1-a)\left(1-p_E\right)\right] rk - \mu\left(1-\varepsilon\right) A_H^{\varepsilon}\left(l_H h\right)\left(s_H k\right)^{-\varepsilon} k + \chi rk && (34)\\
0 &= \left[\left(1-\tau_l\right) - \left(1-p_E\right)\right] w \left(l_G + l_E\right) h + \left[\left(1-\tau_k\right) - \left(1-p_E\right)\right] r \left(s_G + s_E\right) k. && (35)
\end{aligned}
$$

From (35) we get $p_E = \tau_l \frac{wh(l_G+l_E)}{wh(l_G+l_E)+rk(s_G+s_E)} + \tau_k \frac{rk(s_G+s_E)}{wh(l_G+l_E)+rk(s_G+s_E)}$. Assuming $\tau_l = \tau_k = \tau$, then $p_E = \tau$. Then from (31) with the use of (30) and (34) we get $-\dot{\lambda}/\lambda = r\left(1-\tau+r_E\right) - \delta_K$, and from (32) with the use of (30) and (33) it follows that $-\dot{\mu}/\mu = \varepsilon A_H \left(l_H h\right)^{\varepsilon-1} \left(s_H k\right)^{1-\varepsilon} (1-x) - \delta_H$. Using $-\dot{\lambda}/\lambda$ and the derivative of log of (29) with respect to time $\dot{c}/c = r\left(1-\tau+r_E\right) - \delta_K - \rho$, and along the BGP $g = r\left(1-\tau+r_E\right) - \delta_K - \rho$ and variables c, k, and h grow at the rate g.

A.5 Growth Accounting

The best reference for our growth accounting is Baier et al. (2006) who use a new and more comprehensive data set on the growth of output, physical capital and human capital, and input this in a Lucas (1988) -type production function with human capital (as in our economy) to construct the human capital data. Our approach is an extension of this with the added human capital investment sector, related to McGrattan and Prescott (2010). Using the data on physical and human capital growth in the dataset from Baier et al., we apply their growth-accounting procedure to find the total factor productivity growth and the factor productivity growth in each of the two sectors.

The aim is to get postwar estimates for a growth in our model's productivity parameters, A_G and A_H. In the model these are assumed constant for any given BGP. However we consider the move from one BGP to another, while ignoring transition dynamics, by allowing these to be time varying and so seek to estimate from the data $\left(\dot{A}_G/A_G\right)$ and $\left(\dot{A}_H/A_H\right)$. Using the function $F\left(\cdot\right)$ to rewrite in shorthand the production function of the goods sector: $y_{Gt} = A_{Gt} F\left[s_{Gt} k_t, l_{Gt} h_t\right]$, and otherwise using the same notation, the parameter A_{Gt} represents the level of technology, TFP, at time t, whereby $\dot{y}_t/y_t = \left(\dot{A}_{Gt}/A_{Gt}\right) + \left(F_K k_t F\right)\left(\dot{k}_t/k_t\right) + \left(F_H h_t/F\right)\left(\dot{h}_t/h_t\right)$ uses the variables in per worker terms. This implies that $\left(\dot{A}_{Gt}/A_{Gt}\right) = \frac{\dot{y}_t}{y_t} - (1-\beta)\left(\dot{k}_t/k_t\right) - \beta\left(\dot{h}_t/h_t\right)$ where $1-\beta = A_{Gt} F_K k_t/F$ is capital's share of income. For the human capital sector, and ignoring the small magnitude of capital in the evasion sector, similarly rewrite $i_{Ht} = A_{Ht} G\left[\left(1-s_{Gt}\right) k_t, \left(1-l_{Gt}\right) h_t\right]$, with $i_{Ht} = \dot{h}_t + \delta_H h_t$. Expressed in growth rates in per worker terms, this implies $\left(\dot{y}_{Gt}/y_{Gt}\right) = \left(\dot{A}_{Gt}/A_{Gt}\right) + \left(A_{Gt} F_K s_{Gt} k_t/y_{Gt}\right)\left(\left(\dot{\overline{s_{Gt} k_t}}\right)/s_{Gt} k_t\right) + \left(A_{Gt} F_H h_t/h_t\right)\left(\frac{\dot{\overline{l_{Gt} h_t}}}{l_{Gt} h_t}\right)$, and $\left(\dot{i}_{Ht}/i_{Ht}\right) = \left(\dot{A}_{Ht}/A_{Ht}\right) + \left(A_{Ht} G_K \left(1-s_{Gt}\right) k_t/i_{Ht}\right)\left(\left(\dot{\overline{\left(1-s_{Gt}\right) k_t}}\right)/\left(1-s_G\right) k_t\right) +$

$(A_{Ht}G_H(1-l_{Gt})h_t/i_{Ht})\left(((1-\dot{l}_{Gt})h_t)/(1-l_{Gt})h_t\right)$. Assuming competition, CRS production, constant shares of capitals across sectors, s_G and l_G, and let $1-\beta$ and $1-\varepsilon$ denote capital's shares of income in each goods and human capital sectors respectively, then $(\dot{y}_{Gt}/y_{Gt}) = \left(\dot{A}_{Gt}/A_{Gt}\right)+(1-\beta)\left(\dot{k}_t/k_t\right)+\beta\left(\dot{h}_t/h_t\right)$, $(\dot{i}_{Ht}/i_{Ht}) = \left(\dot{A}_{Ht}/A_{Ht}\right)+(1-\varepsilon)\left(\dot{k}_t/k_t\right)+\varepsilon\left(\dot{h}_t/h_t\right)$, and $(i_{Ht}/h_t) = \left(\dot{h}_t/h_t\right)+\delta_H$. Together it results that $\left(\dot{i}_H/h\right)-\left(i_H\dot{h}/h^2\right) = \left(\ddot{h}/h\right)-\left(\dot{h}^2/h^2\right)$. Multiply this last equation through by h/i_H, it results that $\left(\dot{i}_{Ht}/i_{Ht}\right) = \left[\left(\ddot{h}_t/\dot{h}_t\right)+\delta_H\right]/\left[1+\delta_H\left(h_t/\dot{h}_t\right)\right]$. From $\left(\dot{A}_{Gt}/A_{Gt}\right)$ above, the TFP growth rate in the goods sector is $\left(\dot{A}_{Gt}/A_{Gt}\right) = (\dot{y}_{Gt}/y_{Gt})-(1-\beta)\left(\dot{k}_t/k_t\right)-\beta\left(\dot{h}_t/h_t\right)$.From the two equations above in $\left(\dot{i}_{Ht}/i_{Ht}\right)$, the TFP growth rate in the human capital sector is $\left(\dot{A}_{Ht}/A_{Ht}\right) = \left[\left(\ddot{h}_t/\dot{h}_t\right)+\delta_H\right]/\left[1+\delta_H\left(h_t/\dot{h}_t\right)\right]-(1-\varepsilon)\left(\dot{k}_t/k_t\right)-\varepsilon\left(\dot{h}_t/h_t\right)$.

Due to ten year intervals of data, we construct a discrete form of the series for gross investment, with $i_{H,t}= h_t - h_{t-1}\left(1-\delta_H^{10}\right)$ and $g_{I,t} = \left(i_{H,t}/i_{H,t-1}\right)-1$. The baseline calibration is an annual $\delta_H= 7\%$, such that the decade depreciation rate $\delta_{H,10}= 51.6\%$ satisfies $1-\delta_{H,10} = (1-\delta_H)^{10}$.

Using the data set from Baier et al. (2006), and the above methodology within our calibrated economy, Table 5 shows the computed US growth rate of productivity increases for the goods sector and the human capital sector for each decade from 1890 to 2000. These estimates indicate that the growth rate in the human capital investment sector exceeds that of the goods sector in every decade of the 20th century; this gives unbalanced results similar to what McGrattan and Prescott (2010) exploit in a related context. Here the pre-WWII results indicate that the post-WWII results are not abnormal relative to the longer period, making the postwar results more plausible in this sense.

Pre-WWII	1890	1900	1910	1920	1930	1940
$\dot{A}_{Gt}/A_{Gt}$	0.00401	0.00629	0.01281	-0.00833	0.00243	0.00557
$\dot{A}_{Ht}/A_{Ht}$	-0.00522	0.02713	0.01925	0.08448	0.10628	0.13727
Post-WWII	1950	1960	1970	1980	1990	2000
$\dot{A}_{Gt}/A_{Gt}$	0.02370	-0.00221	0.00743	-0.00054	0.00302	0.013921
$\dot{A}_{Ht}/A_{Ht}$	0.03001	0.03840	0.07663	0.00347	0.04298	0.02993

Table 5: US Productivity Estimates 1890-2000

B Online Appendix: Tax Rate Tables

Additional Online Table References:

Piketty, T., E. Saez, and S. Stantcheva (2011). "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities," NBER Working Paper No. 17616, November; forthcoming American Economic Journal: Economic Policy .

Table A1: Top Marginal Tax Rates used in Figure 1

	Individual Income (%)	Corporate Income (%)	Receipts to GDP (%)		Individual Income (%)	Corporate Income (%)	Receipts to GDP (%)
1951	91.00	50.75	16.10	1982	50.00	46.00	19.20
1952	92.00	52.00	19.00	1983	50.00	46.00	17.50
1953	92.00	52.00	18.70	1984	50.00	46.00	17.30
1954	91.00	52.00	18.50	1985	50.00	46.00	17.70
1955	91.00	52.00	16.50	1986	50.00	46.00	17.50
1956	91.00	52.00	17.50	1987	38.50	34.00	18.40
1957	91.00	52.00	17.70	1988	28.00	34.00	18.20
1958	91.00	52.00	17.30	1989	28.00	34.00	18.40
1959	91.00	52.00	16.20	1990	28.00	34.00	18.00
1960	91.00	52.00	17.80	1991	31.00	34.00	17.80
1961	91.00	52.00	17.80	1992	31.00	34.00	17.50
1962	91.00	52.00	17.60	1993	39.60	34.00	17.50
1963	91.00	52.00	17.80	1994	39.60	35.00	18.00
1964	77.00	50.00	17.60	1995	39.60	35.00	18.40
1965	70.00	48.00	17.00	1996	39.60	35.00	18.80
1966	70.00	48.00	17.30	1997	39.60	35.00	19.20
1967	70.00	48.00	18.40	1998	39.60	35.00	19.90
1968	70.00	48.00	17.60	1999	39.60	35.00	19.80
1969	70.00	48.00	19.70	2000	39.60	35.00	20.60
1970	70.00	48.00	19.00	2001	39.10	35.00	19.50
1971	70.00	48.00	17.30	2002	38.60	35.00	17.60
1972	70.00	48.00	17.60	2003	35.00	35.00	16.20
1973	70.00	48.00	17.60	2004	35.00	35.00	16.10
1974	70.00	48.00	18.30	2005	35.00	35.00	17.30
1975	70.00	48.00	17.90	2006	35.00	35.00	18.20
1976	70.00	48.00	17.10	2007	35.00	35.00	18.50
1977	70.00	48.00	18.00	2008	35.00	35.00	17.60
1978	70.00	48.00	18.00	2009	35.00	35.00	15.10
1979	70.00	46.00	18.50	2010	35.00	35.00	15.10
1980	70.00	46.00	19.00	2011	35.00	35.00	15.40
1981	70.00	46.00	19.60	2012	35.00	35.00	15.80

NOTES: * Tax rates are obtained from the Tax Foundation tables, ‘U.S. Federal Individual Income Tax Rates History, 1913-2013 (Nominal and Inflation-Adjusted Brackets)’ and ‘Federal Corporate Income Tax Rates, Income Years 1909-2012’. Reported rates represent the top Federal marginal tax rate on individual and corporate income. Tax receipts as a fraction of GDP are obtained from OMB Historical Table 1.3. The figure for 2012 represents an OMB estimate.

Table A2: Average Tax Rates used in Figure A							
	Personal Income (%)	Corporate Income (%)	Receipts to GDP (%)		Personal Income (%)	Corporate Income (%)	Receipts to GDP (%)
1951	-	52.17	16.10	1982	41.19	34.02	19.20
1952	-	50.29	19.00	1983	40.45	36.03	17.50
1953	-	50.50	18.70	1984	40.19	37.79	17.30
1954	-	46.81	18.50	1985	39.60	40.20	17.70
1955	-	46.13	16.50	1986	37.39	48.11	17.50
1956	-	45.77	17.50	1987	40.09	43.54	18.40
1957	-	46.03	17.70	1988	34.32	39.47	18.20
1958	-	46.28	17.30	1989	35.01	41.11	18.40
1959	-	45.36	16.20	1990	34.23	38.82	18.00
1960	56.03	46.02	17.80	1991	36.22	35.90	17.80
1961	-	46.76	17.80	1992	36.51	34.44	17.50
1962	56.74	44.48	17.60	1993	42.67	36.45	17.50
1963	-	44.69	17.80	1994	44.09	34.36	18.00
1964	57.44	41.85	17.60	1995	44.75	34.21	18.40
1965	-	40.17	17.00	1996	44.16	34.30	18.80
1966	56.45	40.05	17.30	1997	41.97	33.55	19.20
1967	52.81	40.34	18.40	1998	41.34	37.31	19.90
1968	57.46	44.08	17.60	1999	40.80	36.91	19.80
1969	56.34	45.24	19.70	2000	39.80	40.25	20.60
1970	59.45	44.43	19.00	2001	39.82	34.27	19.50
1971	58.01	42.60	17.30	2002	-	28.74	17.60
1972	56.53	41.02	17.60	2003	-	30.90	16.20
1973	56.54	39.48	17.60	2004	33.93	28.68	16.10
1974	54.25	37.90	18.30	2005	-	28.45	17.30
1975	56.41	36.92	17.90	2006	-	28.99	18.20
1976	56.76	37.77	17.10	2007	-	30.40	18.50
1977	55.65	36.97	18.00	2008	-	30.20	17.60
1978	52.61	36.11	18.00	2009	-	21.33	15.10
1979	52.69	35.40	18.50	2010	-	26.39	15.10
1980	51.11	36.59	19.00	2011	-	26.78	15.40
1981	49.12	35.22	19.60				

NOTES: * Personal income tax rates are obtained from Piketty and Saez (2007) and apply to the top 5% of incomes; data is not available prior to 1960, for 1961, 1963, 1965, 2002-03, and from 2005 onwards. * Corporate income tax rates have been calculated from BEA NIPA table 6.17, 'Corporate Profits Before Tax by Industry', and table 6.18, 'Taxes on Corporate Income by Industry'. * Tax receipts as a fraction of GDP are obtained from OMB Historical Table 1.3. The figure for 2012 represents an OMB estimate..

Table A3: Weighted Average Tax Rates used in Figure A

	Marginal Rates		Average		Marginal Rates		Average
	Unadjusted (%)	Adjusted (%)	Rates (%)		Unadjusted (%)	Adjusted (%)	Rates (%)
1951	75.12	75.12	-	1982	49.43	49.43	40.16
1952	74.72	74.15	-	1983	49.54	49.54	39.94
1953	75.36	74.77	-	1984	49.36	49.36	39.80
1954	74.75	74.75	-	1985	49.38	49.38	39.69
1955	76.05	76.05	-	1986	49.39	49.39	39.03
1956	75.66	75.66	-	1987	37.71	38.76	40.70
1957	76.44	76.44	-	1988	29.14	29.14	35.30
1958	76.72	76.72	-	1989	29.13	29.13	36.15
1959	78.52	78.52	-	1990	29.01	29.01	35.00
1960	77.54	77.54	52.58	1991	31.52	31.52	36.16
1961	77.88	77.88	-	1992	31.52	31.52	36.15
1962	78.88	78.88	52.93	1993	38.55	38.55	41.50
1963	78.82	78.82	-	1994	38.65	38.65	42.09
1964	68.20	68.20	52.35	1995	38.64	38.64	42.54
1965	62.46	62.46	-	1996	38.65	38.65	42.12
1966	62.26	62.26	50.68	1997	38.69	38.69	40.31
1967	62.17	62.17	48.38	1998	38.74	38.74	40.59
1968	63.53	68.68	53.53	1999	38.80	38.80	40.12
1969	63.50	69.85	53.06	2000	38.82	38.82	39.88
1970	64.15	65.79	55.46	2001	38.56	38.56	39.09
1971	64.79	64.79	54.36	2002	38.07	38.07	-
1972	64.43	64.43	52.60	2003	35.00	35.00	-
1973	64.28	64.28	52.10	2004	35.00	35.00	32.93
1974	64.60	64.60	50.24	2005	35.00	35.00	-
1975	64.51	64.51	51.54	2006	35.00	35.00	-
1976	64.74	64.74	52.22	2007	35.00	35.00	-
1977	64.32	64.32	50.84	2008	35.00	35.00	-
1978	64.53	64.53	48.50	2009	35.00	35.00	-
1979	64.43	64.43	48.67	2010	35.00	35.00	-
1980	64.97	64.97	48.07	2011	35.00	35.00	-
1981	65.77	64.78	46.67	2012	35.00	35.00	-

NOTES:* Marginal tax rates are obtained from the Tax Foundation tables, ‘U.S. Federal Individual Tax Rates History, 1913-2013 (Nominal and Inflation-Adjusted Brackets)’ and ‘Federal Corporate Income Tax Rates, Income Years 1909-2012’. Rates represent the topFederal marginal tax rate on individual and corporate income. * ‘Adjusted’ marginal tax rates apply corrections surtaxes and mid-year rate changes (as in Piketty, et al., 2011). * Average personal income tax rates are obtained from Piketty and Saez (2007) and apply to the top 0.5% of incomes; data is not available prior to 1960, or for ’61, ’63, ’65, 2002-03, and from 2005 onwards. Average corporate income tax rates have been calculated from BEA NIPA table 6.17, ‘Corporate Profits Before Tax by Industry’, and table 6.18, ‘Taxes on Corporate Income .by Industry’ * Tax rates are weighted using OMB Historical Table 2, ‘Percentage Compostition of Receipts by Source: 1934-2018’. Other sources of revenue are disregarded so that the contribution of individual and corporate taxation to total tax revenue sums to 100%.

Table A4: Tax Weights used in Figure 1, Appendix Figure A					
	Personal taxes, share (%)	Corporate taxes, share (%)		Personal taxes, share (%)	Corporate taxes share (%)
1951	60.55	39.45	1982	85.77	14.23
1952	56.80	43.20	1983	88.58	11.42
1953	58.39	41.61	1984	84.05	15.95
1954	58.32	41.68	1985	84.44	15.56
1955	61.66	38.34	1986	84.70	15.30
1956	60.67	39.33	1987	82.44	17.56
1957	62.68	37.32	1988	80.92	19.08
1958	63.37	36.63	1989	81.23	18.77
1959	67.99	32.01	1990	83.24	16.76
1960	65.48	34.52	1991	82.65	17.35
1961	66.36	33.64	1992	82.58	17.42
1962	68.93	31.07	1993	81.25	18.75
1963	68.77	31.23	1994	79.37	20.63
1964	67.39	32.61	1995	79.02	20.98
1965	65.72	34.28	1996	79.30	20.70
1966	64.83	35.17	1997	80.24	19.76
1967	64.43	35.57	1998	81.39	18.61
1968	70.60	29.40	1999	82.65	17.35
1969	70.44	29.56	2000	82.94	17.06
1970	73.40	26.60	2001	86.78	13.22
1971	76.32	23.68	2002	85.27	14.73
1972	74.67	25.33	2003	85.74	14.26
1973	74.01	25.99	2004	80.98	19.02
1974	75.46	24.54	2005	76.96	23.04
1975	75.04	24.96	2006	74.70	25.30
1976	76.08	23.92	2007	75.88	24.12
1977	74.20	25.80	2008	78.96	21.04
1978	75.12	24.88	2009	86.83	13.17
1979	76.80	23.20	2010	82.34	17.66
1980	79.06	20.94	2011	85.71	14.29
1981	82.38	17.62	2012	83.10	16.90

NOTES: * Weights are calculated using shares obtained from OMB Historical Table 2.2,'Percentage Composition of Receipts by Source: 1934–2018'. 2012 shares are based upon an OMB estimate. * Other sources of tax revenue are disregarded so that the shares sum to 100%.

Table A5: Simple Average Tax Rates (Not Shown in Figures)

	Marginal Rates		Average		Marginal Rates		Average
	Unadjusted (%)	Adjusted (%)	Rates (%)		Unadjusted (%)	Adjusted (%)	Rates (%)
1951	70.88	70.88	-	1982	48.00	48.00	37.60
1952	72.00	71.50	-	1983	48.00	48.00	38.24
1953	72.00	71.50	-	1984	48.00	48.00	38.99
1954	71.50	71.50	-	1985	48.00	48.00	39.90
1955	71.50	71.50	-	1986	48.00	48.00	42.75
1956	71.50	71.50	-	1987	36.25	39.25	41.82
1957	71.50	71.50	-	1988	31.00	31.00	36.89
1958	71.50	71.50	-	1989	31.00	31.00	38.06
1959	71.50	71.50	-	1990	31.00	31.00	36.53
1960	71.50	71.50	51.03	1991	32.50	32.50	36.06
1961	71.50	71.50	-	1992	32.50	32.50	35.48
1962	71.50	71.50	50.61	1993	36.80	36.80	39.56
1963	71.50	71.50	-	1994	37.30	37.30	39.23
1964	63.50	63.50	49.64	1995	37.30	37.30	39.48
1965	59.00	59.00	-	1996	37.30	37.30	39.23
1966	59.00	59.00	48.25	1997	37.30	37.30	37.76
1967	59.00	59.00	46.58	1998	37.30	37.30	39.32
1968	59.00	64.05	50.77	1999	37.30	37.30	38.85
1969	59.00	64.90	50.79	2000	37.30	37.30	40.03
1970	59.00	60.50	51.94	2001	37.05	37.05	37.05
1971	59.00	59.00	50.30	2002	36.80	36.80	-
1972	59.00	59.00	48.77	2003	35.00	35.00	-
1973	59.00	59.00	48.01	2004	35.00	35.00	31.31
1974	59.00	59.00	46.07	2005	35.00	35.00	-
1975	59.00	59.00	46.66	2006	35.00	35.00	-
1976	59.00	59.00	47.27	2007	35.00	35.00	-
1977	59.00	59.00	46.31	2008	35.00	35.00	-
1978	59.00	59.00	44.36	2009	35.00	35.00	-
1979	58.00	58.00	44.04	2010	35.00	35.00	-
1980	58.00	58.00	43.85	2011	35.00	35.00	-
1981	58.00	57.40	42.17	2012	35.00	35.00	-

NOTES: * Marginal tax rates are obtained from the Tax Foundation tables, 'U.S. Federal Individual Tax Rates History, 1913-2013 (Nominal and Inflation-Adjusted Brackets)' and 'Federal Corporate Income Tax Rates, Income Years 1909-2012'. Reported rates represent the top Federal marginal tax rate on individual and corporate income. * 'Adjusted' marginal tax rates apply corrections for surtaxes and mid-year rate changes (as in Piketty et al., 2011). * Average personal income tax rates are obtained from Piketty and Saez (2007) and apply to the top 0.5% of incomes; data is not available prior to 1960, or for '61, '63, '65, 2002-03, and from 2005 onwards. Average corporate income tax rates have been calculated from BEA NIPA table 6.17, 'Corporate Profits Before Tax by Industry', and table 6.18.