

Neural Computation and the Computational Theory of Cognition¹

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Abstract

We argue that neural processes are neither analog nor digital computations; they constitute a third kind of computation. Analog computation is the processing of continuous signals; digital computation is the processing of strings of digits. But current neuroscientific evidence indicates that typical neural signals, such as spike rates, are graded like continuous signals but are constituted by discrete elements (spikes); thus typical neural signals are neither continuous signals nor strings of digits. It follows that neural computation is sui generis. This has three important consequences. First, understanding neural computation requires a specially designed mathematical theory (or theories) rather than the mathematical theories of analog or digital computation. Second, several popular views about neural computation turn out to be incorrect. Third, computational theories of cognition that rely on non-neural notions of computation ought to be replaced or reinterpreted in terms of neural computation.

[A]ny [psychological] theory that is incompatible with known facts of neurology must be, ipso facto, unacceptable. (Fodor 1965, p. 176)

The brain is an analogical machine, not digital. (Lashley 1958, quoted in Beach et al. 1960, p. 539)

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Warren McCulloch and Walter Pitts (1943) argued that neural activity is computation and that neural computation explains cognition (Piccinini 2004). McCulloch and Pitts argued that neural computation is digital; Karl Lashley and others countered that it is analog. We will refer to the thesis that neural computation (simpliciter) explains cognition as computationalism or the computational theory of cognition. Over the last six decades, computationalism—in its various classicist, connectionist, and neurocomputational incarnations²—has been the mainstream theory of cognition. As a consequence, today many cognitive scientists consider it commonsensical to say that neural activity is computation and that computation explains cognition. But they still disagree about *which type* of computation explains cognition. In addition, computationalism itself remains controversial.³

In this paper, we propose to push the debate forward by establishing a sound neurocomputational basis for the explanation of cognition. First, we distinguish computationalism from a number of other theses that are often conflated with it. Second, we distinguish between several important kinds of computation. Third, we defend a weak version of computationalism—neural processes are computations in a generic sense. Fourth, we reject the common assimilation of neural computation to either analog or digital computation—neural computation is *sui generis*. Finally, we highlight some important consequences of our conclusion.

1. Five Theses about the Nervous System

To prepare the ground, let us distinguish computationalism properly so called from other related theses it is often conflated with.

(1) The nervous system is an *input-output* system.

This thesis holds that neural activity generates responses by processing stimuli. Inputs come into the nervous system, are processed, and such processing affects the output. This is obviously true but says nothing about the nature of the inputs, outputs, and internal processes. In this sense, the nervous system is analogous to any system that

² Roughly speaking, classicism is the view that neural computation is closely analogous to digital computation as it occurs in digital computers (Fodor and Pylyshyn 1988); both connectionism and computational neuroscience maintain that cognition is explained by (non-classical) neural network activity but they differ in whether they constrain their models primarily using behavioral evidence (connectionism) or also emphasizing neuroanatomical and neurophysiological evidence (computational neuroscience).

³ The literature is enormous. Classic and influential defenses of computationalism include McCulloch and Pitts 1943, Wiener 1948, von Neumann 1958, Marr 1982, and Churchland and Sejnowski 1992; a similar thesis, with less emphasis on the nervous system, is defended by Fodor 1975, Newell and Simon 1976, Pylyshyn 1984, Newell 1990, and others; more recent defenses of computationalism include Edelman 2008, Fodor 2008, Gallistel and King 2009, and Schneider 2011; influential critiques on computationalism include Lucas 1961, Dreyfus 1979, Searle 1980, Perkel 1990, Globus 1992, Penrose 1994, van Gelder 1998, and Fodor 2000.

generates outputs in response to inputs. If the notion of input and output is liberal enough, any physical system belongs in the class of input-output systems. If so, then thesis (1) is equivalent to the trivial thesis that the brain is a physical system. But even if the notion of input and output is more restricted—for example, by specifying that inputs and outputs must be of a certain kind—thesis (1) remains weak. It is equivalent to stating that the brain processes inputs and outputs of the specified kind.

(2) The nervous system is a *functionally organized* input-output system.

Thesis (2) strengthens (1) by adding a further condition: neural activity has the *function* of generating specific responses from specific stimuli. In other words, the nervous system is functionally organized to exhibit certain capacities and not others. Under this stronger thesis, the nervous system belongs to the specific class of physical systems that are functionally organized. Systems of this type include computers as well as systems that intuitively do not perform computations, such as engines, refrigerators, and stomachs. This second thesis is stronger than the first one but remains weak. Again, there is no reason to doubt it.

(3) The nervous system is a *feedback control*, functionally organized, input-output system.

Thesis (3) is stronger than (2) because it specifies the function that is peculiar to neural activity: the nervous system controls the behavior of an organism in response to stimuli coming from the organism's body and environment that include the effects of the nervous system itself. The specific function of neural activity differentiates the nervous system from other functionally organized systems. Under thesis three, the nervous system belongs in the class of feedback control systems together with, for instance, autopilot systems. Thesis (3) is not trivial and took serious scientific work to establish rigorously (Cannon 1932). Nevertheless, it is by now uncontroversial.⁴

(4) The nervous system is an *information-processing*, feedback control, functionally organized, input-output system.

Thesis (4) is stronger than (3) because it specifies the means by which the nervous system exerts its feedback control function. Those means are the processing of information. The term "information" is ambiguous between several senses. We will briefly mention four.

In the thermodynamic sense, information is closely related to entropy. Everything carries information and every physical process is an instance of information

⁴ Of course, there are different kinds of feedback control mechanism, some of which are more powerful than others. Therefore, it remains to be determined which feedback control mechanisms are present or absent within the nervous system. For some options, see Grush 2003.

processing in this sense. This notion of information is too general to be directly relevant to theories of cognition. We will leave it aside.

In a first relevant sense, “information” means mutual information in the sense of communication theory (Shannon and Weaver 1949). Mutual information is a measure of the statistical dependency between a source and a receiver. It is experimentally well established that neural signals such as firing rate are statistically dependent on other variables inside and outside the nervous system. Thus, mutual information is often used to quantify the statistical dependence between neural signals and their sources and to estimate the coding efficiency of neural signals (Dayan and Abbott 2001, Chap. 4; Baddeley et al. 2000). It should be uncontroversial that the nervous system processes signals that carry information in this sense, although this is still a fairly weak notion of information *processing*.

In a second relevant sense, “information” means what we call “natural semantic information,” which is approximately the same as “natural meaning” (Grice 1957). A signal carries natural semantic information *about* a source if and only if it causally, or at least reliably, correlates with the source (Dretske 1981). This second sense of “information” is different from the first because it is a kind of *semantic* content, whereas mutual information is not a measure of semantic content. Nevertheless, the two senses are conceptually related because whenever there is mutual information between a signal and a source, there is by definition a reliable correlation between source events and receiver events. Thus, the receiver acquires natural semantic information *about* the source.

Once again, it is both evident to naïve observation and experimentally well established that neural variables carry natural semantic information about other internal and external variables (Adrian 1928; Rieke et al. 1999; cf. Garson 2003). Much contemporary neurophysiology centers on the discovery and manipulation of neural variables that correlate reliably—and often usefully—with variables external to the nervous system.

Given (3) and the complexity of the nervous system’s control functions, thesis (4) follows: in order to exert sufficiently complex control functions, the only efficient way to perform feedback control is to possess and process internal variables that correlate reliably with relevant external variables. These internal variables must be processed based on the natural information they carry. Thus, it should be uncontroversial that the nervous system processes information in the sense of processing signals based on the natural semantic information they carry.

Caveat: many cognitive scientists appear to use the term “representation” for internal states or variables that carry natural semantic information. This is a weak sense of representation. Representation as usually understood by philosophers allows for the possibility of *misrepresentation* (Dretske 1986; Millikan 2004). For instance, the statement “this is an armadillo” is true of armadillos but false of non-armadillos; it correctly represents armadillos while misrepresenting everything else. But natural semantic information cannot misrepresent. A variable that does not correlate with a source does not misrepresent; it simply fails to carry natural information about the source. Thus, natural semantic information represents only in a weak sense. In this

weak sense, the nervous system is surely a representational system. Most philosophers of cognitive science, however, reserve the term “representation” for states or variables that can be “wrong” or misrepresent. From now on, we will follow this usage.⁵

The stronger notion of representation is related to a third relevant notion of information (Scarantino and Piccinini 2010). In this sense, “information” means what we call “non-natural semantic information,” which is approximately the same as “non-natural meaning” (Grice 1957). A signal carries non-natural semantic information *about* *x* if and only if it represents *x* a certain way, whether or not *x* is the way it’s represented. In our example, the sentence “this is an armadillo” carries non-natural semantic information about whatever “this” refers to, and it represents it as an armadillo. The represented object may be a *source* of the non-natural information carried by a signal but it need not be. In fact, a signal may represent objects that don’t even exist.

Many theories in cognitive science assume that nervous systems possess and process representations (carrying non-natural semantic information). Specifically, it is difficult to explain language processing and other higher cognitive processes without postulating representations. Although some authors doubt that representations are needed in a theory of cognition (e.g., Ramsey 2007), we shall follow the mainstream and assume that nervous systems process representations.

The upshot of our discussion of thesis (4) is that there is at least one important and uncontroversial sense of information processing—meaning the functionally organized processing of signals based on the (either natural or non-natural) semantic information they carry—in which the nervous system processes information.

(5) The nervous system is a computing system.

Thesis (5) is a generic form of computationalism. To evaluate computationalism and assess its relation to (1)-(4) requires saying more about what counts as computation in a physical system such as the nervous system. We will get to that in the next section.

We can already highlight some relationships between the five theses. As we already pointed out, each of (2), (3), and (4) is stronger than its predecessor.

While strictly speaking there can be computing systems without inputs, outputs, or both, we will disregard them here. In addition, following Piccinini (2007a, 2008), we assume that any computing system that performs nontrivial computations is functionally organized to perform its computations. With these assumptions in place, computing (5) entails having inputs and outputs (1) and being functionally organized (2). There is no entailment in the other direction. For there are plenty of systems—including functionally organized systems such as the animals’ locomotive, circulatory, and respiratory systems—that do not perform computations in any relevant sense. So (2) and a fortiori (1) do not entail (5).

⁵ Sometimes the neurons themselves are said to constitute a representation. For instance, the motor cortex is said to carry a representation of the hand whether or not the motor cortex is firing. This notion of representation is analogous to the sense in which a map is a representation whether or not someone is looking at it and using it as such.

Computing (5) entails neither feedback control (3) nor information processing (4). This is because computing systems need not perform any feedback-control or information-processing functions—an example would be a computer programmed to manipulate meaningless digits according to some arbitrary rule. We postpone discussing whether (3) and (4) entail (5) until after we say more about what counts as computing.

2. Computation: Generic, Digital, and Analog

Drawing from previous work on the nature of computation (Piccinini 2007a, 2008, Piccinini and Scarantino 2011), we now sketch what it takes for a physical system to perform (or implement) computations of a certain kind. We distinguish between computation in a generic sense and its two most relevant species: digital and analog computation.

2.1 Generic Computation

Concrete computing systems are functionally organized mechanisms of a special kind—mechanisms that perform concrete computations. A functionally organized mechanism is a system of organized components, each of which has functions to perform (cf. Craver 2007, Wimsatt 2002). When appropriate components and their functions are organized and functioning properly, their combined activities constitute the capacities of the mechanism. Conversely, when we look for an explanation of the capacities of a mechanism, we decompose the mechanism into its components and look for their functions and organization. The result is a mechanistic explanation of the mechanism's capacities—and explanatory strategy familiar to both biologists and engineers.

In order to capture all uses of “computation” in cognitive science, we need a broad notion of computation. We call it *generic computation*. Digital computation and analog computation turn out to be species (among others) of generic computation.

Computation in the generic sense is the processing of vehicles according to rules that are sensitive to certain vehicle properties and, specifically, to differences between different portions (i.e., spatiotemporal parts) of the vehicles. Processing is performed by a functionally organized mechanism, that is, a mechanism whose components are functionally organized to perform the computation. Thus, if the mechanism malfunctions, a miscomputation occurs.

When we define concrete computations and the vehicles that they manipulate, we may consider only the properties that are relevant to the computation. Since concrete computations and their vehicles are described sufficiently abstractly as to be defined independently of the physical media that implement them, we call them “medium-independent”.

In other words, a vehicle is medium-independent if and only if the rules (i.e., the input-output maps) that define a computation are sensitive only to differences between portions of the vehicles along specific dimensions of variation—they are insensitive to any more concrete physical properties of the vehicles. Put yet another way, the rules are functions of state variables associated with a set of functionally relevant degrees of freedom, which can be implemented differently in different physical media. Thus, a

given computation can be implemented in multiple physical media (e.g., mechanical, electro-mechanical, electronic, magnetic, etc.), provided that the media possess a sufficient number of dimensions of variation (or degrees of freedom) that can be appropriately accessed and manipulated and that the components of the mechanism are functionally organized in the appropriate way.

Given this account of computing, feedback control (3) does not entail computing (5); a system may exert feedback control functions without ever transducing the signals into an internal medium solely devoted to computing—i.e., without any medium-independent processing. For instance, a Watt governor exerts feedback control by exploiting specific physical relations that obtain between its variables and the variables it controls.

2.2 Digital Computation

The best-known kind of computation is digital computation—the kind of process that was mathematically characterized by the classical theory of computation begun by Alan Turing and other logicians and now a well-established branch of mathematics (Davis et al. 1994).

Digital computation may be defined both abstractly and concretely. Roughly speaking, abstract digital computation is the manipulation of strings of discrete elements, that is, strings of letters from a finite alphabet. Here we are interested primarily in concrete computation, or physical computation. As we have seen, letters from a finite alphabet may be physically implemented by what we call “digits”. To a first approximation, concrete digital computation is the processing of sequences of digits according to general rules defined over the digits (Piccinini 2007a). Let us briefly consider the main ingredients of digital computation.

The atomic vehicles of concrete digital computation are digits, where a digit is a macroscopic state (of a component of the system) whose type can be reliably and unambiguously distinguished by the system from other macroscopic types. To each (macroscopic) digit type, there correspond a large number of possible microscopic states. For instance, a huge number of distinct arrangements of electrons (microscopic states) correspond to the same charge stored in a capacitor (macroscopic state). Artificial digital systems are engineered so as to treat all those microscopic states in one way—the way that corresponds to their (macroscopic) digit type. To ensure reliable manipulation of digits based on their type, a physical system must manipulate at most a finite number of digit types. Digits need not mean or represent anything, but they can. For instance, numerals represent numbers, while other digits may not represent anything in particular.

Digits can be concatenated (i.e., ordered) to form sequences or *strings*, which serve as the vehicles of digital computations. A digital computation consists in the processing of strings of digits according to rules, which are simply maps from input strings, plus possibly internal states, to output strings. Examples of rules that may figure in a digital computation include addition, multiplication, identity, and sorting.

Digits are unambiguously distinguishable by the processing mechanism under normal operating conditions. Strings of digits are ordered sets of digits, i.e., digits such

that the system can distinguish different members of the set depending on where they lie along the string. The rules defining digital computations are, in turn, defined in terms of strings of digits and internal states of the system, which are simply states that the system can distinguish from one another. No further physical properties of a physical medium are relevant to whether they implement digital computations. Thus, digital computations can be implemented by any physical medium with the right degrees of freedom.

The notion of digital computation here defined is quite general. It should not be confused with three other commonly invoked but more restrictive notions of computation: classical computation in the sense of Fodor and Pylyshyn (1988), algorithmic computation, and computation of Turing-computable functions (see Figure 1).

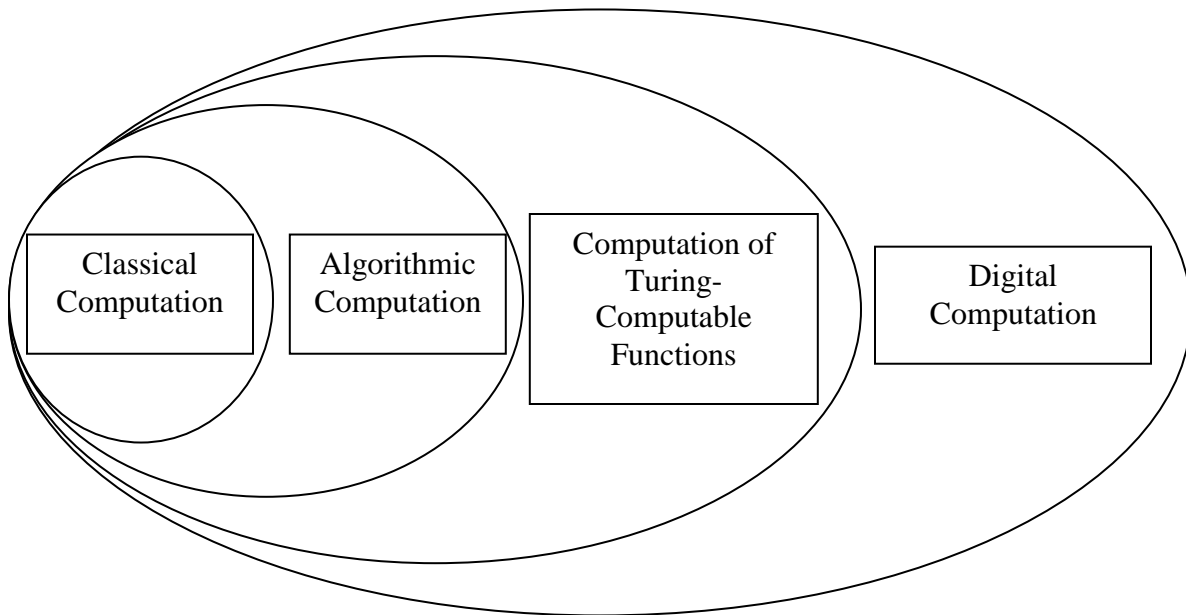


Figure 1. Types of digital computation and their relations of class inclusion.

Many other distinctions may be drawn within digital computation, such as hardwired vs. programmable, special purpose vs. general purpose, and serial vs. parallel (cf. Piccinini 2008b, c). These distinctions play an important role in debates about the computational architecture of cognitive systems.

To summarize, a physical system is a digital computing system if and only if it is a system that is functionally organized to manipulate input strings of digits, depending on the digits' type and their location on the string, in accordance with a rule defined over the strings (and possibly certain internal states of the system).

For present purposes, two points about strings of digits are especially important: (i) whether a particular microscopic state belongs to a digit type is unambiguous relative to the behavior of the system; (ii) the output of a computation depends (either

deterministically or probabilistically) only on the internal state of the system and on the number of input digits, their types, and the way they are concatenated within the string during a given time interval.

Systems that qualify as performing computations in this sense include physical implementations of standard computability theory formalisms such as Turing machines and finite state automata, ordinary computers and calculators, and physical implementations of connectionist or neurocomputational systems that are characterized in terms of the processing of strings of discrete inputs and outputs. The latter category includes McCulloch-Pitts nets (McCulloch and Pitts 1943), perceptrons (Minsky and Papert 1969), Hopfield nets (Hopfield 1982), standard Parallel Distributed Processing networks (Rumelhart and McClelland 1986), among others. Since digital computation is the notion that inspired the computational theory of cognition, it is the most relevant notion for present purposes.

2.3 Analog Computation

Analog computation is often contrasted with digital computation, though it is a vaguer and more slippery concept. The clearest notion of analog computation is that of Pour-El (1974; see also Rubel 1993, Mills 2008). Roughly, abstract analog computers are systems whose function is to manipulate continuous variables—variables that can vary continuously over time and take any real values within certain intervals—so as to instantiate appropriate functional relationships between the variables. A major function of analog computers is solving certain systems of differential equations by instantiating the relations between continuous variables specified in the equations. The most important tool for solving differential equations using classical analog computers is the integrator—a component whose output is the integral of its input variable over a relevant time interval.

Physically implemented continuous variables are quite different vehicles than strings of digits. While a digital computing system can always unambiguously distinguish digits from one another, a physically implemented analog computing system cannot do the same with the exact values of continuous variables, because these variables can only be measured within a margin of error. Primarily due to this, analog computations (in the present, strict sense) are a different kind of process than digital computations. Nevertheless, like digital computers, analog computers operate on medium-independent vehicles.

There are other kinds of computation besides digital and analog computation. One increasingly popular kind, quantum computation, is an extension of digital computation in which digits are replaced with quantum states called qudits (most commonly, binary qudits, which are known as qubits). There is no room here for a complete survey of kinds of computation. Suffice it to conclude that generic computation includes digital computation, analog computation, quantum computation, and more (Figure 2).

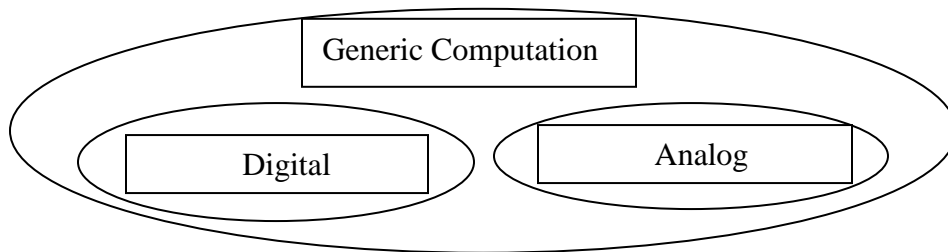


Figure 2. Types of computation and their relations of class inclusion.

3. Two Arguments for Generic Computationalism

We are now ready to give two arguments for generic computationalism—the thesis that neural processes are computations in the generic sense.

The argument from the functional organization of the nervous system

1. Neural processes are defined over medium-independent vehicles.
2. The processing of medium-independent vehicles constitutes computation in the generic sense (as per Section 2.1).
3. Therefore, neural processes are computations in the generic sense.

Current evidence suggests that the primary vehicles manipulated by neural processes are neuronal spikes (action potentials) and that the functionally relevant aspects of neural processes depend on dynamical aspects of the spikes such as spike rates and spike timing. Early studies of neural signaling (Adrian and Zotterman 1926; Adrian 1928) demonstrated that nervous systems transduce different physical stimuli into a common internal medium. Most neural signals follow the *all-or-none* principle: a signal of a certain amplitude—a spike—is transmitted above a certain threshold of stimulus intensity, whereas below that threshold no signal is transmitted. Information about the stimulus can be conveyed through changes in spike rates (*rate coding*). While it has been suggested that information is primarily encoded in spike rates, much recent evidence suggests that precise spike timing can also play a role in information transmission (*temporal coding*). Indeed, spike timing may be critically important when stimuli must be processed more quickly than would be possible if the brain relied on spike rates (VanRullen et al. 2005). Examples of temporal coding can be found in the visual (Bair and Koch, 1996; Meister and Berry, 1999; Gollisch 2009) and somatosensory (Johansson and Birznieks, 2004) systems. The relative importance and prevalence of temporal vs. rate coding remains an issue of intense debate (London et al. 2010).

The functionally relevant aspects of spike trains, such as spike rates and spike timing, are similar throughout the nervous system regardless of the physical properties of the stimuli (i.e., auditory, visual, somatosensory, etc.) and may be implemented either by neural tissue or by some other physical medium, such as a silicon-based circuit (cf. Craver 2010). Thus, spike trains—sequences of spikes such as those produced by neurons in real time—appear to be medium-independent vehicles, thus qualifying as proper vehicles for generic computation. Assuming that brains process spike trains and

that spikes are medium-independent vehicles, it follows that brains perform computations in the generic sense.

The argument from semantic information processing

1. Neural processes process semantic information.
2. The processing of semantic information requires computation.
3. Therefore, neural processes are computations in the generic sense.

The argument for generic computationalism from semantic information processing begins with the observation made in Section 1 that cognition involves the processing of information, in at least three important senses. First, cognition involves processing stochastic signals; this is information in the non-semantic sense of mutual information between a source and a receiver. Second, cognition involves the processing of signals based on their causal correlations with their sources (Adrian 1928; Rieke et al. 1999); we called this natural semantic information. Third, at least some forms of cognition, such as language processing, appear to require manipulating full-blown representations, i.e., internal states that can represent either correctly or incorrectly; we called this non-natural semantic information.

For those who identify computation and information processing, the fact that cognition involves information processing is enough to conclude that cognition involves computation. But, as we pointed out, computation and information processing are conceptually distinct (Piccinini and Scarantino 2011). Nevertheless, the fact that cognition involves information processing does entail that cognition involves computation. This is because information (in the three above-mentioned senses of the term) is a medium-independent notion.

Processing information means processing vehicles on the basis of the information they carry rather than their specific physical properties. And the information carried by a signal is defined independently of the physical medium that implements the signal. Thus, processing information requires processing vehicles that are defined in a medium-independent way. Thus, per Section 2.1, information processing entails generic computation. (The converse does not hold because medium-independent vehicles need not carry any information.)

We conclude that cognition involves computation in the generic sense and nervous systems perform computations in the generic sense. As we shall see, it's much harder to establish that cognition involves a more specific kind of computation.

4. Traditional Objections to Computationalism

Before discussing digital and analog versions of computationalism, we need to briefly see why computationalism is not refuted by existing objections. There are two types of objection: (1) insufficiency objections, based on alleged differences between computations and cognitive processes, and (2) objections from neural realization, based on alleged differences between computations and neural processes (Piccinini 2010).

4.1 Insufficiency Objections

There is a long tradition of arguments to the effect that computation is insufficient for genuine cognition. These are the most discussed objections to computationalism. Their general form is as follows: cognitive phenomena include X , but computation is insufficient for X . Therefore, cognition is not computation. Candidates for X include but are not limited to the human ability to grasp mathematical truth (Lucas 1961; Penrose 1994), conscious experience (Block 1978), the human ability to represent or understand (Searle 1980), the human ability to perform explanatory inferences such as abduction (Fodor 2000), embodiment or the role played by the body in cognition (Thompson 2007), and embeddedness or the role played by the environment in cognition (van Gelder 1998).

Each insufficiency objection has met resistance. Whether computation is sufficient for any proposed X remains controversial. But the most relevant observation for present purposes is that insufficiency objections do not refute computationalism because they are at best partial: even if it is granted that computation is insufficient for X , it doesn't follow that neural processes are not computations or that computation plays no role in explaining cognition. All that follows is that explaining X requires something else *in addition to computation*. For all that insufficiency objections establish, computation may well be sufficient for cognitive phenomena other than X , may be part of the explanation for X , or both.

4.2 Objections from Neural Realization

There are also objections to computationalism based on alleged differences between neural processes and computations.

4.2.1 Non-electrical Processes

Like signals in electronic computers, neural signals include the propagation of electric charges. But neural signals also include the diffusion of chemical substances such as neurotransmitters and hormones, which ultimately mediate or modulate the electrical signals. Therefore, according to this objection, neural processes are not computations (cf. Perkel 1990).

While this objection correctly points out a disanalogy between nervous systems and electronic computers, the objection does not impugn computation in general. Computation may be realized using chemical signals, since it is a medium-independent process.

4.2.2 Temporal Constraints

Neural processes are temporally constrained, whereas computations are not; hence, neural processes are not computations (cf. Globus 1992, van Gelder 1998). This objection is based on the observation that *algorithmic digital* computations can be defined and individuated in terms of computational steps, independently of how much time it takes to complete a step.

But this abstraction from temporal constraints is a feature not of computations themselves, but of (some of) their descriptions. The same abstraction can be performed on any dynamical process, whether computational or not. When a temporal scale is

specified, computational descriptions must take temporal constraints into account just as much as any other descriptions. In fact, even defenders of classical computationalism have been concerned with temporal constraints on the computations that, according to their theory, explain cognitive phenomena (Pylyshyn 1984; Newell 1990; Vera and Simon 1993). And temporal constraints are a main concern, of course, in the neurocomputational literature—especially on motor control (VanRullen et al. 2005).

4.2.3 Spontaneous Activity

Neural processes are not the effect of inputs alone because they also include a large amount of spontaneous activity; hence, neural processes are not computations (cf. Perkel 1990). The obvious problem with this objection is that computations may certainly be the effect of inputs *together with* internal states. In fact, only relatively simple computations are the direct effect of inputs; in the more interesting cases, internal states also play a role.

4.2.4 Analog vs. Digital

Neural processes are analog, whereas computations are digital; hence, neural processes are not computations (e.g., Dreyfus 1979, Perkel 1990). This is one of the oldest and most repeated objections from neural realization.

One problem with this objection is the mistaken assumption that all computation is digital. (Another problem is the assumption that neural computation is analog; more on that presently.) While digital computation appears to be the most versatile type of computation, and while analogies between neural processes and digital computations were the original source of computationalism (McCulloch and Pitts 1943), the notion of computation is broader than that of digital computation. This observation leads directly to analog computationalism: the view that neural processes are analog computations.

5. Why Neural Processes Are Not Analog Computations

Some authors have argued that neural processes are analog computations. In fact, the original proponents of the analog-vs.-digital objection did not offer it as a refutation of computationalism simpliciter, but only of McCulloch and Pitts's *digital* computationalism (Gerard 1951).

But analog computationalism does not immediately follow from the claim that neural processes are analog. Depending on what is meant by “analog,” an analog process may or may not be an analog computation.

In a broad sense, “analog” refers to processes that can be characterized as the dynamical evolution of real (i.e., continuous, or analog) variables in real time. Some authors who discuss whether the nervous system is analog seem to employ this broad notion (e.g., Churchland and Sejnowski 1992, van Gelder 1998). Surely neural processes are analog in this sense, but so are all or most physical processes according to current

physical theory.⁶ Yet most systems that are analog in this sense, such as the weather, planetary motion, or digestion, are not analog computers in any interesting sense. Even more importantly, given a sufficiently fine-grained description, ordinary digital computers are analog in this sense too! Thus, the trivial fact that neural mechanisms are analog in this broad sense does nothing to either refute computationalism or establish an analog version of it.

In another sense, “analog” refers to representations that bear some positive analogy to what they represent. There is evidence that nervous systems contain and manipulate analog models of their environment (Waskan 2003, Grush 2004). Examples include visual (Hubel and Wiesel, 1962) and auditory (Schreiner and Winer, 2007) receptive fields, as well as the somatosensory representation of the world touched by a rodent's whiskers in the “barrel cortex” (Inan and Crair, 2007). But analog *models* need not be represented and manipulated by analog *computers*.

Finally, in the most relevant sense, “analog” refers to analog computers properly so called (Pour-El 1974, Rubel 1993, Mills 2008, and see Section 2.3 above). Claiming that neural systems are analog computers is a strong empirical hypothesis. Analog computationalism is committed to the view that the functionally significant signals manipulated by the nervous system are continuous variables and that the main tool for processing neural signals are integrators.

To be sure, brains and analog computers both appear to have an essentially continuous dynamic, that is, their vehicles must be assumed to vary over real time in order to construct successful scientific theories of them (for the case of brains, see Dayan and Abbott 2001, Ermentrout and Terman 2010). Neuronal inputs—i.e., neurotransmitters and hormones—are most usefully modeled as continuous variables; in addition, their release and uptake is modulated by chemical receptors that operate continuously in real time. Also, neuronal dendrites and at least some neuronal axons transmit graded potentials—that is, more or less continuously varying voltage changes. In these respects, brains appear to be more similar to analog computers than to digital ones. Finally, as Rubel (1985) points out, neural integrators do play an important role in at least some processes, such as oculomotor control.

But there is at least one crucial disanalogy between brains and analog computers. Typically, if the total graded input signal received by a neuron is above a certain threshold, the neuron fires a spike. If the total signal received by a neuron is below the threshold, no spike is fired. As a consequence of the all-or-none character of the spike, computational neuroscience focuses on firing rates and spike timing as the most significant functional variables at the level of circuit and network.

Thus, while firing threshold is subject to modulation by continuous variables (e.g., ion concentrations), firing rates vary in a graded way, and spike timing (in real time) may be functionally significant, none of these aspects of neural signals amounts to

⁶ It has been argued that at the fundamental physical level, everything is discrete (e.g., Toffoli 1984). This remains a very speculative hypothesis for which there is no direct evidence. Moreover, it would still be true that our best science of midsize objects, including digital computers at certain levels of organization, is carried out by assuming that the relevant variables evolve continuously and are characterized as the dynamical evolution of real variables in real time.

the use of continuous variables as vehicles within analog computers. Furthermore, neural integrators are specialized components that operate on firing rates (as opposed to continuous variables), which means they operate on a different type of vehicle than the integrators of analog computers. Finally, unlike integrators within analog computers, neural integrators do not appear to be the most important component of every neural system. Thus, in spite of some similarities, neural systems are not analog computers.

6. How Spike Trains *Might Have Been* Strings of Digits

The mathematical modeling of neural processes can be traced back to the mathematical biophysics pioneered by Nicolas Rashevsky and his associates (Rashevsky 1938, Householder and Landahl 1945). They employed integral and differential equations to describe and analyze processes that might explain neural and cognitive phenomena. In their explanations, they used neither the notion of computation nor the mathematical tools of computability theory and computer science. Those tools were introduced into theoretical neuroscience by the youngest member of Rashevsky's group, Walter Pitts, in collaboration with Warren McCulloch.

Like Rashevsky's mathematical biophysicists, McCulloch and Pitts (1943) attempted to explain cognitive phenomena in terms of putative neural processes. But McCulloch and Pitts introduced concepts and techniques from logic and computability theory to model what is functionally relevant to the activity of neurons and neural nets, in such a way that (in our terminology) a neural spike can be treated mathematically as a digit, and a set of spikes can be treated as a string of digits.

McCulloch and Pitts's empirical justification for their theory was the similarity between digits and spikes—spikes *appear* to be discrete or digital, i.e., to a first approximation, their occurrence is an unambiguous event relative to the functioning of the system. This was the main motivation behind McCulloch's identification between spikes and atomic mental events, leading to his formulation of digital computationalism (Piccinini 2004). We will leave aside the question of whether spikes correspond to atomic mental events and limit our discussion to whether spikes are digits and whether sets of spikes are strings of digits.

The assumption that sets of spikes are strings of digits requires that spikes be concatenated. In the case of spike trains from a single neuron, the temporal ordering of spikes is a natural candidate for a concatenation relation. In the case of sets of spikes from different neurons occurring within well defined time intervals, a concatenation relation might be defined by first identifying a relevant set of neurons and then taking all spikes occurring within the same time interval as belonging to the same string. The set of neurons might be defined purely anatomically or by a combination of anatomical and functional criteria. For example, one could try to identify, electrophysiologically or histologically, which neurons are presynaptic and postsynaptic to which.

The suggestive analogy between spikes and digits—based on the all-or-none character of spikes—is far from sufficient to treat a spike as a digit, and even less sufficient to treat a set of spikes as a string of digits. In order to build their theory,

according to which neural activity is digital computation, McCulloch and Pitts made numerous other assumptions.

A crucial assumption was that a fixed number of neural stimuli are always necessary and sufficient to generate a neuron's pulse. They knew this was false: they explicitly mentioned that the "excitability" of a neuron does vary over time (McCulloch and Pitts 1943, p. 20-21). In fact, neurons exhibit both absolute and relative refractory periods as well as varying degrees of plasticity ranging from short to long term potentiation and depression. To this, it should be added that spiking is not deterministic but probabilistic. Indeed, the flow of ions through transmembrane channels has a stochastic component (White et al. 1998; Hille 2001), as does neurotransmitter release (Bennet and Kearns 2000; Hille 2001). Even when neurons fire with almost pacemaker-like regularity, their natural frequencies retain some variability (Le Bon-Jego and Raichle 2007; Yassin et al. 2010) due to natural fluctuations in physiological parameters which, in turn, affect the time at which the transmembrane potential reaches the firing threshold.

Another crucial assumption was that all neurons within a network are synchronized so that all relevant events—conduction of the impulses along nerve fibers, refractory periods, and synaptic delays—occur within temporal intervals of fixed and uniform length, which are equal to the time of synaptic delay. In addition, according to McCulloch and Pitts, all events within one temporal interval only affect the relevant events within the following temporal interval. This assumption makes the *dynamics* of the net discrete: it allows the use of logical functions of discrete inputs or states to fully describe the transition between neural events.

As McCulloch and Pitts pointed out, these assumptions are not based on empirical evidence. The reason for these assumptions was, presumably, that they allowed the mathematics of computation to describe, at least to a first approximation, what they presumed to be most functionally relevant to neural activity.

The similarity between digits and spikes can hardly be doubted. But there are significant dissimilarities as well, and there are further dissimilarities between spike sets—either spike trains from the same neuron or sets of more or less synchronous spikes from different neurons—and strings of digits. In 1943, relatively little was empirically known about the functional properties of neural spikes and spike trains. Since then, neurophysiology has made great progress, and it is past time to bring what is currently known about neural processes to bear on the question of whether they are digital computations.

Although McCulloch and Pitts's theory became fundamental to computer science and computability theory (Piccinini 2004), as a theory of neural activity it was soon abandoned in favor of more sophisticated and neurally plausible mathematical models such as that of Hodgkin and Huxley (1952). From a mathematical point of view, the models employed by theoretical neuroscientists after McCulloch and Pitts are closer to those of Rashevsky's original mathematical biophysics, in that they do not employ concepts and techniques from computability theory. Note that by "computability theory" we mean the theory of recursive functions and their computation (Davis et al. 1994); we do *not* include concepts such as Shannon's information and entropy

measures, which have been used extensively by computational neuroscientists over recent years (Rieke et al. 1999; Dayan and Abbott 2001; Borst and Theunissen 1999; Pereda et al. 2005). In the models currently employed by theoretical neuroscientists, spikes are not treated as digits, and spike sets are not treated as strings of digits. The rest of this paper is devoted to identifying principled reasons, based on empirical evidence, for why this is so.

The point of the exercise is not to beat a long-dead horse—McCulloch and Pitts’s theory. The point is that if we reject McCulloch and Pitts’s assumptions, which were the original motivation for calling neural activity digital computation, it remains to be seen whether there is any way to replace McCulloch and Pitts’s assumptions with assumptions that are both empirically plausible and allow us to retain the view that neural activity is digital computation in the relevant sense. If no such assumptions are forthcoming, digital computationalism needs to be abandoned.

7. Why Spikes Are Not Digits and Spike Sets Are Not Strings

We’ll now argue that current neuroscientific evidence suggests that typical neural processes are not digital computations. The structure of the argument will be as follows:

- (1) Digital computation is the manipulation of strings of digits [as per Section 2.2];
- (2) Neural spikes are not digits, and even if they were digits, spike sets would not be strings;
- (3) Therefore, the manipulation of spike sets is not digital computation.

Our argument challenges not only classical computationalism, which is explicitly committed to digital computation, but also any form of connectionist or neurocomputational theory that is either explicitly or implicitly committed to the thesis that neural activity is digital computation.

We stress that the present argument is not formulated in terms of the notion of representation. It rests on the notion of digits as discrete entities that can be concatenated into strings. The digits discussed in this paper may be vehicles for many types of representations. Our argument does not depend on which if any notion of representation is adopted. Specifically, it is indifferent to the distinctions between digital vs. analog (Maley 2010), localized vs. distributed, and “symbolic” vs. “sub-symbolic” representation (Rumelhart and McClelland 1986, Smolensky 1989). In fact, our argument is consistent both with representationalism and its denial. The truth value of representationalism is simply not affected by our argument.

For a class of particular events to constitute digits within a computing mechanism, it must be possible to type them in an appropriate way. In the next subsection we discuss the difficulties in doing so for spikes. For a set of digits to constitute a string, it must be possible to determine, at the very least, which digits belong to which strings. Later we will discuss the difficulties in doing so for sets of spikes.

7.1 Typing Spikes as Digits

Digits belong to finitely many types, which must be unambiguously distinguishable by the system that manipulates them. So for spikes and their absence to be digits, it must be possible to type them into finitely many types that are unambiguously distinguishable by neural mechanisms.

Spikes are all-or-none events; either they occur or not. So there might be two types of digits: the *presence* or *absence* of a spike at some time. This was McCulloch and Pitts's proposal. For this proposal to be adequate, more needs to be said about the time during which the presence or absence of spikes is to be counted.

The relevant time cannot be instantaneous. Otherwise, there would be uncountably many digits in any finite amount of time, which is incompatible with the notion of digit. Besides that, spikes do take some time (about 1 msec) to occur. McCulloch and Pitts's proposal was to divide time into intervals whose length was equal to synaptic delay. Their proposal suffers from a number of shortcomings. First, it assumes that synaptic delay is constant; recent studies show it to be variable (e.g., Lin and Faber 2002). Second, any time interval separating two spikes is assumed to be equal to an exact multiple of the master time interval, whereas there is no empirical evidence of any such master time interval. Third, in order to count the presence or absence of spikes from different neurons as belonging to the same computation, it assumes that neurons are perfectly synchronized, in the sense that their relevant time intervals begin and end in unison. The third assumption is also relevant to the concatenation of digits into strings, so we will discuss it in subsequent subsections. In what follows, we will focus on the first two assumptions.

The lack of evidence for the first two assumptions could be remedied by choosing a fixed time interval of unambiguous physiological significance. But there is no evidence that such precise, finite time intervals exist. Spiking is affected in varying degrees by events (such as input spikes) that occur at different times in the neuron's past history (e.g., Sheperd 1999, p. 126), and there is no known principled way of typing both relevant causes and the time intervals during which they occur into finitely many types. Moreover, spikes are *probabilistic* events.

By itself, that spikes are probabilistic need not undermine the analogy between spikes and digits, since digital computations may be probabilistic as well as deterministic. In a probabilistic digital computation, there are a finite number of possible outcomes, each of which has a certain probability of occurring. In other words, the sample space of a probabilistic digital computation is finite. This seems superficially analogous to the neural spiking, which occurs with a certain probability at a time (given a certain input).

But there is a crucial disanalogy. Whereas digital computational events are temporally discrete, spike times are continuous variables. In the case of a probabilistic digital computation, events occur within fixed functionally significant time intervals; this makes it possible to specify the probabilities of the finitely many outcomes effectively, outcome by outcome, so that each constitutes a digit. By contrast, there appear to be no functionally significant time intervals during which spikes are to be counted, and as a result, spikes must be counted over real time. Because of this, the probability that a spike occurs at any instant of real time is zero. In other words, the sample space for

spiking events during a time interval is uncountably infinite. Spike probabilities need to be given by probability densities, which specify the probability that a spike occurs within an infinitesimal time interval (Dayan and Abbott 2001, p. 24). In conclusion, since there can only be finitely many digit types but there are an uncountable number of possible spiking events, spiking events cannot be digits.

There is another, independent and perhaps deeper reason for denying that the presence or absence of a spike can constitute a digit. Until now, the discussion has been based on the implicit assumption that the physiological significance of the presence or absence of *individual* spikes is functionally relevant to the processing of neural signals. But this assumption is unwarranted in many cases. Although the presence or absence of spikes does affect neural systems differentially, individual spikes rarely seem to have unambiguous functional significance for the processing of neural signals (though see VanRullen et al. 2005).

First, neurons exhibit spontaneous activity, not triggered by an external stimulus; indeed, spontaneous neural activity accounts for about 75% of the brain's energy consumption. This spontaneous activity need not be stochastic: spontaneous firing of neocortical neurons has been shown to have regular, pacemaker-like activity (Le Bon-Jego and Yuste 2007). Some have suggested that neural signals consist in relatively small deviations from a background level of activity. This is what makes the hypothesis of a "default network", suggested by BOLD fMRI imaging studies, so compelling (Raichle et al. 2001; Zheng and Raichle 2010) and so controversial (Morcom and Fletcher 2007). Finally, even a noisy, stochastic spontaneous firing state can have some function; witness the role of noise in enhancing weak subthreshold signals in the nervous system via stochastic resonance (Douglass et al. 1993; Levin and Miller 1996; Moss et al. 2004).

Functional, noisy, or neither, spontaneous activity per se is not an objection to digital computationalism. For digital computations can depend on both inputs and internal states. In principle, spontaneous activity could be analogous to the internal state of a digital computing mechanism. But the internal states of digital computing mechanisms are such that their contribution to a computation can be described as digits (discrete states) and specified independently of the contribution given by the input. By contrast, spontaneous neural activity has all the characteristics already noted for spikes in general, which led to the conclusion that spikes are not digits.

Second, regardless of whether a spike is caused by spontaneous activity, a sensory input, or both, what appears to have functional significance is often not any individual spike, but rather the firing *rate* of a neuron. Consider the simple case of the crayfish photoreceptor, which fires at a rate of about 5 Hz in the dark, and speeds up to 30 Hz when exposed to visible light (Bruno and Kennedy 1962). In such clear cases of rate coding, individual spikes have no functional significance on their own. For that matter, many individual spikes might in principle be added to or removed from a spike train without appreciably altering its functional significance (Dayan and Abbott 2001, Chap. 1). In fact, neuroscientists typically assess the functional significance of neural

activity by computing average firing rates, namely by averaging the firing rates exhibited by a neuron over many similar trials.⁷

Thus, there are principled reasons to doubt that spikes and their absence can be treated mathematically as digits, to be concatenated into strings. But individual spikes are not the only candidates to the role of digits. Before abandoning the issue of digits, we should briefly look at two other proposals.

One possibility is that digits are to be found in precise patterns of several spikes. There have been serious attempts to demonstrate that some repeating triplets or quadruplets of precisely timed (≈ 1 msec) spikes from single neurons have functional significance. But these attempts have failed: rigorous statistical analysis indicates that both the number and types of precisely timed spike patterns are those that should be expected by chance (Oram et al. 1999). In fact, in many cases signals coming from individual neurons are too noisy to constitute the minimal units of neural processing; according to some estimates, the minimal signaling units in the human brain are of the order of 50-100 neurons (Shadlen and Newsome 1998).

Another possibility is that spike rates themselves should be taken to be digits of sorts. This was von Neumann's proposal (von Neumann 1958). Unlike the previous proposal, this one is based on variables that are functionally significant for the processing of neural signals. But this proposal raises more problems than it solves. Again, there is the problem of finding a significant time interval. Spike rates vary continuously, so there seems to be no non-arbitrary way to parse them into time intervals and count the resulting units as digits. It's also unclear how spike trains from individual neurons, understood as digits, and could be concatenated with other spike trains from other neurons to form strings. Perhaps most importantly, spike rates cannot be digits in the standard sense, because they cannot be typed into finitely many types of unambiguous physiological significance. As von Neumann noted, there is a limit to the precision of spike rates, so if we attempt to treat them as digits, there will be a significant margin of error as to which type any putative digit belongs to; the same problem occurs with individual spike timings. As a consequence, computability theory as normally formulated does not apply to spike rates—a different mathematical theory is needed. Von Neumann pointed out that he didn't know how to do logic or arithmetic over such entities as spike rates, and hence he didn't know how the brain could perform logic or arithmetic using such a system. As far as we know, no one after von Neumann has addressed this problem. And without a solution to this problem, von Neumann's proposal remains an empty shell.

7.2 Fitting Spikes into Strings

In the previous section, we saw that there is no reason to believe that spikes, or sets thereof, can be treated as the digits of a computation. But even if they could, it wouldn't follow that neural mechanisms perform computations in any useful sense. For although some trivial computations are defined over single digits or pairs of them (i.e.,

⁷ Individual animals must react quickly to external stimuli, however, and do not have the luxury of averaging trials; this is one argument used for the importance of temporal coding (VanRullen et al., 2005)

those performed by ordinary logic gates), nontrivial computations are defined over strings of digits. Strings of digits are entities with finitely many digits concatenated together, in which there is no ambiguity as to which digits belong to a string. If sets of spikes were to count as strings of digits, there should be a way to unambiguously determine which spikes belong to which string. But there is no reason to believe that this can be done. The two natural candidates for strings are sets of spikes from synchronous neurons and spike trains from individual neurons.

7.2.1 Strings and Synchronous Spikes

In the case of sets of synchronous spikes, the concatenation relation might be given by spatial and temporal contiguity within a structure defined by some combination of anatomical and functional properties. For instance, synchronous spikes from a cortical column may be seen as belonging to the same string. Implicitly, this is how inputs and outputs of many connectionist systems are defined.

A first difficulty with this proposal is the looseness of the boundaries between neural structures. It doesn't appear that neural structures, such as cortical columns, are defined precisely enough that one can determine, neuron-by-neuron, whether each neuron belongs to one structure or another. The boundaries are fuzzy, so that neurons can be recruited to perform different functions as they are needed, and the functional organization of neural structures is reshaped as a result (Recanzone, Merzenich, and Schreiner 1992, Elbert and Rockstroh 2004). This first problem may not be fatal, however. Perhaps the boundaries between neural structures are fuzzy, and yet at any time, there is a way to assign each spike to one and only one string of spikes synchronous to it.

But there is a fatal problem: synchrony between spikes is a matter of degree. Just as there is no meaningful way to divide the activity of a neuron into discrete functionally significant time intervals (see Section 7.1), there is no meaningful way to divide the activity of neural ensembles into discrete functionally significant time intervals to determine in an absolute manner whether their spikes are synchronous. Even if such absolute time determinations were possible, the inherent noisiness in neural firing times would result in a varying degree of synchrony over time. Synchrony between spikes has been studied intensely by the nonlinear dynamics community over the past few decades, in the context of the synchronization of coupled oscillators (Pikovsky et al. 2003) and has been applied to the analysis of neural pathologies such as epilepsy (Wong et al. 1986; Uhlhaas and Singer 2006) and Parkinson's disease (Tass et al. 2003); it may also play a role in normal brain functions such as attention (Steinmetz et al. 2000; Fries et al. 2001, 2008; Roy et al. 2007). But the interesting question about neural synchrony is not *whether*, in an absolute sense, there is synchrony in a neural process, but rather *how much* synchrony there is. The typical definition of synchrony used within the neurophysiological community is that of *stochastic phase synchronization*, which measures the degree to which the phase difference between neurons remains constant; thus, neurons can be synchronized even with a (relatively) constant phase lag (Pikovsky et al. 2003).

As a result, even if neural spikes were digits (which they aren't), sets of synchronous spikes would not be strings of digits.

7.2.2 Strings and Spike Trains

In a spike train from a single neuron, the concatenation relation may be straightforwardly identified with the temporal order of the spikes. Nevertheless, in order to treat a spike train as a string, its beginning (first spike) and end (end spike) must be identified unambiguously. Again, two independent problems undermine this proposal.

The first problem is that the question of which spikes belong to a string appears to be ill defined. Generally, spike trains are the combined effect of spontaneous activity of a neuron plus that neuron's response to its inputs. There doesn't seem to be any functionally significant way to divide spontaneous activity into discrete units of independent functional significance. Likewise, there doesn't seem to be any useful way to divide the neural responses to inputs into subsets with unambiguous beginnings and ends. The same point applies to the two components taken together.

The second problem is that given the amount of noisiness in neural activity, individual spike trains are not useful units of functional significance. What has functional significance is not a specific train of spikes, each of which occurs during a particular time interval, but something held in common by many different spike trains that are generated by the same neuron in response to the same stimulus. The only known way to find useful units of functional significance is to average spike trains from single neurons over many trials and use average spike trains as units of functional significance.

For these reasons, even if neural spikes could be treated as digits (which they can't), spike trains could not be meaningfully treated as strings of digits.

7.3. What About the Neural Code?

A final relevant issue is the relationship between neural codes and (digital) computational codes. In both cases, the same term is used, which suggests an analogy between the two. Is there such an analogy?

The expression "neural code" usually refers to the functional relationship between properties of neural activity and variables external to the nervous system (e.g., see Dayan and Abbott 2001). Roughly speaking, the functional dependence of some property of a neural response on some property of an environmental stimulus is called neural encoding, whereas the inference from some property of a neural response to some property of the stimulus is called neural decoding. Neural encoding is a measure of how accurately a neural response reflects a stimulus; neural decoding is a measure of how accurately a neural mechanism can respond to a stimulus.

By contrast, in the theory of (digital) computation, "coding" refers to one-one mappings between classes of strings of digits (e.g., see Rogers 1967, pp. 27-29). Given each member of a (countably infinite) class of strings over some alphabet, any algorithm that produces a corresponding member of a different class of strings (possibly over a different alphabet) is called an encoding. This kind of encoding is always fully reversible:

by definition, for any encoding algorithm there is a decoding algorithm, which recovers the inputs of the encoding algorithm from its outputs.

Neural coding is only loosely analogous to computational coding. Both are relationships between two classes of entities, and both give procedures for retrieving one class of entities from the other. But the disanalogies are profound. Whereas neural encoding is only approximate and approximately reversible, (digital) computational coding is (by definition) exact and exactly reversible. Whereas neural encoding is asymmetric (neural responses encode stimuli but not vice versa), computational coding is symmetric (if a encodes b , then b encodes a). Most importantly, whereas neural encoding relates physical quantities in the environment to neural responses, neither one of which is likely to be accurately described as a class of digital strings, computational coding is a relationship between classes of digital strings. For these reasons, neural coding should not be confused with (digital) computational coding. The existence of a neural code has no consequences on whether neural mechanisms are digital computational.

7.4. Against Digital Computationalism

Given current evidence, the most functionally significant units for the purpose of understanding the processing of neural signals are neural spikes. We have argued that neither spike trains from single neurons nor sets of synchronous spikes from multiple neurons are viable candidates for strings of digits. Without strings, we can't identify operations defined over strings. And without operations defined over strings, we can't have digital computations.

The discrete nature of spikes was the main empirical motivation for digital computationalism. Once the original assumptions about spikes that supported digital computationalism are rejected, the burden is on supporters of digital computationalism to find evidence that some neural variables are suited to being treated as strings of digits, and that this has something to do with explaining mental phenomena.

8. Some Consequences

While few neuroscientists take typical neural processes to be digital computations, many psychologists and philosophers are still building their theories of cognition using constructs that rely either implicitly or explicitly on digital computation, and some of them still defend digital computationalism (e.g., Fodor 2008, Gallistel and King 2009, Schneider 2011). Moreover, there remains considerable confusion in the literature—including the neuroscience literature—about what computationalism does and does not entail. We will now examine some widespread fallacies. After that, we will reiterate that understanding neural computation requires specially designed mathematical tools (Dayan and Abbott 2001, Ermentrout and Terman 2010) rather than the mathematics of analog or digital computation. We will conclude with a plea to psychologists who rely on non-neural notions of computation to replace or reinterpret their theories in terms of neural computation.

8.1 Avoiding Some Fallacies

Computationalism properly so called is often conflated with other theses that are weaker than or logically independent of it. At least two issues are relevant here.

The first issue has to do with the relationship between computationalism and representationalism. Many authors believe that the mind (or brain) has semantic content, and therefore that the mind (or brain) computes: “It is useful ... to class certain functions in the brain as computational because nervous systems *represent* and they respond on the basis of representations” (Churchland, Koch, and Sejnowski 1990, 48; emphasis original). These authors typically believe that computation requires representation.

We agree that many spike trains are representations, at least in the weak sense that they reliably correlate with variables external to the nervous system. Much theoretical neuroscience is devoted to studying how these correlations originate and how neural representations are processed. We also argued that processing representations entails computing. But the converse does not hold, because computation need not operate on representations.

The second issue has to do with the relationship between computationalism and mechanistic, or functional, explanation. Computationalism and mechanism have been run together in the literature. For example, David Marr dubbed the mathematical analysis of the functions of neural mechanisms “computational theory,” by which he meant theory of neural computations (Marr 1982). From this, it follows that neural processes are computations. But thus understood, computationalism ceases to be an empirical hypothesis about the specific functional nature of neural processes, to become a trivial consequence of the definition of mechanistic explanation. Suppose, instead, that we begin our investigation with a rich notion of mechanistic explanation (see Craver 2007). Under such a notion, computation is one kind of mechanistic process among others. Then, we may ask—with McCulloch and Pitts and many other computationalists—whether neural processes are computations and in which sense, and, as we argued, we can find empirical evidence that neural computation is *sui generis*.

Computationalism is often used to derive various consequences, which may or may not follow from digital computationalism but certainly do not follow from generic computationalism: (i) that in principle we can capture what is functionally relevant to neural processes in terms of some formalism taken from (digital) computability theory (such as Turing machines), (ii) that it is possible to design computer programs that are functionally equivalent to neural processes in the sense in which computer programs can be computationally equivalent to each other, (iii) that the study of neural (or mental) computation is independent of the study of neural implementation, (iv) that the Church-Turing thesis applies to neural activity in the sense in which it applies to digital computers. None of these alleged consequences of computationalism follow from the view that neural activity is computation in the generic sense.

If neural activity is not digital computation, as we have argued, then there is no reason to suppose that formalisms that manipulate strings of digits can capture what is functionally relevant to neural processes. They might be used to build computational

models of neural processes, but the explanation of neural processes will be given by the constructs of theoretical neuroscience, not computability theory.

The relationship between computer programs that model neural processes and the processes they simulate is significantly different from the relationship between computer programs that simulate each other. When a computer program simulates another program, (some aspect of) the computations of the simulating program encode (in the sense of computer science) the computations of the simulated program. By contrast, when a computer program implements a mathematical model of a neural process, the computations of the simulating program are a means for generating approximate representations of the states that the mathematical model attributes to the neural process. This kind of model attempts to capture something essential to a neural process based on what is known about the process by employing suitable abstractions and idealizations. There is no sense in which such modeling work can proceed independently of the empirical study of neural processes. There is no room here for an in-depth treatment of the methodology of computational modeling (Humphreys 2004, Craver 2006, Piccinini 2007b, Winsberg 2010). For present purposes, what matters is that computational modeling is a different activity, with different rules, from the simulation of one computer program by another program.

Finally, the Church-Turing thesis is a very complex issue, and there is no room here to do it justice. For present purposes, the following brief remark will suffice. To a first approximation, the Church-Turing thesis states that any function that is computable in an intuitive sense is computable by some Turing machine. Many authors have argued that the Church-Turing thesis proves computationalism to be true (e.g., Churchland 2007). But as recent scholarship has shown, this argument commits the fallacy of conflating mechanism and a form of computationalism restricted to the computations that fall under the Church-Turing thesis (Copeland 2000, Piccinini 2007c).

When mechanism and computationalism are kept distinct, as they should be, two separate questions about neural computation arise. One question—the most discussed in the literature—is whether neural computations are computable by Turing machines. If McCulloch and Pitts's theory of the brain (or a similar theory) were correct, then the answer would be positive. Another possibility is that neural computations are digital but more powerful than those of Turing machines—neural mechanisms might be hypercomputers (Copeland 2000, Bringjord 1998, Siegelmann 2003). The more fundamental question, however, is whether neural activity is digital computation in the first place. If, as we have argued here, neural activity is *not* digital computation, then the question of whether neural computation is computable by Turing machines doesn't even arise. Of course, there remains the question of whether computing humans can do more than Turing machines. People can certainly perform computations, as anyone who has learned basic arithmetic can attest. And presumably there are neural processes that explain human computing. But there is no evidence that humans are computationally more powerful than Turing machines. No human has ever been able to compute functions, such as the halting function, that are not computable by Turing machines. In this limited sense, the Church-Turing thesis appears to be true.

8.2 Neural Computation is Sui Generis

Given the present state of the art, a defense of digital computationalism (or analog computationalism, for that matter) as a general account of neural computation is not only implausible—it is unmotivated. For unlike in 1943, when digital computationalism was initially proposed, today we have an advanced science of neural mechanisms and processes, based on sophisticated mathematical concepts and techniques that are, in turn, grounded in sophisticated experimental concepts and techniques. We are referring to what is often called theoretical or computational neuroscience. Theoretical neuroscientists build mathematical models of neural mechanisms and processes and use them to explain neural and cognitive phenomena. Philosophers and psychologists have unduly neglected the development of this discipline. Specifically, they have rarely noticed that in realistic mathematical models of neural processes (which can be tested against experiments), the explanatory role of (digital) computability theory and (digital) computer design is nil.

In light of this, why do so many neuroscientists (e.g., Hopfield 1982, Churchland and Sejnowski 1992, Shadlen and Newsome 1998, Koch 1999) profess to believe that neural activity is computation? We think the view that captures what most of them mean, or at least what the empirical evidence supports, is *generic* computationalism, without any commitment to digital or analog computationalism.

8.3 From Cognitive Science to Cognitive Neuroscience

If neural computation is *sui generis*, what happens to computational theories—many of which rely on digital computation—in psychology? Mental models (e.g., Johnson-Laird 1983, 2010), production systems (e.g., Newell 1990), and theories (e.g., Murphy 2002) are examples of the many computational constructs employed by psychologists to explain cognitive phenomena. For over sixty years, various forms of *digital* computationalism—either classical or connectionist—have provided the background assumption against which many computational constructs in psychology have been interpreted and justified. Computational psychological theories are legitimate, the assumption goes, because the digital computations they postulate can be realized by neural computations. But if neural computations are not digital (or if digital neural computations are an exception rather than a rule), what should we do with computational psychological theories?

One tempting response is that psychological explanations remain unaffected because they are *autonomous* from neuroscientific explanations. According to a traditional view, psychological theories are not directly constrained by neural structures; psychology offers functional analyses, not mechanistic explanations like neuroscience (Fodor 1968, Cummins 1983). But on closer examination, functional analysis turns out to be an elliptical or partial mechanistic explanation (Piccinini and Craver 2011). Thus, psychological explanation is not autonomous from neuroscientific explanation, because psychological explanation is a type of mechanistic explanation, directly constrained by neural structures.

Another tempting response is to eliminate psychological explanations, but that would be premature. Computational theories in psychology are often our current best way of capturing and explaining psychological phenomena.

What needs to be done, instead, is to gradually reinterpret or replace computational constructs in psychology in terms of theoretical constructs that can be realized by known neural processes, such as the spike trains of neuronal ensembles. The shift that psychology is currently undergoing, from classical cognitive psychology to cognitive neuroscience, goes in this direction. Much groundbreaking work in psychology over the past two decades may be seen as contributing to this project (e.g., Anderson 2007, Barsalou 1999, Gazzaniga 2009, Kalat 2008, Kosslyn et al. 2006, O'Reilly and Munakata 2000, Posner 2004). We are on our way to explaining more and more neural and cognitive phenomena in terms of neural mechanisms and processes. There is no reason why we should restrict our rich discipline of theoretical neuroscience and psychology to the narrow and incorrect framework of digital computationalism.

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