f. The total fine structure correction

We may write the total fine-structure correction as

\[ E_{FS} = E_T^{(1)} + E_{SO}^{(1)} \]

where it is understood that if \( \ell = 0 \) then \( E_{SO}^{(1)} \) represents the Darwin term. We have then

\[ E_{FS} = -\frac{1}{2}\mu c^2 \frac{\alpha^2}{n^2} \left[ \frac{n}{(j + \frac{1}{2})} - \frac{3}{4} \right] = E_n^{(0)} \frac{\alpha^2}{n^2} \left[ \frac{n}{(j + \frac{1}{2})} - \frac{3}{4} \right] \]

Notice that, although the three separate contributions depend on \( \ell \), the total shift, \( E_{FS} \), does not. It depends only on \( j \) the total angular momentum quantum number. The figure below shows the different splittings for the \( n = 2 \) non-relativistic level as well as the total splitting for \( n = 2 \).

The magnitudes of the total fine structure splitting of the non-relativistic \( n = 1, 2 \) and 3 levels are shown in the figure below.
Note that our estimate of $10^{-4}$ times the non-relativistic energy, 10 eV $\approx$ 80650 cm$^{-1}$, was reliable. We also examine the splittings due to each of the terms.