CHAPTER 2
ANGULAR MOMENTUM - TWO SOURCES

a. Introduction

So far we have discussed a single source of angular momentum, that is, a single angular momentum vector. Even a single particle can, however, have associated with it two angular momenta, for example, spin and orbital angular momenta. Thus, there are a number of quantized variables that describe the system. For two angular momenta corresponding to the two operators, \( J_1 \) and \( J_2 \), there are four quantum numbers that might be used to describe the system, \((j_1, m_{j1})\) and \((j_2, m_{j2})\). In addition, the total angular momentum operators

\[
J = J_1 + J_2 \quad \text{and} \quad J_z = J_{1z} + J_{2z}
\]

having quantum numbers \((j, m_j)\) can also be considered. What is needed is to find sets of commuting operators that will provide a suitable description of the system.

We will find that there are two sets of mutually commuting operators that lead to good quantum numbers that describe a given system. Each set of operators leads to a different set of angular momentum eigenkets and eigenvalues. Each set of eigenfunctions constitute a basis set upon which a wave function may be expanded. Thus, the conditions of the problem determine which basis set is the most convenient. Moreover, we would like to know how to convert from one basis set to the other.

There are two fundamental questions that can be asked:

1. What do the commutation relations for angular momentum imply about the \textit{total} angular momentum of the system?

2. If the angular momentum state of one part of the system is fully specified, can the state of the other part be specified simultaneously?