The Stern-Gerlach Experiment

The Stern-Gerlach experiment had monumental consequences for the development of the modern quantum theory. It showed definitively that angular momentum was quantized, and, in particular, intrinsic angular momentum was quantized. A schematic diagram of the apparatus and the result are shown in the figure.

The original experiment was performed with Ag atoms (total spin 1/2). We may imagine that the experiment is being performed with electrons and ignore the effects of the magnetic field on the charged electrons.

If the electrons emerging from the oven are "unpolarized", i.e. if the spins are randomly oriented, then, from the pattern on the screen, we have

\[ |\psi\rangle = \frac{1}{\sqrt{2}} |\alpha\rangle + \frac{1}{\sqrt{2}} |\beta\rangle \]

where SGz represents a Stern-Gerlach device with the inhomogeneous magnet field oriented in the z-direction. In other words, the SGz device sorts out the particles in the beam according to their z-components of intrinsic angular momentum. The wave function representing the unpolarized incoming electrons is

Recall that \( |\alpha\rangle \) and \( |\beta\rangle \) are eigenkets of \( S_z \) and we are defining the direction of the magnetic field to be the z-direction. Now, suppose we block one of the emerging
beams and pass the remaining one through another SGz device. The result can be represented pictorially.

In other words, the particles that constitute the remaining beam, those in the $|\alpha>\,$ eigenstate, remain in the $|\alpha>\,$ eigenstate, as expected. Suppose, however, that instead of passing the output beam of the first SGz device into another SGz device we pass it through an SGx device, a Stern-Gerlach apparatus having the magnetic field oriented at right angles to the field in the first apparatus. Pictorially, we have

To determine the fate of the pure $|\alpha>\,$ beam when it passes through the SGx device we must express $|\alpha>\,$ in terms of $|\pm>\,$. We have already found $|\pm>\,$ in terms of $|\alpha>\,$ and $|\beta>\,$.

$$|\pm>\,_x = \frac{1}{\sqrt{2}} |\alpha> + \frac{1}{\sqrt{2}} |\beta> \quad \& \quad |\mp>\,_x = \frac{1}{\sqrt{2}} |\alpha> - \frac{1}{\sqrt{2}} |\beta>$$

Adding these equations we eliminate $|\beta>\,$ and solve for $|\alpha>\,$

$$|\alpha> = \frac{1}{\sqrt{2}} |\mp>\,_x + \frac{1}{\sqrt{2}} |\pm>\,_x$$

Thus, the SGx device sorts the particles in the beam according to their $x$-components of spin. The $|\alpha>\,$ beam is split into two equal beams.
Now, what happens if we block the $|\rightarrow_x\rangle$ beam and pass the $|\rightarrow\rangle_x$ beam through a SGz device? Since

$$|\rightarrow\rangle_x = \frac{1}{\sqrt{2}} |\alpha\rangle + \frac{1}{\sqrt{2}} |\beta\rangle$$

the beam will be split into two equal parts again. The situation is represented graphically below.