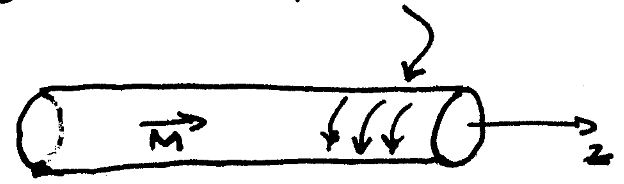
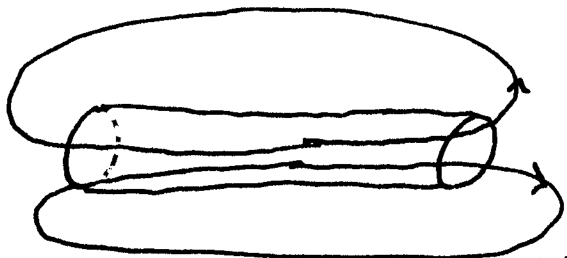
6.9 & 6.14

$\vec{J}_b = \nabla \times \vec{M} = 0; \quad \vec{K}_b = \vec{M} \times \hat{n} = M\hat{z} \times \hat{s} = M\hat{\varphi}$




Since the only current that flows is in the $\hat{\varphi}$ direction this is like a finite solenoid. From the direction of $\vec{K}_b$ the B-field must be in the z-direction inside the magnet. B will have an s-component because the solenoid is finite in length.

We have



B-field

Now, how about $\vec{H}$?

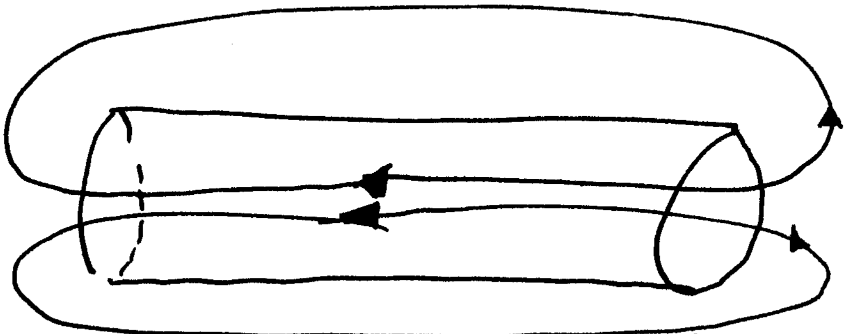
$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$ $\therefore$ We must determine the direction of $\vec{H}$ by examining the relative magnitudes of $\frac{1}{\mu_0}\vec{B}$ and $\vec{M}$ because of the minus sign. Outside $\vec{H} = \frac{1}{\mu_0}\vec{B}$ because $\vec{M} = 0$ so it is in the same direction as $\vec{B}$

Inside $\vec{B} < \vec{B}_{\substack{\text{infinite} \\ \text{solenoid}}} = \mu_0 n I$ because B-lines emerge from the ends. In other words the B-line density is lower in the finite solenoid than in an infinite solenoid.

Let's temporarily assume $B = \mu_0 n I$ where $nI \to K_b = M$ for this permanent magnet. Then

$$\vec{H}_{inside} = \frac{1}{\mu_0}(\mu_0 M) - M \equiv 0$$

Because $B_{inside} < \mu_0 M$, $H_{inside}$ must be in the opposite direction of M and $B_{inside}$.



H-field

H is discontinuous at the end caps of the cylinder. It is as though there are sources of H which would mean that $\nabla \cdot \vec{H} \neq 0$ and that there is fictitious magnetic charge on the end caps.

$$\nabla \cdot \vec{H} = \nabla \cdot \left( \frac{1}{\mu_0} \overbrace{\vec{B}}^{\to \text{zero}} - \vec{M} \right)$$

$\nabla \cdot (\vec{H} + \vec{M}) = 0$ which shows again that $\vec{H}$ and $\vec{M}$ are in opposite directions.