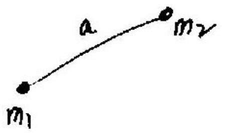
8-5



Consider the motion to be that of a single particle of mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$. The force of attraction is gravitational

$$F_{GRAV} = G\frac{m_1 m_2}{a^2} = \mu\frac{v^2}{a} = \mu a \omega^2$$
$$= \mu a\left(\frac{2\pi}{\tau}\right)^2$$

$$\therefore \quad a = \left[\frac{G m_1 m_2 \tau^2}{4\pi^2 \mu}\right]^{1/3}$$

After the circular motion stops the particle of mass μ moves toward the force center. From conservation of energy

$$-G\frac{m_1 m_2}{a} = TME = \tfrac{1}{2} m \dot{x}^2 - G\frac{m_1 m_2}{x}$$

so $\dot{x} = \left[\frac{2Gm_1m_2}{\mu}\left(\frac{1}{x} - \frac{1}{a}\right)\right]^{1/2} = \frac{dx}{dt}$

$$t = \int dt = -\int_a^0 dx \Big/ \left[\frac{2Gm_1m_2}{\mu}\left(\frac{1}{x} - \frac{1}{a}\right)\right]^{1/2}$$

time increases as x decreases.

Rearranging

$$t = \sqrt{\frac{a\mu}{2Gm_1m_2}}\,\underbrace{\int_0^a \frac{\sqrt{x}}{\sqrt{a-x}}\,dx}_{I}\,; \quad \text{Let } x = y^2;\ dx = 2y\,dy$$

$$I = 2\int_0^{\sqrt{a}} \frac{y^2}{\sqrt{a-y^2}}\,dy = 2\left[\frac{y\sqrt{a-y^2}}{2} - \frac{a}{2}\sin^{-1}\left(y/\sqrt{a}\right)\right]_0^{\sqrt{a}}$$
$$= -\frac{\pi a}{2}$$

$$\therefore \quad t = \sqrt{\frac{a\mu}{2Gm_1m_2}}\,\frac{a\pi}{2} = \tau/4\sqrt{2}$$