

8-31 The condition for stability is 8.93

$$\frac{F'(p)}{F(p)} + \frac{3}{p} > 0 \quad \text{with } F(p) = -\frac{h}{p^2} - \frac{h'}{p^4}$$

$$\frac{F'(p)}{F(p)} + \frac{3}{p} = \frac{\frac{2h}{p^3} + \frac{4h'}{p^5}}{-\frac{h}{p^2} - \frac{h'}{p^4}} + \frac{3}{p}$$

$$= -\frac{2hp^2 + 4h'}{hp^3 + h'p} + \frac{3}{p}$$

$$= \frac{-2hp^3 - 4h'p + 3hp^3 + 3h'p}{p(hp^3 + h'p)}$$

$$= \frac{hp^3 - h'p}{p(hp^3 + h'p)} = \frac{1}{p} \frac{(hp^2 - h')}{(hp^2 + h')}$$

The denominator is positive definite so the condition for this quantity to be positive is

$$hp^2 - h' > 0 \Rightarrow hp^2 > h'$$

Note that it was not necessary to calculate  $p$ .