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Equation of the orbit $\frac{a}{r} = 1 + \epsilon \cos \theta$

We require \dot{r} so take the time derivative of the equation of the orbit.

$$\begin{aligned}\frac{\dot{r}}{r^2} &= \frac{\epsilon}{a} \dot{\theta} \sin \theta \\ &= \frac{\epsilon l}{\mu r^2} \sin \theta\end{aligned}$$

$$\begin{aligned}(8.45) \quad \tau &= \frac{2\mu}{l} A \\ &= \frac{2\mu}{l} (\pi a b)\end{aligned}$$

but, use (8.43) $b = \frac{a}{\sqrt{1-\epsilon^2}}$ so

$$\tau = \frac{2\mu}{l} \pi a \left(\frac{a}{\sqrt{1-\epsilon^2}} \right) \Rightarrow \frac{l}{a} = \frac{2\pi\mu a}{\sqrt{1-\epsilon^2}}$$

From the \dot{r} equation above occurs when $\theta = \frac{\pi}{2}$

$$\begin{aligned}|\dot{r}|_{\text{max}} &= \frac{\epsilon}{\mu} \cdot \frac{l}{a} = \frac{\epsilon}{\mu} \cdot \frac{2\pi\mu a}{\sqrt{1-\epsilon^2}} \\ &= \frac{2\pi a \epsilon}{\sqrt{1-\epsilon^2}}\end{aligned}$$