8-23 Equation of the orbit $\frac{\alpha}{r} = 1 + \epsilon\cos\theta$

We require $\dot{r}$ so take the time derivative of the equation of the orbit.

$$\frac{\dot{r}}{r^2} = \frac{\epsilon}{\alpha}\dot{\theta}\sin\theta$$

$$= \frac{\epsilon l}{\alpha\mu r^2}\sin\theta$$

(8.45) $$\tau = \frac{2\mu}{l}A$$

$$= \frac{2\mu}{l}(\pi ab)$$

but, use (8.43) $b = \frac{\alpha}{\sqrt{1-\epsilon^2}}$ so

$$\tau = \frac{2\mu}{l}\pi a\left(\frac{\alpha}{\sqrt{1-\epsilon^2}}\right) \Rightarrow \frac{l}{\alpha} = \frac{2\pi\mu a}{\tau\sqrt{1-\epsilon^2}}$$

From the $\dot{r}$ equation above occurs when $\theta = \pi/2$

$$|\dot{r}|_{max} = \frac{\epsilon}{\mu}\cdot\frac{l}{\alpha} = \frac{\epsilon}{\mu}\cdot\frac{2\pi\mu a}{\tau\sqrt{1-\epsilon^2}}$$

$$= \frac{2\pi a\epsilon}{\tau\sqrt{1-\epsilon^2}}$$