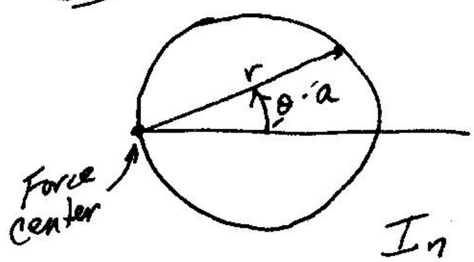
8-11



Equation of the orbit:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{\ell^2} F(r)$$

In this case the equation of the orbit is

$$r = 2a\cos\theta \implies \cos\theta = \frac{r}{2a}$$

The orbit equation becomes

$$\underbrace{\frac{1}{2a}\frac{d^2}{d\theta^2}\left(\frac{1}{\cos\theta}\right)} + \frac{1}{2a}\frac{1}{\cos\theta} = -\frac{4a^2\mu}{\ell^2}\cos^2\theta\, F(r)$$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{\cos\theta}\right) = \frac{d}{d\theta}\left(\frac{\sin\theta}{\cos^2\theta}\right) = \frac{1}{\cos\theta} + \frac{2\sin^2\theta}{\cos^3\theta}$$

so

$$\frac{1}{\cos\theta} + \frac{2\sin^2\theta}{\cos^3\theta} + \frac{1}{\cos\theta} = -\frac{8a^3\mu}{\ell^2} F(r)\cos^2\theta$$

solve for $F(r)$

$$F(r) = -\frac{\ell^2}{8a^3\mu}\frac{1}{\cos^2\theta}\left[\frac{2}{\cos\theta}\left(1 + \frac{\sin^2\theta}{\cos^2\theta}\right)\right]$$

$$= -\frac{\ell^2}{8a^3\mu}\frac{2}{\cos^5\theta}$$

$$= -\frac{\ell^2}{8a^3\mu}\left(\frac{2a}{r}\right)^5 = -\frac{8a^2\ell^2}{\mu}\cdot\frac{1}{r^5}$$