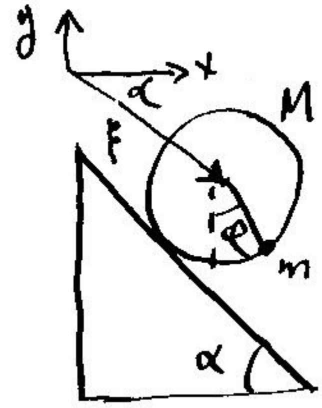
7-9



Note that this problem would be a bit more complicated if the inclined plane were moving.

For disk:

$x = \xi\cos\alpha \, ; \; y = -\xi\sin\alpha$

For bob:

$x = l\sin\varphi + \xi\cos\alpha$

$y = -l\cos\varphi - \xi\sin\alpha$

$$T = T_{disk} + T_{bob} = \frac{1}{2}M\dot{\xi}^2 + \frac{1}{2}I\dot{\varphi}^2 + \frac{1}{2}m\left(\dot{x}_{bob}^2 + \dot{y}_{bob}^2\right)$$

$$= \frac{1}{2}(m+M)\dot{\xi}^2 + \frac{1}{2}I\dot{\varphi}^2 + \frac{1}{2}m\ell^2\dot{\varphi}^2 + m\ell\dot{\varphi}\dot{\xi}\cos(\varphi+\alpha)$$

$$U = U_{disk} + U_{bob} = Mg\,y_{disk} + mg\,y_{bob}$$

$$= -(M+m)g\xi\sin\alpha - mg\ell\cos\varphi$$

Now $\xi = R\varphi$ & $I = MR^2$

$$L = T - U = \left(\frac{3}{4}M + \frac{1}{2}m\right)\dot{\xi}^2 + \frac{1}{2}m\ell^2\dot{\varphi}^2 + m\ell\dot{\varphi}\dot{\xi}\cos(\varphi+\alpha)$$

$$+ (M+m)g\xi\sin\alpha + mg\ell\cos\varphi$$

Which leads to

$$\left(\frac{3}{2}M + m\right)\ddot{\xi} - (M+m)g\sin\alpha + m\ell\left[\ddot{\varphi}\cos(\varphi+\alpha) - \dot{\varphi}^2\sin(\varphi+\alpha)\right] = 0$$

$$\ddot{\varphi} + \frac{1}{\ell}\ddot{\xi}\cos(\varphi+\alpha) + \frac{g}{\ell}\sin\varphi = 0$$