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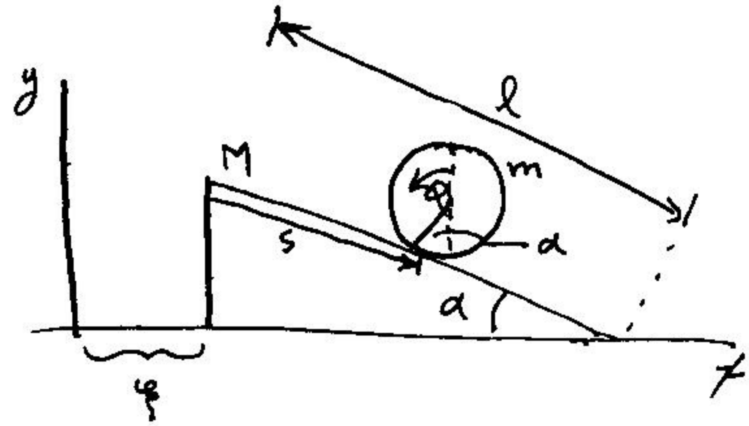

First write the x, y coordinates of the center of the hoop.

$$x = \xi + s\cos\alpha + R\sin\alpha \; ; \quad \dot{x} = \dot{\xi} + \dot{s}\cos\alpha$$

$$y = (l - s)\sin\alpha + R\cos\alpha \; ; \quad \dot{y} = -\dot{s}\sin\alpha$$

$$T_{hoop} = \tfrac{1}{2} m(\dot{x}^2 + \dot{y}^2) + \tfrac{1}{2} I \dot{\phi}^2 = \tfrac{1}{2}\left[m(\dot{\xi} + \dot{s}\cos\alpha)^2 + \dot{s}^2\sin^2\alpha\right] + \tfrac{1}{2}(mR^2)\frac{\dot{s}^2}{R^2}$$

$$= \tfrac{1}{2} m\left[2\dot{s}^2 + \dot{\xi}^2 + 2\dot{\xi}\dot{s}\cos\alpha\right]$$

$$T = T_{total} = T_{hoop} + \tfrac{1}{2} M \dot{\xi}^2$$

$$U = mgy = mg\left[(l - s)\sin\alpha + R\cos\alpha\right]$$

$L = T - U$ from which we get two Lagrange eqns:

$$\left\{\begin{aligned} 2m\ddot{s} + m\ddot{\xi}\cos\alpha - mg\sin\alpha &= 0 \\ (m+M)\ddot{\xi} + m\ddot{s}\cos\alpha &= 0 \end{aligned}\right\}$$

These are easily uncoupled to get $\ddot{x}$ and $\ddot{\xi}$.

More interesting part of the problem:

The second of these equations can be re-written as:

$$\frac{d}{dt}\Big[\underbrace{(m+M)\dot{\xi}}_{\text{x-component of linear momentum of the system.}} + \underbrace{m\dot{s}\cos\alpha}_{\text{x-component of linear momentum of the hoop wrt the inclined plane.}}\Big] = 0$$

$\therefore$ x-component of total momentum is conserved.

y-component of total momentum not conserved.