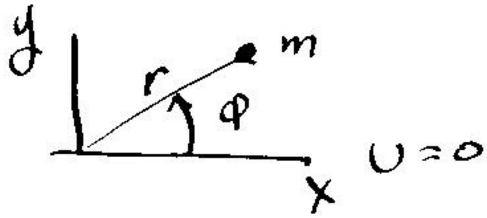
1-5



$x = r\cos\varphi$
$y = r\sin\varphi$

$$T = \frac{1}{2}m(\dot{x}^2+\dot{y}^2) = \frac{1}{2}m(\dot{r}^2+r^2\dot{\varphi}^2)$$

Since $f = -Ar^{\alpha-1}$ the potential energy associated with this force is $U(r) = \frac{A}{\alpha}r^\alpha$

$$\therefore U(r) = \frac{A}{\alpha}r^\alpha + mgy$$
$$= \frac{A}{\alpha}r^\alpha + mgr\sin\varphi$$

$$L = T - U = \frac{1}{2}m(\dot{r}^2+r^2\dot{\varphi}^2) - \frac{A}{\alpha}r^\alpha - mgr\sin\varphi$$

Lagrange eqn for r:

$$mr\dot{\varphi}^2 - Ar^{\alpha-1} - mg\sin\varphi - m\ddot{r} = 0$$
(don't need this)

Lagrange eqn for φ:

$$-mgr\cos\theta - \frac{d}{dt}\underbrace{(mr^2\dot{\varphi})}_{\text{ang. momentum}} = 0$$

Since $\frac{d}{dt}(\text{ang. mom.}) \neq 0$

ang mom is not conserved.

The reason is that the gravitational force produce an external torque.

Another way of saying this is the the potential energy is not a central potential, ie.

$$U(r,\varphi) = \frac{A}{\alpha}r^\alpha + mgr\sin\varphi \neq U(r\text{ only})$$