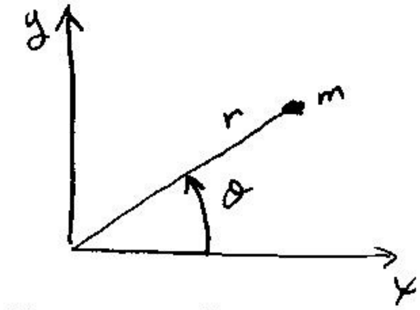
1-4



$x = r\cos\theta$

$y = r\sin\theta$

Use r, θ as generalized coordinates.

$$T = \tfrac{1}{2} m(\dot{x}^2 + \dot{y}^2) = \tfrac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2)$$

$$f = -\frac{\partial U}{\partial r} = -A r^{\alpha-1}$$

so $U = \dfrac{A}{\alpha} r^{\alpha}$ where $U(r=0) = 0$

Then

$$L = \tfrac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{A}{\alpha} r^{\alpha}$$

Lagrange's eqn for r:

$$m\ddot{r} - mr\dot{\theta}^2 + Ar^{\alpha-1} = 0$$

Lagrange's eqn for θ:

$$\frac{d}{dt}(\underbrace{mr^2\dot{\theta}}) = 0$$

$p_\theta = \ell =$ angular momentum which *is* conserved.

We can now eliminate $\dot{\theta}$ from the first Lagrange equation using $\ell = mr^2\dot{\theta}$. We have

$$m\ddot{r} - \frac{\ell^2}{mr^3} + Ar^{\alpha-1} = 0$$

multiply through by $\dot{r}$

$$m\dot{r}\ddot{r} - \dot{r}\frac{\ell^2}{mr^3} + A\dot{r}r^{\alpha-1} = 0 \quad \text{which is the same as}$$

$$\frac{d}{dt}\left[\tfrac{1}{2} m\dot{r}^2\right] + \frac{d}{dt}\left[\frac{\ell^2}{2mr^2}\right] + \frac{d}{dt}\left[\frac{A}{\alpha} r^{\alpha}\right] = 0$$

or

$$\frac{d}{dt}\Big[\underbrace{\tfrac{1}{2} m\dot{r}^2 + \underbrace{\frac{\ell^2}{2mr^2}}_{\frac{1}{2}mr^2\dot{\theta}^2}}_{T} + \underbrace{\frac{A}{\alpha} r^{\alpha}}_{U(r)}\Big] = 0$$

so $\dfrac{d}{dt}[T + U] = 0$ and TME is conserved.