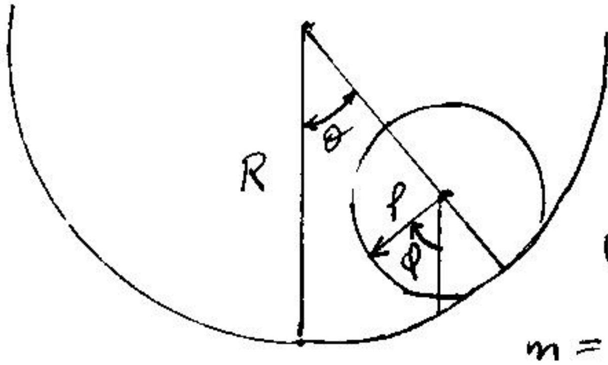
1-3



Take θ and φ as the generalized coordinates so

$$T = \frac{1}{2}m[(R-\rho)\dot{\theta}]^2 + \frac{1}{2}I\dot{\varphi}^2$$

where I = moment of inertia of the sphere about a diameter.

$$U = [R - (R-\rho)\cos\theta]\, mg$$

m = mass of the sphere

$I = \frac{2}{5}m\rho^2$

$$\therefore\ L = T - U = \frac{1}{2}m(R-\rho)^2\dot{\theta}^2 + \frac{1}{5}m\rho^2\dot{\varphi}^2 - [R-(R-\rho)\cos\theta]\, mg$$

rolling w/o slipping gives the equation of constraint,

$$f(\theta,\varphi) = R\theta - \rho\varphi = 0$$

$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) + \lambda\frac{\partial f}{\partial \theta} = 0$ becomes

$$-mg(R-\rho)\sin\theta - m(R-\rho)^2\ddot{\theta} + \lambda R = 0$$

$\frac{\partial L}{\partial \varphi} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) + \lambda\frac{\partial f}{\partial \varphi} = 0$ becomes

$$0 - \frac{2}{5}m\rho^2\ddot{\varphi} - \lambda\rho = 0 \Rightarrow \lambda = -\frac{2}{5}m\rho\ddot{\varphi}$$

$$= -\frac{2}{5}mR\ddot{\theta} \quad \text{(eqn of constraint)}$$

putting this into the first Lagrange eqn:

$$-mg(R-\rho)\sin\theta - m\left[(R-\rho)^2 + \frac{2}{5}R\right]\ddot{\theta} = 0$$

or $\ddot{\theta} + \omega^2\sin\theta = 0$ for small θ $\sin\theta \approx 0$

and ω^2 = frequency of small oscillations

$$= (R-\rho)g \Big/ \left[(R-\rho)^2 + \frac{2}{5}R\right]$$