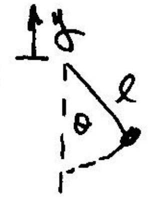
7-26 a)



$$L = T - U = \frac{1}{2} m\ell^2 \dot{\theta}^2 - mgy$$

$$= \frac{1}{2} m\ell^2 \dot{\theta}^2 + mg\ell\cos\theta$$

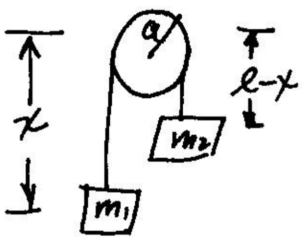
$$P_\theta = \frac{\partial L}{\partial \dot{\theta}} = m\ell^2\theta^2 \implies \dot{\theta} = P\theta / m\ell^2$$

Since U is independent of velocity and the coordinate transformations are independent of time, the Hamiltonian is the total energy.

$$H = T + U = \frac{P_\theta^2}{2m\ell^2} - mg\ell\cos\theta$$

Eqns of motion: $\dot{\theta} = \frac{\partial H}{\partial P_\theta} = \frac{P_\theta}{m\ell^2}$; $\dot{P}_\theta = -\frac{\partial H}{\partial \theta} = -mg\ell\sin\theta$

b)



$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{x}^2 + \frac{1}{2} I \left(\frac{\dot{x}^2}{a^2}\right)$$

$$U = -m_1 g x - m_2 g(\ell - x)$$

$$P_x = \frac{\partial L}{\partial \dot{x}} = \left[m_1 + m_2 + \frac{I}{a^2}\right]\dot{x}$$

$$\dot{x} = P_x / \left[m_1 + m_2 + I/a^2\right]$$

$$H = T + U = \left[P_x^2 / 2\left[m_1 + m_2 + I/a^2\right]\right] - m_1 g x - m_2 g(\ell - x)$$

Eqns of motion

$$\dot{x} = \frac{\partial H}{\partial p_x} = \frac{P_x}{\left[m_1 + m_2 + I/a^2\right]}$$

$$\dot{p}_x = -\frac{\partial H}{\partial x} = m_1 g - m_2 g = (m_1 - m_2) g$$