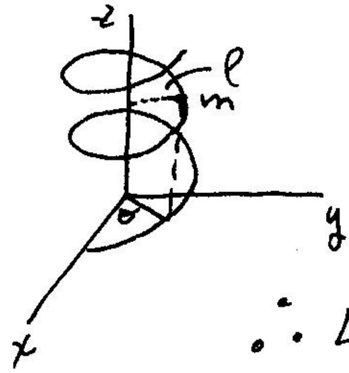
7-25



$$T = \frac{1}{2}m\left[\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2\right]$$

$$U = mgz$$

But $r = \text{constant}$ & $z = k\theta$

$$\therefore L = \frac{1}{2}m\left[\frac{r^2}{k^2}\dot{z}^2 + \dot{z}^2\right] - mgz$$

and $$P_z = \frac{\partial L}{\partial \dot{z}} = m\left[\frac{r^2}{k^2} + 1\right]\dot{z}$$

so $$\dot{z} = \frac{P_z}{m\left[\frac{r^2}{k^2}+1\right]}$$

$$H = P_z\dot{z} - L = P_z\frac{P_z}{m\left[\frac{r^2}{k^2}+1\right]} + mgz$$

$$= T + V$$

Hamilton's equations of motion

$$\dot{P}_z = -\frac{\partial H}{\partial z} \quad \text{\&} \quad \dot{z} = \frac{\partial H}{\partial P_z}$$

$$\dot{P}_z = -mg \quad \text{\&} \quad \dot{z} = \frac{P_z}{m\left[\frac{r^2}{k^2}+1\right]}$$

differentiate the second and insert into the first

$$\ddot{z} = -\frac{g}{\left[\frac{r^2}{k^2}+1\right]}$$