

$$1-23 \quad H = \sum_j P_j \dot{q}_j - L$$

in rectangular coordinates

$$L = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - U$$

Linear momentum components, i.e. canonical momenta

$$P_x = \frac{\partial L}{\partial \dot{x}} = m \dot{x}; \quad P_y = \frac{\partial L}{\partial \dot{y}} = m \dot{y}; \quad P_z = \frac{\partial L}{\partial \dot{z}} = m \dot{z}$$

Then

$$H = m \dot{x} \cdot \dot{x} + m \dot{y} \cdot \dot{y} + m \dot{z} \cdot \dot{z} - \left[\frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - U \right]$$

$$= \frac{1}{2} \left[\frac{(m \dot{x})^2}{m} + \frac{(m \dot{y})^2}{m} + \frac{(m \dot{z})^2}{m} \right] - U$$

$$= \frac{1}{2m} (P_x^2 + P_y^2 + P_z^2) - U$$

$$\dot{q}_k = \frac{\partial H}{\partial P_k} \dot{q}_k - \dot{P}_k = \frac{\partial H}{\partial q_k}$$

so (using the second of these)

$$\left. \begin{aligned} \dot{P}_x &= -\frac{\partial U}{\partial x} = F_x \\ \dot{P}_y &= -\frac{\partial U}{\partial y} = F_y \\ \dot{P}_z &= -\frac{\partial U}{\partial z} = F_z \end{aligned} \right\} \text{Newton's 2nd law}$$