

1-22 Since $F(x,t) = \frac{h}{x^2} e^{-t/\tau}$; $F = -\frac{\partial U}{\partial x}$

$$U(x,t) = \frac{h}{x} e^{-t/\tau}$$

$$\therefore L = T - U = \frac{1}{2} m \dot{x}^2 - \frac{h}{x} e^{-t/\tau}$$

Hamiltonian:

$$H = p_x \dot{x} - L = \dot{x} \frac{\partial L}{\partial \dot{x}} - L$$

$$= \dot{x} \cdot m \dot{x} - \frac{h}{x} e^{-t/\tau}$$

But $H = H(q,p)$

$$p_x = \frac{\partial L}{\partial \dot{x}} = m \dot{x} \text{ so}$$

$$H = \frac{p_x^2}{2m} - \left(\frac{1}{2} \frac{p_x^2}{m} - \frac{h}{x} e^{-t/\tau} \right)$$

$\left(\dot{x} \frac{\partial L}{\partial \dot{x}} \right) \quad \underbrace{\hspace{10em}}_L$

$$= \frac{p_x^2}{2m} + \frac{h}{x} e^{-t/\tau} = \text{total energy}$$

because U does
not depend on
velocity.

But - the total energy is not conserved
because H contains t explicitly.