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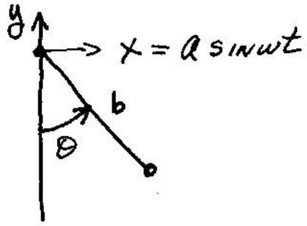

For the mass m:

$$x = a\sin\omega t + b\sin\theta$$
$$y = -b\cos\theta$$
$$\dot{x} = a\omega\cos\omega t + b\dot{\theta}\cos\theta$$
$$\dot{y} = b\dot{\theta}\sin\theta$$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2); \quad U = mgy$$

$$L = \frac{1}{2}m\left(a^2\omega^2\cos^2\omega t + 2ab\,\omega\dot{\theta}\cos\omega t\cos\theta + b\dot{\theta}^2\right) + mgb\cos\theta$$

Lagrange's eqn for θ:

$$\frac{d}{dt}\left(mab\omega\cos\omega t\cos\theta + mb^2\dot{\theta}\right) = -mab\omega\dot{\theta}\cos\omega t\sin\theta - mgb\sin\theta$$

which reduces to

$$\ddot{\theta} + \frac{g}{b}\sin\theta - \frac{a}{b}\omega^2\sin\omega t\cos\theta = 0$$

Note that if there is no driving force, i.e. $a = 0$, then the differential equation reduces to that for a simple pendulum.