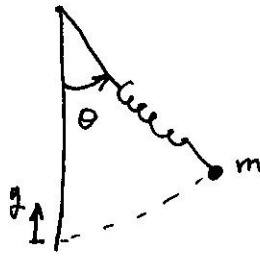


7-15



$b$  = unextended length of spring  
 $l$  = variable length of spring

$$T = \frac{1}{2} m (\dot{l}^2 + l^2 \dot{\theta}^2)$$

$$U = \frac{1}{2} k (l-b)^2 + mgl$$

$$= \frac{1}{2} k (l-b)^2 + mg(b - l \cos \theta)$$

$$L = T - U = \frac{1}{2} m (\dot{l}^2 + l^2 \dot{\theta}^2) - \frac{1}{2} k (l-b)^2 + mgl \cos \theta$$

Lagrange's eqn for  $l$ :

$$m l \dot{\theta}^2 - k(l-b) + mg \cos \theta - m \ddot{l} = 0$$

Lagrange's eqn for  $\theta$ :

$$-mg l \sin \theta - m l^2 \ddot{\theta} - 2ml \dot{l} \dot{\theta} = 0$$

These reduce to

$$\ddot{l} - l \dot{\theta}^2 + \frac{k}{m} (l-b) - g \cos \theta = 0$$

$$\ddot{\theta} + \frac{2}{l} \dot{l} \dot{\theta} + \frac{g}{l} \sin \theta = 0$$