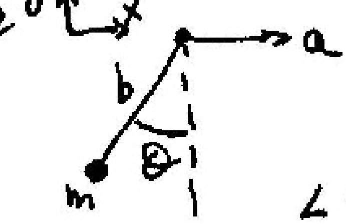
7.13

a)



$x = \frac{1}{2}at^2 - b\sin\theta$; $\dot{x} = at - b\dot{\theta}\cos\theta$

$y = -b\cos\theta$; $\dot{y} = b\dot{\theta}\sin\theta$

$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgy$

$= \frac{1}{2}m(a^2t^2 - 2atb\dot{\theta}\cos\theta + b^2\dot{\theta}^2) + mgb\cos\theta$

Lagrange's eqn is:

$$\frac{d}{dt}\left[-matb\cos\theta + mb^2\dot{\theta}\right] = matb\dot{\theta}\sin\theta - mgb\sin\theta$$

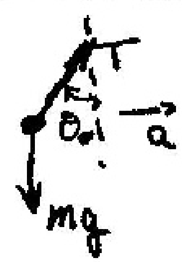
or $$\ddot{\theta} + \frac{g}{b}\sin\theta - \frac{a}{b}\cos\theta = 0$$

Note that if $a = 0$ we get the familiar eqn of motion for a simple pendulum.

b) Because $a \neq 0$ there will be an equilibrium angle θ_0 about which the pendulum swings. We must therefore expand $\sin\theta$ and $\cos\theta$ about θ_0.

First find θ_0:

at equilibrium $\rightarrow$



$T\cos\theta_0 = mg$

$T\sin\theta_0 = ma$

$\tan\theta_0 = a/g$

Then, since $f(\theta) \simeq f(\theta_0) + (\theta - \theta_0)\left[f'(\theta)\right]_{\theta=\theta_0}$

$\sin\theta \simeq \sin\theta_0 + (\theta - \theta_0)\cos\theta_0 \simeq \frac{1}{\sqrt{a^2+g^2}}(a + g\theta - g\theta_0)$

$\cos\theta \simeq \cos\theta_0 - (\theta - \theta_0)\sin\theta_0$

$= \frac{1}{\sqrt{a^2+g^2}}(g - a\theta + a\theta_0)$; put these into the eqn of motion gives

$$\ddot{\theta} + \underbrace{\frac{\sqrt{g^2+a^2}}{b}}_{\omega^2 \text{ for SHM}}\theta - \left(\frac{g^2+a^2}{b}\right)\theta_0$$

$$T = \frac{2\pi}{\omega} = 2\pi b^{1/2}/(g^2+a^2)^{1/4}$$

Note that $T = 2\pi\sqrt{b/g}$ if $a = 0$.