

5-4 For this force the PE is

$$U = -\int F dx = m h^2 \int \frac{dx}{x^3} = -\frac{m h^2}{2x^2}$$

Then

$$TME = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} m \frac{h^2}{x^2} = -\frac{1}{2} m \frac{h^2}{d^2}$$

so

$$\left(\frac{dx}{dt}\right)^2 = \frac{h^2}{x^2} - \frac{h^2}{d^2} = h^2 \left(\frac{1}{x^2} - \frac{1}{d^2}\right)$$

This differential equation is separable

$$\frac{dx}{dt} = \pm h \sqrt{\frac{1}{x^2} - \frac{1}{d^2}} \quad \text{so}$$

$$dt = \frac{dx}{\pm h \sqrt{\frac{1}{x^2} - \frac{1}{d^2}}}$$

Now we must choose the sign on the r.h.s.
In this problem, when x decreases, t increases.
 \therefore we must choose the minus sign because
as x decreases the denominator increases
so t decreases unless we choose the
minus sign.

$$\begin{aligned} \therefore \int_0^t dt = t &= -\frac{1}{h} \int_d^0 \frac{dx}{\sqrt{\frac{d^2 - x^2}{x^2 d^2}}} \\ &= \frac{1}{2h} \int_0^d \frac{2x dx}{\sqrt{d^2 - x^2}} \\ &= \frac{d}{h} \left. \frac{1}{2} \left(\frac{d^2 - x^2}{d^2} \right)^{1/2} \right|_0^d \\ &= d/h \end{aligned}$$