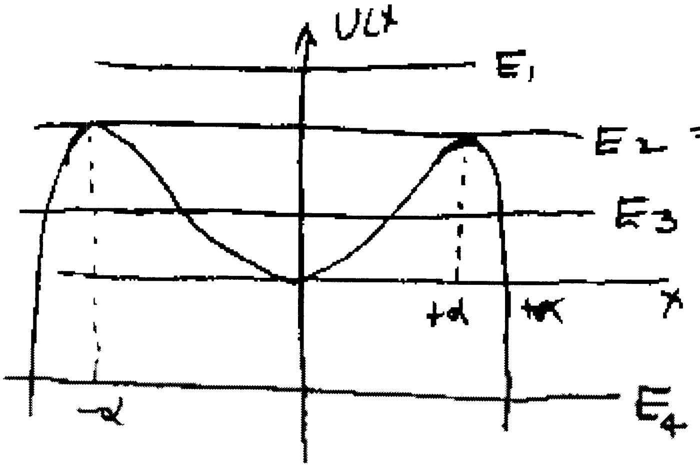
2-43 $F(x) = -kx + k\, x^3/\alpha^2$

$U(x) = -\int F(x)dx = \frac{1}{2}kx^2 - \frac{1}{4}k\frac{x^4}{\alpha^2}$

$\frac{dU}{dx} = kx - kx^3/\alpha^2 = 0 \Rightarrow$ for extrema $x = 0, \pm\alpha$

$\frac{d^2U}{dx^2} = k - 3k\frac{x^2}{\alpha^2}$
$= k\left[1 - 3\frac{x^2}{\alpha^2}\right]$

x	sign of $\frac{d^2U}{dx^2}$	
0	+	concave up (stable)
$\pm\alpha$	−	concave down (unstable)



E_1: unbounded motion

E_2: unstable equilibrium at $x = \pm\alpha$

E_3: inside the well oscillatory motion
outside the well the particle "hits" the potential energy curve and bounces back to $\pm\infty$

E_4: the particle can be on either side, but not within the potential energy curve.

What happens at $E = \frac{1}{4}k\alpha^2$? At $E = \frac{1}{4}k\alpha^2$, $x = \pm\alpha$. ∴ Unstable equilibrium at these points. If inside the well, it oscillates to the very top of the well. If outside it almost makes it inside the well.