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Converting From A Present Value Lump Sum To A Future Payment Stream

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I. Introduction

The objective of our original research was to correct a problem in the calculation of hedonic damages. We have found several applications in forensic economics, which are discussed below. They require the derivation of a start-of-the-period, annual amount, which when projected by a known (net) discount rate produces a present value that is also known.

Let us illustrate the problem with an application that occurs (but rarely), derive the formula, and then move to several applications. Assume that, as an economist for the defense, one is reviewing the conclusions of a plaintiff's economist about the present value of a lost earnings stream. You know the economist's projected present value, that he or she is using a constant wage growth factor of 4% per year and a discount rate of 5.5%. You also know that the economist is using a worklife expectancy of 20 years. What you don't know and need is the value of the base starting-income figure in the current year, from which he or she projects the future income stream and the present value. This base figure may be critical to criticisms of the plaintiff's economic loss projections.

II. The Algebraic Derivation of the Formula

It should be emphasized, and is clear in what follows, that we are developing a formula for a *beginning* of the period value in the first, forecast period.[1] Start with the present value formula for a payment stream with a constant growth factor g, a constant discount rate i, a number of annual payments n, and a base starting figure a, as follows:

$$1) \qquad PV = \sum_{j=0}^{n-1} ar^j$$

where r represents the net discount rate[2], or

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[1]A different result is produced by specifying equation 1) for payments at other than the beginning of the period.

[2]While this algebraic expression for the net discount rate has been widely used in the literature of forensic economics, it should be noted for the newcomer that the net discount rate is the wage growth rate less the interest (discount) rate. Whether or not the inflation rate is considered *does* affect the estimate See the technical appendix to Chapter 3 of Michael L.

2) $$r = \frac{1+g}{1+i}$$

Given the values of PV, g, i, and n, solve for a, and rearranging 1),

3) $$PV = a\sum_{j=0}^{n-1} r^j$$

and note that,

4) $$r\sum_{j=0}^{n-1} r^j = \sum_{j=1}^{n} r^j = \sum_{j=0}^{n-1} r^j - 1 + r^n$$

therefore,

5) $$r\sum_{j=0}^{n-1} r^j = \sum_{j=0}^{n-1} r^j - 1 + r^n$$

6) $$(r-1)\sum_{j=0}^{n-1} r^j = -1 + r^n$$

7) $$\sum_{j=0}^{n-1} r^j = \frac{1-r^n}{1-r}$$

Thus,

8) $$PV = a\left(\frac{1-r^n}{1-r}\right),$$

and solving for *a*

9) $$a = \frac{PV(1-r)}{(1-r^n)}$$

Thus, if PV = $350,000, n = 20 years, g = 4%, and i = 5.5%, r is the net discount rate as in 2) and equals 0.98578.

10) $$a = \frac{\$350{,}000(1-0.98578)}{1-(0.98578)^{20}} = \$19{,}982$$

Table 1 shows the annual payments and cumulative present values derived from this solution for the value of the base at $19,982 in 1993, with 20 annual payments commencing in 1993. As indicated in the table, the cumulative present value for the 20-year period is $350,006, the $6.00 be-

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Brookshire and Stan Smith, *Economic/Hedonic Damages. The Practice Book for Plaintiff and Defense Attorneys*, Cincinnati: Anderson, 1990, Chapter 3

ing a result of tiny rounding errors. The fact that the calculation achieves the starting present value of $350,000 is the test of the validity of the formula.

Table 1
Annual Payments from a 1993 Base $19,982*
With a 4% Wage Increase and 5.5% Discount Rate

Year	Index	Earnings	Present Value	Cumulative
1993	1	$19,982	$19,982	$19,982
1994	2	$20,782	$19,698	$39,681
1995	3	$21,613	$19,418	$59,099
1996	4	$22,478	$19,142	$78,241
1997	5	$23,377	$18,870	$97,111
1998	6	$24,312	$18,602	$115,713
1999	7	$25,284	$18,337	$134,050
2000	8	$26,296	$18,077	$152,127
2001	9	$27,347	$17,819	$169,946
2002	10	$28,441	$17,566	$187,512
2003	11	$29,579	$17,316	$204,828
2004	12	$30,762	$17,070	$221,899
2005	13	$31,993	$16,827	$238,726
2006	14	$33,272	$16,588	$255,314
2007	15	$34,603	$16,352	$271,667
2008	16	$35,987	$16,120	$287,787
2009	17	$37,427	$15,891	$303,677
2010	18	$38,924	$15,665	$319,342
2011	19	$40,481	$15,442	$334,784
2012	20	$42,100	$15,222	$350,006

*It should be noted that spreadsheets are also available which, in lieu of the formula discussed in this article, allow the calculation of the initial, annual value. Key inputs are the net discount rate and the cumulative present value at the end of the period.

III. Some Applications

Besides the defense-side use of our conversion formula to determine the plaintiff's annual base, several other applications of this tool are as follows:

1) Correcting a calculation problem that is not addressed in *Economic/Hedonic Damages* (Brookshire and Smith, 1990).[3]
2) Conversion of a settlement offer into an implied payment stream with an annual CPI adjustment for each future year.

[3]Ibid , Chapter 9.

Table 2
Present Value of the Lost Pleasure of Life of Jack Doe 1989-2028
With a 1.29% Real Growth Rate and 3.13% Real Discount Rate

Year	Age	Value of Life	Discount Factor	Present Value	Cumulate
1989	36	$86,764	1.00000	$86,764	$86,764
1990	37	$87,883	0.96965	$85,216	$171,979
1991	38	$89,017	0.94022	$83,695	$255,675
1992	39	$90,165	0.91169	$82,202	$337,877
1993	40	$91,328	0.88402	$80,735	$418,612
1994	41	$92,506	0.85719	$79,295	$497,907
1995	42	$93,700	0.83117	$77,880	$575,788
1996	43	$94,908	0.80594	$76,491	$652,278
1997	44	$96,133	0.78148	$75,126	$727,404
1998	45	$97,373	0.75777	$73,786	$801,190
1999	46	$98,629	0.73477	$72,469	$873,659
2000	47	$99,901	0.71247	$71,176	$944,836
2001	48	$101,190	0.69084	$69,906	$1,014,742
2002	49	$102,495	0.66988	$68,659	$1,083,401
2003	50	$103,817	0.64955	$67,434	$1,150,835
2004	51	$105,157	0.62983	$66,231	$1,217,066
2005	52	$106,513	0.61072	$65,049	$1,282,116
2006	53	$107,887	0.59218	$63,889	$1,346,004
2007	54	$109,279	0.57421	$62,749	$1,408,753
2008	55	$110,689	0.55678	$61,629	$1,470,383
2009	56	$112,117	0.53988	$60,530	$1,530,912
2010	57	$113,563	0.52350	$59,450	$1,590,362
2011	58	$115,028	0.50761	$58,389	$1,648,751
2012	59	$116,512	0.49220	$57,347	$1,706,099
2013	60	$118,015	0.47726	$56,324	$1,762,423
2014	61	$119,537	0.46278	$55,319	$1,817,742
2015	62	$121,079	0.44873	$54,332	$1,872,075
2016	63	$122,641	0.43512	$53,363	$1,925,438
2017	64	$124,223	0.42191	$52,411	$1,977,849
2018	65	$125,826	0.40910	$51,476	$2,029,324
2019	66	$127,449	0.39669	$50,557	$2,079,882
2020	67	$129,093	0.38465	$49,655	$2,129,537
2021	68	$130,758	0.37297	$48,769	$2,178,307
2022	69	$132,445	0.36165	$47,899	$2,226,206
2023	70	$134,153	0.35068	$47,045	$2,273,251
2024	71	$135,884	0.34004	$46,205	$2,319,456
2025	72	$137,637	0.32972	$45,381	$2,364,837
2026	73	$139,412	0.31971	$44,571	$2,409,408
2027	74	$141,211	0.31001	$43,776	$2,453,185
2028	75	$143,032	0.30060	$42,995	$2,496,180
2029	76	$144,878	0.29147	$42,228	$2,538,408
2030	77	$146,746	0.28263	$41,475	$2,579,882
2031	78	$148,639	0.27405	$40,735	$2,620,617
2032	79	$150,557	0.26573	$40,008	$2,660,625
2033	80	$152,499	0.25767	$39,294	$2,699,919

3) Demonstration of how closely a proposed property allocation to a wife in a divorce action compares to her lost earnings through forbearance of career earning capacity during the marriage. Each of these three applications is briefly discussed below.

IV. Hedonic Damages Calculations

While the two authors disagree on several aspects of "value of life" testimony, an important application of the above is to correctly derive a stream of annual hedonic losses from a present value sum that represents hedonic losses. The problem may be seen with reference to the "Sample Death Case" from *Economic/Hedonic Damages* (Brookshire and Smith, 1990). The original calculation table is replicated in the Appendix, and corrected values under our suggested conversion are shown in Table 2 of this paper.

In the Brookshire-Smith hypothetical, $800,000 in lost earnings have been subtracted from a hypothetical "value of life" estimate of $3,500,000 to obtain a remaining value of $2,700,000 for the lost enjoyment of life. This value is then divided by an assumed life expectancy of 45 years to generate an annual "hedonic" life value of $60,000 per year. This $60,000 per year figure is then assumed to have an annual real growth rate of 1.29% and is discounted at a real discount rate of 3.13%.

The problem here is that the $2,700,000 is a present value sum, derived from studies showing workers' and/or consumers' current willingness to pay to be equal to $2,700,000 to preserve the enjoyment of one human life, presumably in the near future. Since this figure is already a present value, it should already be equal to the value of life enjoyment for a person with an average life expectancy, as in the example. By dividing by 45 and then adjusting for a growth factor and a discount rate, the present value in the example turns out to be $1,709,842. However, using our formulas, the value of r in our equation 2) should be 0.98216 and the value of a in equation 9) should be $86,764. When we replace $60,000 in the example with $86,764, we find a present value of $2,496,180 instead of $1,709,842.

Our Table 2 continues for five years beyond the life expectancy of Jack Doe in the hypothetical example. This is simply to show that if we assumed that Jack Doe had an average life expectancy of 45 years instead of a 40-year life expectancy, our total present value of $2,699,919 virtually equals the $2,700,000 value of life estimate from which our annual life enjoyment figures were calculated.

V. An Assessment Tool for Settlement Offers

Forensic economists are sometimes called upon to evaluate settlement offers. Our simple formulas can be used to simulate an expenditure stream from a specific settlement offer so that a plaintiff's attorney could reasonably assess whether or not a given settlement offer could meet his client's needs. Assume the following circumstance: The defense has offered a cash settlement of $800,000. For a permanently disabled plaintiff, what rate of sustainable annual expenditures in constant purchasing power over 33 years of remaining life expectancy would this settlement offer imply? Assume further that the forensic economist believes that a net discount rate of 2% is the appropriate difference between the cost of living increase factor and the appro-

priate discount rate. This would imply that r in equation 2) equals 0.98039 and that a in equation 9) equals

$$11) \qquad \frac{\$800,000 \times (1-0.98039)}{1-(0.98039)^{33}}$$

or $32,696 per year in current dollars.

This would mean that the forensic economist could offer the plaintiff and the plaintiff's attorney the suggestion that the $800,000 settlement offer would be equivalent to an offer of $32,696 per year in current purchasing power, which would then increase by the real rate of growth that lies behind the 2% net discount rate.

VI. Divorce Forbearance Valuation

A somewhat similar application might be found with respect to a given proposal for maintenance in light of a homemaker's forbearance of career development opportunity. Suppose, for example, that the attorney for the husband has proposed that a husband make five years of maintenance payments at $10,000 each. The attorney for the wife wishes to have the judge view that proposal as being equivalent to a wife's reduction in earning power during the marriage. Assume further that the wife is at age 45 and that the comparison is being based on an assumption that the wife would retire at age 65. Make the further assumption that wages will be expected to increase at a nominal 4% per year and that the selected discount rate is 6% per year.

The first calculation will be to determine the present value of the maintenance proposal. The value of these payments at a 6% discount rate is $42,124. This then becomes the PV value in equation 9). The value of r in equation 2) is 0.981132. Therefore, the value of a equals

$$12) \qquad \frac{\$42,124 \times (1-0.981132)}{1-(0.981132)^{20}},$$

or $2,508 per year in forbearance in the current year. If it had already been shown by a vocational expert, for example, that the wife's loss in earning power would start at $8,000 per year, the above demonstration that a given proposal was equivalent to only $2,508 per year might be useful.

VII. Summary and Conclusion

A formula has been provided for converting from a present value lump sum to a future payment stream, and four applications in forensic economics have been identified and discussed. It is likely that readers will identify other applications for this formula.

Appendix

Present Value of the Lost Pleasure of Life of Jack Doe 1989-2028
With a 1.29% Real Growth Rate and a 3.13% Real Discount Rate

Year	Age	Value of Life	Discount Factor	Present Value	Cumulate
1989	36	$52,500	1.00000	$52,500	$52,500
1990	37	$60,774	0.96965	$58,930	$111,430
1991	38	$61,558	0.94022	$57,878	$169,308
1992	39	$62,352	0.91169	$56,846	$226,154
1993	40	$63,156	0.88402	$55,831	$281,985
1994	41	$63,971	0.85719	$54,835	$336,820
1995	42	$64,796	0.83117	$53,856	$390,676
1996	43	$65,632	0.80594	$52,895	$443,571
1997	44	$66,479	0.78148	$51,952	$495,523
1998	45	$67,337	0.75777	$51,026	$546,549
1999	46	$68,206	0.73477	$50,116	$596,665
2000	47	$69,086	0.71247	$49,222	$645,887
2001	48	$69,977	0.69084	$48,343	$694,230
2002	49	$70,880	0.66988	$47,481	$741,711
2003	50	$71,794	0.64955	$46,634	$788,345
2004	51	$72,720	0.62983	$45,801	$834,146
2005	52	$73,658	0.61072	$44,984	$879,130
2006	53	$74,608	0.59218	$44,181	$923,311
2007	54	$75,570	0.57421	$43,393	$966,704
2008	55	$76,545	0.55678	$42,619	$1,009,323
2009	56	$77,532	0.53988	$41,858	$1,051,181
2010	57	$78,532	0.52350	$41,112	$1,092,293
2011	58	$79,545	0.50761	$40,378	$1,132,671
2012	59	$80,571	0.49220	$39,657	$1,172,328
2013	60	$81,610	0.47727	$38,950	$1,211,278
2014	61	$82,663	0.46278	$38,255	$1,249,533
2015	62	$83,729	0.44874	$37,573	$1,287,106
2016	63	$84,809	0.43512	$36,902	$1,324,008
2017	64	$85,903	0.42191	$36,243	$1,360,251
2018	65	$87,011	0.40911	$35,597	$1,395,848
2019	66	$88,133	0.39669	$34,961	$1,430,809
2020	67	$89,270	0.38465	$34,338	$1,465,147
2021	68	$90,422	0.37298	$33,726	$1,498,873
2022	69	$91,588	0.36166	$33,124	$1,531,997
2023	70	$92,769	0.35068	$32,532	$1,564,529
2024	71	$93,966	0.34004	$31,952	$1,596,481
2025	72	$95,178	0.32972	$31,382	$1,627,863
2026	73	$96,406	0.31971	$30,822	$1,658,685
2027	74	$97,650	0.31001	$30,272	$1,688,957
2028	75	$68,831	0.30342	$20,885	$1,709,842

Source: Michael L. Brookshire and Stan V. Smith. *Economic/Hedonic Damages:: The Practice Book for Plaintiff and Defense Attorneys*, Cincinnati: Anderson Publishing Company, 1990, 170-172. Reproduced with permission.