

# Thermodynamic Possibility Puzzlers

Find here examples of how 1st and 2nd Law intuition tells us what is likely NOT (and what may be) possible, without delving into details about how to pull it off. The focus is on energy and uncertainty flows into, and out of, "steady-state engines" of virtually any type. The equations treat the world outside the engine as a contained universe. This is possible since steady-state engines (by definition) process energy and information (often cyclicly) while their own state of being is maintained (i.e. at the end of each cycle, the state of the engine itself is to first order unchanged). This page helps set up the problems, writing out color-coded terms for 1st and 2nd law equations, and it provides the numeric answer for one example problem in each case as a reality check for your own deductions. **However, there is a catch!** We ask that you work out for yourself the final, generally useful, formula for solving each problem type.

*Can you do it?*

The templates provided may, of course, help solve possibility problems we haven't thought of as well. Suggestions for problem types to add to this list are invited. E-mail your suggestions to [pfraudorf@umsl.edu](mailto:pfraudorf@umsl.edu).

**"Science of the possible" equations, for flows into and out of steady state engines, follow from...**

**1st Law:** Heat\_OUT plus Work\_OUT equals Heat\_IN plus Work\_IN  
**2nd Law:** Uncertainty\_INCREASE minus Correlation\_INCREASE equals Surprisal\_Irreversibly\_Lost, which is greater than or equal to **Zero**.

Note that Uncertainty\_INCREASE can often be expressed as...  
Heat\_OUT/Temperature\_OUT minus Heat\_IN/Temperature\_IN,  
and Correlation is information on the relation between subsystems in [J/K] or [bits].

**...which allow us to put limits (for example) on:**

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[\[work available\]](#) [\[work required\]](#) [\[heating costs\]](#) [\[irreversibility losses\]](#) [\[information engines\]](#)

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**1). Work available from a heat engine (e.g. operating on fossil fuels or via photosynthesis)...**

$$Q_{\text{exhaust}} + W_{\text{out}} = Q_{\text{hot}}, \text{ and } Q_{\text{exhaust}}/T_{\text{exhaust}} - Q_{\text{hot}}/T_{\text{hot}} \geq 0,$$

implies that *Carnot Efficiency*  $W_{\text{out}}/Q_{\text{hot}} \leq e[T_{\text{exhaust}}/T_{\text{hot}}]$ , where  $e[x] = ( \quad )$ .

*Case Study:* An automobile engine, with  $T_{\text{hot}} = 429\text{K}$

and  $T_{\text{exhaust}} = 300\text{K}$ , has an efficiency limit of 30%.

**2). Work needed to keep the frost on a 6-pack: Refrigerator coefficient of performance...**

$$Q_{\text{room}} = Q_{\text{cold}} + W_{\text{in}}, \text{ and } Q_{\text{room}}/T_{\text{room}} - Q_{\text{cold}}/T_{\text{cold}} \geq 0,$$

implies a *Refrigerator C.O.P.* of  $Q_{\text{cold}}/W_{\text{in}} \leq \frac{T_{\text{cold}}}{T_{\text{room}} - T_{\text{cold}}}$ .

*Case Study:* For each joule of electricity, a freezer in a 295K room may remove up to 9.9 joules of heat from its 268K air.

**3). Work needed to pump winter heat from the outside in, or heat pump C.O.P....**

$$Q_{\text{room}} = Q_{\text{cold}} + W_{\text{in}}, \text{ and } Q_{\text{room}}/T_{\text{room}} - Q_{\text{cold}}/T_{\text{cold}} \geq 0,$$

implies a *Heat Pump C.O.P.* of  $Q_{\text{room}}/W_{\text{in}} \leq \frac{T_{\text{room}}}{T_{\text{room}} - T_{\text{cold}}}$ .

*Case Study:* For each joule of electricity, heat pumps might bring inside up to 7.4 joules of heat from a 0 F backyard.

**4). Reversible home heating with a flame: Getting lots more BTU's for the buck...**

$$Q_{\text{room}} = Q_{\text{flame}} + Q_{\text{cold}}, \text{ and } Q_{\text{room}}/T_{\text{room}} - (Q_{\text{flame}}/T_{\text{flame}} + Q_{\text{cold}}/T_{\text{cold}}) \geq 0,$$

implies a *Reversibility Gain* of  $Q_{\text{room}}/Q_{\text{flame}} \leq e^{\frac{T_{\text{room}}}{T_{\text{flame}} - T_{\text{cold}}}}$

*Case Study:* When  $T_{\text{outside}}=273\text{K}$ , a furnace irreversibly heating a room to  $T_{\text{room}}=298\text{K}$  with  $T_{\text{flame}}=1000\text{K}$  uses 8.67 times the fuel actually required.

**5). Zero energy ovens for eskimos, or how to cook food in cold weather for free...**

$$Q_{\text{oven}} = Q_{\text{room}} + Q_{\text{cold}}, \text{ and } Q_{\text{oven}}/T_{\text{oven}} - (Q_{\text{room}}/T_{\text{room}} + Q_{\text{cold}}/T_{\text{cold}}) \geq 0,$$

implies a *Heat Transfer Ratio* of  $Q_{\text{oven}}/Q_{\text{room}} \leq e^{\frac{T_{\text{oven}}}{T_{\text{oven}} - T_{\text{room}}}}$ .

*Case Study:* After "the turkey is done", this transfer may be spontaneously reversed (see previous example) for no net heat loss!

**6). Reversible home cooling with a flame...**

$$Q_{\text{warm}} = Q_{\text{flame}} + Q_{\text{room}}, \text{ and } Q_{\text{warm}}/T_{\text{warm}} - (Q_{\text{flame}}/T_{\text{flame}} + Q_{\text{room}}/T_{\text{room}}) \geq 0,$$

implies a *Reversibility Gain* of  $Q_{\text{warm}}/Q_{\text{flame}} \leq e^{\frac{T_{\text{warm}}}{T_{\text{flame}} - T_{\text{room}}}}$

*Case Study:* When  $T_{\text{warm}}=305\text{K}$ , a flame AC reversibly cooling a room to  $T_{\text{room}}=295\text{K}$  with  $T_{\text{flame}}=1000\text{K}$  can pump up to 21.5 [J] out for every [J] of flame expended.

**7). Surprisal losses from an oven leaking heat...**

$$Q_{\text{room}} = Q_{\text{oven}}, \text{ and } S_{\text{irr}} \geq (Q_{\text{room}}/T_{\text{room}} - Q_{\text{oven}}/T_{\text{oven}}),$$

implies surprisal losses of  $S_{\text{irr}} \geq Q_{\text{oven}}( \quad )$

*Case Study:* A 473K oven irreversibly raises state uncertainty by nearly  $10^{20}$  nats, per joule of heat leaked to a 295K room.

**8). Surprisal losses from ice melting in the North Sea (or a glass of iced tea).**

$$Q_{\text{ice}} = Q_{\text{liquid}}, \text{ and } S_{\text{irr}} \geq (Q_{\text{ice}}/T_{\text{melt}} - Q_{\text{liquid}}/T_{\text{melt}}),$$

implies surprisal losses of  $S_{\text{irr}} \geq Q_{\text{ice}}( \quad )$ .

*Case Study:* Chipped ice in an ice-cold slurry melts reversibly, resulting in  $S_{\text{irr}} = 0$ .

**9). Surprisal value on earth of a joule of solar photons...**

$$Q_{\text{ambient}} + W_{\text{out}} = Q_{\text{sun}}, \text{ and } Q_{\text{ambient}}/T_{\text{ambient}} - Q_{\text{sun}}/T_{\text{sun}} \geq 0,$$

implies that the *Net\_Surprisal*  $I_{\text{net}} = W_{\text{out}}/T_{\text{ambient}} \leq Q_{\text{sun}}( \quad )$ .

*Case Study:* Insolation of 1000 [J/(sec m<sup>2</sup>)] from a 6000K sun onto a 295K earth means  $I_{\text{net}} \leq 3.22$  [J/K] and  $W_{\text{out}} \leq 951$  [J], per second per square meter.

**10). Surprisal losses as your coffee gets cold...**

$$dQ_{\text{room}} = dQ_{\text{coffee}}, \text{ and } dS_{\text{irr}} \geq dQ_{\text{room}}/T_{\text{room}} - dQ_{\text{coffee}}/T_{\text{coffee}}, \text{ can be integrated}$$

from  $T_{\text{coffee}}$  down to  $T_{\text{room}}$  using  $dQ_{\text{coffee}} = \text{HeatCapacity } dT_{\text{coffee}}$ ,

to get *Surprisal\_Loss*  $\geq \text{HeatCapacity } f[T_{\text{coffee}}/T_{\text{room}}]$ , where  $f[x] = \quad$ .

*Case Study:* For water cooled from a boil, with dimensionless

$\text{HeatCapacity} = 9/\text{molecule}$ ,  $S_{\text{irr}} \geq 0.298$  nat/molecule.

**11). Ice water invention: Turn hot water to cold reversibly, work-free with ambient exhaust...**

$$dQ_{\text{room}} = dQ_{\text{hot}} + dQ_{\text{cold}}, \text{ and } dQ_{\text{room}}/T_{\text{room}} - (dQ_{\text{hot}}/T_{\text{hot}} + dQ_{\text{cold}}/T_{\text{cold}}) \geq 0 \text{ can be integrated}$$

to  $T_{\text{room}}$  for  $dQ_{\text{hot}}$  from  $T_{\text{hot}}$ , and for  $dQ_{\text{cold}}$  from  $T_{\text{cold}}$ , using  $dQ_{\text{water}} = \text{HeatCapacity } dT_{\text{water}}$ ,

to get  $\text{HeatCapacity} (f[T_{\text{hot}}/T_{\text{room}}] - f[T_{\text{cold}}/T_{\text{room}}]) \geq 0$ , so that  $T_{\text{room}} \leq ( \quad )/\ln[ \quad ]?$

*Case Study:* For water cooled from a boil,

this invention will make ice water as long as  $T_{\text{room}} \leq 100/\ln[373/273] = 320.4$ [K] or 47.4[C].

**12). Energy required to clear one's mind (or a quantum computer's memory)...**

$$Q_{\text{ambient}} = W_{\text{in}}, \text{ and } Q_{\text{ambient}}/T_{\text{ambient}} - I_{\text{open}} \geq 0,$$

implies work to erase old data (and clear space for new) of  $W_{\text{in}} \geq I_{\text{open}}$  .

*Case Study:* At room temperature, nature thus requires  $W_{\text{in}} = 1/40$  eV per nat of data erased.

**13). Maximum astrophysical (or other) observation rates, per observer per meal...**

$$Q_{\text{ambient}} = W_{\text{food}}, \text{ and } Q_{\text{ambient}}/T_{\text{ambient}} - I_{\text{recorded}} \geq 0,$$

limits mutual information created to  $I_{\text{recorded}} \leq W_{\text{food}}$  .

*Case Study:* Human observers with typical caloric intake must therefore themselves create less than  $10^{21}$  Gigabytes of correlation information per day.

(Aside: Some of us, yours truly included, produce MUCH LESS!)