An experience model for anyspeed motion

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Extreme physics data and simulations (e.g. of high speeds and small sizes), delivered modeling-workshop style with self-discovery rather than rote-learning the goal, can provide explorers of any age with clues to how science adapts to new phenomena. In particular, high speed airtrack simulations offer clues to relativistically informed patterns of thought (e.g. local rather than global time), and data for non-trivial experiments describing motion at any speed. For example, with a compressable spring plus a "fast" clock-equipped glider and timer gate-pair, Minkowski's spacetime version of Pythagoras' theorem (the flat-space metric equation) is there to be found. With a second glider, "anyspeed" expressions for momentum and kinetic energy follow easily as well.

I. INTRODUCTION

The physics of worlds outside the range of normal human experience, like those exemplified in George Gamow's "Mr. Tompkins" series[1] among others, is both fascinating and increasingly relevant to our day to day existence. Today's students may already be relying on global positioning system devices whose accuracy relies on special and general relativistic local-time corrections[2], as well as on nano-engineered materials and catalysts in everything from their contact lenses to their auto exhaust. Nanoscale processes, which depend on the size of Planck's constant, are becoming relevant in growing numbers of employment areas as well.

One way to assist work on integrating these developments into the introductory curriculum is by giving students a taste, here and there, of these extreme physics phenomena and the new thinking required to take them into account. Exercises directed toward "informing motion studies to developments at high speeds", or "informing our understanding of machines to developments on small size scales", show particular promise because Newtonian treatments of motion and machines already inhabit most introductory courses.

Here we discuss specifically how experience with data from high speed airtrack simulations might challenge students to think about time in new ways, and in the process give them insight into both the way scientific thinking evolves, and into the principles that underlie special and general relativity. For students unfamiliar with modeling workshop strategies[3,4], we propose a simple thought experiment involving quantitative consideration of experimental data (provided by the teacher) that shows clocks experiencing different elapsed times. Since this is primarily a graphing and discussion exercise, it's qualitative application could easily be extended to middle school students seeking to understand how scientific inquiry accomodates new and unexpected data.

For students already familiar with modeling strategies, we suggest exercises with specific quantitative curriculum goals. Data from a single glider can facilitate discovery of the flat-space metric equation (Minkowski's space-time version of Pythagoras' theorem). Even short exposure of high school modelers to quantitative evidence for the metric equation, in context of a single frame of reference, might inspire them to look into calculus-based explorations of curved spacetime[5], as well as into the mastery of more abstract tools. Moreover, use of the metric equation and single-frame perspectives lie at the heart of modern pedagogical approaches to relativity[6,7], as well as of modern studies of gravity[8] and other forces[9].

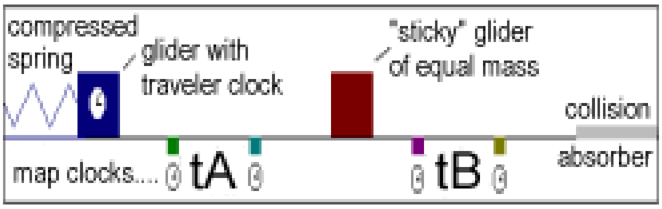

FIG. 1: Schematic of the simulation.

II. DIALOG ON DIFFERING CLOCKS

Ask students to imagine the first half of the airtrack experiment illustrated in Fig 1. Here a spring is compressed by distance x_0, and on release allowed to transfer kinetic energy K_0 to a glider. The kinetic energy of the glider may be inferred from information on the spring's mass, displacement, and spring constant, or from information on the energy used to compress the spring. Although compression distance is easier for introductory students to understand, use of kinetic energy (hence the amount of work done compressing a massless spring) as the independent variable is consistent with trends in physics education to introduce conservation of energy (e.g. Moore[10]) as early as possible. It is also consis-

TABLE I: "Flexible spring data" with some random measurement error, using 1[kg] gliders, gate separations of 1[foot] or 0.305[m], and a spring constant of 1[N/m].

x_0[cm]	K_0[μJ]	t_A[s]	τ_A[s]	t_B[s]	τ_B[s]
1	50	30.5	30.7	60.9	60.9
2	200	15.4	15.2	30.6	30.5
5	1250	6.15	6.13	12.2	12.3
10	5000	3.08	3.03	6.13	6.07
20	20000	1.53	1.54	3.03	3.07
50	125000	0.61	0.60	1.23	1.22
100	500000	0.31	0.30	0.61	0.60

TABLE II: "Tight spring data" with some random measurement error, using 1[kg] gliders, gate separations of 0.305[m], and a spring constant of 1×10^{18}[N/m].

x_0[cm]	K_0[TJ]	t_A[ns]	τ_A[ns]	t_B[ns]	τ_B[ns]
1	50	30.4	30.3	61.4	60.9
2	200	15.4	15.1	30.8	30.3
5	1250	6.22	6.11	12.3	12.1
10	5000	3.17	2.99	6.11	6.04
20	20000	1.78	1.44	3.08	2.88
50	125000	1.11	0.466	1.38	0.936
100	500000	1.03	0.158	1.06	0.314

tent with the American Assocation for the Advancement of Science benchmarks[11], which propose energy as a " major exception to the principle that students should understand ideas before being given labels for them". In any case these independent variables are used simply to vary traverse times, which are the sole quantitative focus except in the final section of this paper.

The time interval elapsed as this glider then crosses gate-pair A is measured using map-clocks (t_A) and a traveling-clock (τ_A). Assume conditions under which glider motion is essentially frictionless. Values for t_A and τ_A are listed for a rather flexible spring in Table I, and for an extremely tight spring in Table II. The gate separation (x_A) is 1 foot or 0.305 meters. The data contain some measurement errors in the times, although we've been careful to eliminate systematic errors, and to keep the random variations in measurement of the same time-interval at or about the 1 percent level.

Note that in the flexible spring case (Table I), the glider clock and the gate timers agree on the time it takes to traverse gate pair A. However, in the tight spring case (Table II), the gate timers suggest that traversing the distance between gates is never less than a nanosecond, while the glider time elapsed during the gate traverse seems to get arbitrarily small as the energy of the launch is increased. This is shown graphically in Fig 2.

The question for students is: What can this mean? What concepts, not needed for low speed motion, might be helpful here (e.g. lightspeed or time-dilation)? Also, is there some simple way to predict glider time elapsed, given the gate time elapsed, or vice versa? Is one of the two clocks wrong, and if so, which one? How would you check this, experimentally (e.g. switching clocks)?

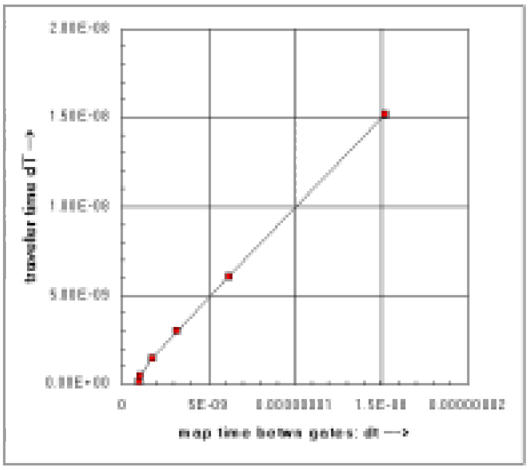

FIG. 2: Plot of glider time versus lab time using data like that in Table II

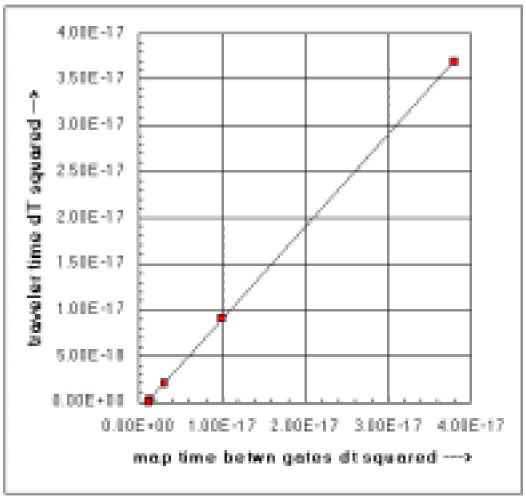

FIG. 3: The squares of glider time versus lab time using data like that in Table II

Fortunately, it is probably now safe to report that moving objects do not behave according to conventional wisdom. Sometimes it isn't. For example, Galileo's discussions of motion and celestial observation, in 16th century Italy, were written into the dialogs of three characters (for, against, and comic relief) in a play so that the author could deny believing them himself. You might recommend the plays[12] as an interesting read about both the importance of careful observation, and of velocity changes, when studying motion.

At the end of the discussion, you can tell students that this data might have been purely an exercise in imagination to generate discussion. In fact it is based on information from real experiments. You might also illustrate how plotting the data points in different ways can lead to simpler visual and symbolic relationships, like those discussed in the section below.

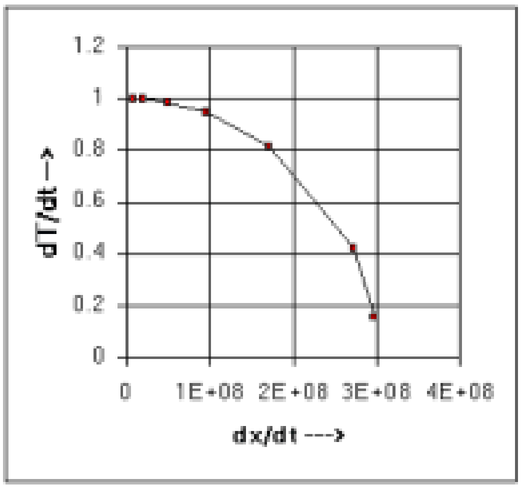

FIG. 4: Plot of rates $\Delta\tau/\Delta t$ versus $\Delta x/\Delta t$, using data like that in Table II

III. MODELING THE METRIC EQUATION

For students familiar with ways to model data, a set of added goals may be attempted given two class sessions, separated by time for at-home analysis and reflection.

A. Objectives:

- See how coordinate velocities ($v \equiv \frac{\Delta x}{\Delta t}$) have an upper limit of lightspeed c. Optional: Note that proper velocities ($w \equiv \frac{\Delta x}{\Delta \tau}$) have no such limit.
- Discover any of three equivalent relations: the flat-space metric equation $(c\Delta\tau)^2 = (c\Delta t)^2 - (\Delta x)^2$, the sum-of-squares relationship between v and $c\frac{\Delta\tau}{\Delta t}$ (outlined later), or the familiar "gamma" equality between $\frac{\Delta t}{\Delta\tau}$ and $\frac{1}{\sqrt{1-(v/c)^2}}$.

B. Prerequisite Abilities:

- Record data (elapsed times and gate separations) on paper or in a spreadsheet.
- Graph various functions of the data, like $(\Delta\tau)^2$ versus $(\Delta t)^2$.
- Attempt to explain the data, with equations and underlying physical mechanisms.

C. Resources:

- Tools (e.g. graph paper, calculators or spreadsheets) for students to graph data on elapsed times, which has either been provided by the instructor, or obtained by the student.
- Optional: Access to a relativistic airtrack (unlikely), or a computer simulation thereof, so that students can acquire their own data.

D. Motivation:

The introduction to this paper may provide some motivation. More extensive lesson plan material is also available on line, as well as information on algorithms and simulators for taking data[13].

E. Creating a dataset: What to put in?

To carefully examine the transition from normal to relativistic speeds, we recommend providing students with (or asking them to take) data like that in Table II, i.e. data in which time elapsed on the glider clock (between gates separated by a foot) ranges between 0.1 and 10 nanoseconds. That's because lightspeed (the constant that relates our units for distance with our units for time) is about 1 foot per nanosecond. Perhaps six data points in this range, with only a few points lying outside, will suffice (unless experimental precision is something the student wants to work on explicitly).

F. Analyzing the data: What to plot?

The first thing to plot once again might be times elapsed (say glider clock time τ versus laboratory clock time t) during passage through the first set of gates, as discussed above for Fig. 2. Since this is not a trivially-shaped curve, the next step is to ask: Can we plot functions of these variables so that simpler shapes (e.g. a straight lines) result. There are many possibilities. For example, one may plot powers or roots or products or ratios or sums or differences of the measured variables, versus other powers or roots, etc.

If one plots the square of time on the glider clock, versus the square of time on the lab clocks, the straight line of Fig. 3 results. Given this, one can write down an equation. Since it is a line of slope 1 with an x-intercept at 10^{-18} [sec^2], an equation that works is $(\Delta\tau)^2 = (\Delta t)^2 - 10^{-18}$ [sec^2]. What can this mean? If one assumes that multiplying the gate separation Δx by a factor of n will increase both times by the same factor (reasonable for constant speed motion), that rightmost constant will be multiplied by n^2 and thus must be proportional to Δx^2. Hence the metric equation follows, written as $(\Delta\tau)^2 = (\Delta t)^2 - (\frac{\Delta x}{c})^2$ with $c \simeq 3 \times 10^8$[m/s].

If instead the student were to take the tact of plotting various ratios of the experimental parameters (in effect, speeds of various sorts), they might have stumbled upon the circular plot in Fig 4. This illustrates an alternate consequence of the metric equation, namely that when not moving through space we "travel at the speed of light through time", with a speed liberally defined in velocity units as c times $\frac{\Delta\tau}{\Delta t}$. As our speed through space ($\frac{\Delta x}{\Delta t}$) increases, the above "rate at which we travel through time" is decreased, so that the sum of squares of the two remains at lightspeed. Thus traveling through space at lightspeed brings the traveler's clock (relative to times measured in the lab frame) to a halt.

G. Connecting data to underlying mechanisms

What concepts might be useful for cashing in on the insight from this experiment? Aristotle might have said to pay attention to motive force and speed. Galileo in the 1500's might have said that accelerations should be considered[12]. Newton in the 1600's might have suggested the concept of momentum, and James Clerk Maxwell may have suggested considering energy[14]. Einstein in 1906 might have agreed, but then stressed the importance of specifying frame of reference when measuring positions and times.

More to the point: Take a look at the concepts used by the students in "explaining" the results. Are they loosely defined, or precisely defined? Do they follow the strategy of others, or are they a bit off the wall? Do they allow one to predict what would happen in future experiments with the same gate separations? How about with different gate separations? Do the "explanations" yield predictions which might be tested by future experiments, or which are relevant to phenomena in everyday life? Do the explanations show too little, or too much, caution? All of these are questions about scientific assertions that the students might want to learn to ask themselves.

H. Assessment:

The question here is perhaps not "Did the students identify the correct relationships, or did they use the correct concepts?". Consider instead "What processes did they use, and how far did they take those processes?". Try applying this question to relative student performance in: (i) data acquisition, (ii) data analysis, and (iii) data interpretation. Of course, then be ready to provide them with your own answers, and perhaps answers others might have provided, to these challenges as well.

IV. DISCUSSION

The second glider in Fig. 1 provides data on conservation of momentum at high speeds. Data on energy conservation is present as well, but due to limits on both classroom time and publication space, these are appropriate subjects for elaboration elsewhere but not here.

Lastly, a caution: These exercises expose students to high speed motion in context of a single lab (or map) reference frame. The moving glider, of course, reports it's clock readings but makes no attempt to relate them to a separate extended framework of moving yardsticks and synchronized clocks. Hence students are exposed to key elements of space-time (e.g. the flat-space metric equation), without being exposed to multi-frame concepts like frame-dependent simultaneity, length contraction, etc. The easy way for the instructor to avoid the pitfalls of everyday vernacular in this larger arena is to limit class discussion to only distances measured in the lab frame, and to only stationary synchronized clocks (e.g. gate pairs). Time on the glider clock itself is thankfully one of the most sharable quantities in relativity: a scalar invariant. We also recommend staying with uni-directional motion, and away from concepts of changing relativistic mass[15], but that is another tale.

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