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Masses in the Weinberg-Salam Model

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CHAPTER I

INTRODUCTION

The remarkable experimental success of the gauge theory of quantum electrodynamics (QED) led in the 60's and early 70's to the idea of extending the gauge symmetry principle [1] to the description of other known interactions. A gauge theory (see [2] for a review) is constructed by requiring the Lagrangian to be locally invariant under a group of internal (gauge) symmetries (i.e. symmetries that do not involve the space-time coordinates). This naturally leads to the introduction of vector fields (gauge bosons) in a number equal to the number of generators of the symmetry group. The structure of their self-couplings (if the group is non-abelian) as well as that of their couplings to matter are then completely determined, by the gauge symmetry, in terms of the gauge couplings. In QED for instance, the Lagrangian is invariant under a set of local U(1) transformations whose generator, Q, is the electric charge operator. Thus, only one vector field is introduced that corresponds to the electromagnetic potential A_μ and the corresponding vector boson is the photon. The quantum field theory of the strong interactions, quantum chromodynamics (QCD), is a gauge theory based on the group SU(3) (it is experimentally required that there be <u>three</u> "color" degrees of freedom [3]). This group has eight generators, therefore eight vector bosons (called gluons) must be introduced.

In the Glashow-Weinberg-Salam (GWS) theory [4], the weak and electromagnetic interactions are described by a local gauge theory based on the group SU(2) x U(1). The group has four generators (T_i (i = 1, 2, 3) for SU(2) and Y for U(1)), thus four vector bosons must be introduced: the photon and the recently detected [5] charged vector bosons $W^\pm$ and neutral vector boson Z. If the gauge symmetry were an exact symmetry of the vacuum, all four vector bosons would be massless, as a mass term for them in the Lagrangian violates the gauge symmetry. However, due to the short range nature of the weak interactions, the $W^\pm$ and Z must be very heavy, thus the symmetry must be broken. A gauge symmetry can be spontaneously broken by elementary scalar fields introduced in the Lagrangian which acquire non-zero expectation values in the vacuum state. With this mechanism [6] the Lagrangian remains invariant under the gauge symmetry, whereas the ground state does not; the result of this is the generation of masses for some of the vector bosons (those that do not correspond to the generators of the symmetries that are left unbroken) accompanied by one or more massive physical scalars (the short range nature of the strong interactions cannot be explained by spontaneous symmetry breaking [7]. Instead, it is postulated in QCD that the gluons are massless and that the long distance, non-perturbative behavior of the theory is responsible for the limited range of the strong interactions [7]). The standard analogy is ferromagnetism, where the Hamiltonian has rotational invariance, but the ground state (below the critical temperature) does not.

In the GWS model, the SU(2) x U(1) symmetry is spontaneously broken down to the $U(1)_{e.m.}$ theory of electromagnetism, and the $W^\pm$ and the Z acquire mass. Fermions are included in the model as left-handed SU(2) doublets and (with the possible exception of the neutrinos) right-handed singlets. As a mass term for them in the Lagrangian violates the gauge symmetry, they acquire masses through their Yukawa couplings to the elementary scalars. These masses, as well as that of the scalars, are arbitrary except for limits to be discussed in this thesis.

The limits to be discussed come primarily from cosmological constraints on the $SU(2) \times U(1) \rightarrow U(1)_{e.m.}$ phase transition. Over the past decade, we have seen remarkable connections emerge between cosmology and astrophysics on the one hand and particle physics and quantum field theory (QFT) on the other. The combined GWS and QCD theories are the standard model of the strong, weak and electromagnetic interactions. The gauge group is $SU(3) \times SU(2) \times U(1)$, which breaks down to the low energy $SU(3) \times U(1)_{e.m.}$ at scales of order 100 GeV. All three interactions, however, can be unified into a gauge theory based on a simple group, with one coupling, which breaks down to $SU(3) \times SU(2) \times U(1)$ at scales of order 10^{15} GeV. These so-called grand unified theories (GUT's) predict baryon number violating processes that can, in principle, explain the observed baryon number to entropy ratio. These theories have dramatic consequences for the evolution of the universe as well; phase transitions occurring at temperatures of the order of the scale of the breakdown lead to a period of exponential expansion of the universe (the so-called inflationary universe [8]) which solves two major cosmological puzzles, the horizon and flatness problems (see Section 2.4), and might be able to explain the origin of galaxies. In turn, standard cosmology provides many constraints on particle physics models. For instance the sum of the masses, m_ν, of light, stable neutrinos has an upper bound [9], $\Sigma\, m_\nu < 40$ eV, obtained from the experimental upper limit on the density of the universe. Also, the requirement that the axion energy density be less than the upper limit on the density of the universe and that they do not carry too much energy away from red giants implies $10^{-5} < m_a/eV < 10^{-2}$ (the axion [10] is a light pseudoscalar associated with the spontaneous breakdown of a symmetry that Peccei and Quinn [11] postulated as the explanation of the absence of CP violation in QCD).

In this thesis we present a detailed discussion of the existing bounds on the otherwise arbitrary masses of scalars and fermions in the GWS model. As it turns out, these constraints come primarily from cosmology and the self-consistency of perturbative grand unification. In Section II we present a brief review of gauge theories (2.1) and spontaneous symmetry breaking (2.2). We then present the GWS model (2.3) and discuss GUT's briefly (2.4). In Section III we review the QFT tools needed to discuss the GWS phase transition in various cases. The effective potential is introduced in 3.1 to discuss symmetry breaking when radiative effects become important. Finite temperature field theory is introduced in Section 3.2 to discuss the GWS transition at finite temperature. As it turns out, metastable phases can occur and it becomes necessary to discuss their decay to energetically favored phases. The decay of false vacua is discussed in 3.3. In Section IV we review the renormalization group equation (RGE) for the effective potential, which allows one to determine the effective potential over a wide range of scales. Finally, in Sections V and VI we discuss in detail the theoretical and experimental bounds on scalar and fermion masses.

CHAPTER II

QUANTUM FIELD THEORIES OF ELEMENTARY PARTICLES

2.1 Gauge Theories

Symmetry principles have long been an essential ingredient of the physical description of nature as well as a powerful constraint on the mathematical theories that are used to describe it. In 1905 Einstein identified the group of symmetries of space and time under which physical laws must be invariant. This is a global symmetry, for which the symmetry transformations are the same for all points in space-time. A more powerful kind of symmetry is a local symmetry in which one requires invariance under transformations that vary from point to point in space-time. Local space-time invariance led to Einstein's General Relativity.

Local invariance under a group of internal symmetries (i.e. symmetries that do not refer to space and time) leads to gauge theories. A local gauge theory is defined by requiring that the Lagrangian be invariant under the local set of transformations (see [2] for a review)

$$\Phi(x) \rightarrow \exp(-i\vec{L} \cdot \vec{\beta}(x))\, \Phi(x) = U(\vec{\beta})\Phi(x) \qquad (2.1)$$

where Φ is a column vector that represents all the matter fields (fermion and boson). $\vec{L} \cdot \vec{\beta}(x) = \Sigma\, \beta_i(x) L_i$, where N is the number of generators, L_i, of the gauge group G (which is equal to the dimension of G) and the β_i's specify an arbitrary element of G at an arbitrary space-time point x (with the restriction that they be non-singular functions of x). The existence of this local symmetry implies the existence of vector (or gauge) bosons, one for each generator of the symmetry group, whose self-interactions, as well as their interactions with matter fields, are completely determined by the gauge invariance. To see this we note that under a gauge transformation

$$\partial_\mu \phi \rightarrow U(\partial_\mu \phi) + (\partial_\mu U)\phi \qquad (2.2)$$

i.e. $\partial_\mu \phi$ does not transform covariantly, therefore a kinetic energy term in the Lagrangian would not be invariant. If we require a derivative term to be covariant

$$D_\mu \phi \rightarrow U(D_\mu \phi) \qquad (2.3)$$

then we are forced to introduce N vector bosons $A_\mu^i (i=1,\ldots,N)$ so as to cancel the extra term above. Thus one defines

$$D_\mu = \partial_\mu - ig\vec{A}_\mu \cdot \vec{L} \qquad (2.4)$$

Demanding D_μ to transform covariantly as in (2.3) implies

$$\vec{A}_\mu \cdot \vec{L} \rightarrow U\vec{A}_\mu \cdot \vec{L}U^{-1} - i/g(\partial_\mu U)U^{-1} \qquad (2.5)$$

The gauge invariant kinetic energy terms for fermions and complex scalar fields are then

$$\mathcal{L}_{kin} = \bar{\psi} i \not{D} \psi + (D_\mu \phi)^\dagger D^\mu \phi \tag{2.6}$$

We see that the gauge invariance determines the form of the couplings between the gauge fields and the matter fields. However, for the A^i_μ not to be just auxiliary fields, we must add a gauge invariant kinetic energy term

$$\mathcal{L}_{kin} = - {}^1\!/\!_4 F_{\mu\nu} F^{\mu\nu} \tag{2.7}$$

where $F_{\mu\nu} \equiv \vec{F}_{\mu\nu} \cdot \vec{L} \equiv \partial_\mu \vec{A}_\nu \cdot \vec{L} - \partial_\nu \vec{A}_\mu \cdot \vec{L} - ig[\vec{A}_\mu \cdot \vec{L}, \vec{A}_\nu \cdot \vec{L}]$. This last term in $F_{\mu\nu}$ is very important; it implies the existence of self-interactions between the gauge fields if $\vec{L} \times \vec{L} = 0$. In the abelian case ($\vec{L} \times \vec{L} = 0$) we see that the vector boson does not carry the charge it couples to, whereas in the nonabelian case they do.

A mass term $\frac{1}{2} M_{ij} A^i_\mu A^{\mu j}$ for the gauge bosons is not gauge invariant, thus it is not allowed. This we know is the case of QED (the upper limit on the photon's mass is 6×10^{-22} MeV), but the weak interactions are short-ranged, thus we know that the gauge bosons of the theory cannot be massless (in QCD the gauge bosons are massless, but color confinement (presumably due to the increase in the color charge at large momenta) gives a short range interaction. The possibility of the weak interactions being strong has been discussed in the literature by Abbott and Farhi [12]). Therefore the symmetry must be broken.

This brings us to the idea of spontaneous symmetry breaking (ssb) to which we now turn for a quick review.

2.2 Spontaneous Symmetry Breaking

The oldest idea of symmetry breaking is that of "approximate" symmetries. One supposes that there are terms in the Lagrangian that violate the symmetry but that they are, in some sense, "small". However, breakage of gauge invariance in the equations of motion completely spoils the renormalizability of the theory and one could not make sense out of the theory beyond the tree-level approximation (another possibility is anomalous symmetry breaking, but here again if the gauge current has an anomaly one cannot consistently build a quantized renormalizable gauge theory.)

However, it is possible that the symmetries of the equations of motion are not respected by their stable solutions. The lowest energy stable solution, the vacuum, may thus be invariant under a smaller subgroup H of the group G of symmetries of the Lagrangian. Then one says that G is spontaneously broken down to H. A simple example is a ferromagnet: the equations of motion are rotationally invariant, yet the spins in a real ferromagnet are aligned in a definite direction. In general, if the vacuum of a gauge field theory has a non-zero distribution of the charge associated with a given generator, then the associated gauge boson will constantly interact with this charge and will develop an effective mass proportional to the expectation value of the charge.

A simple mechanism to implement ssb is the Higgs mechanism [6]. One introduces spin-zero fields into the theory which transform in a non-trivial way under the gauge symmetry. If the vacuum expectation value (vev) of one of these fields is non-zero, then all of the gauge bosons for which that field has non-zero charge will be massive. To see how this takes place, consider a simple U(1) gauge theory with a self-interacting scalar field

$$\mathcal{L} = |D_\mu \phi|^2 - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - U(\phi) \tag{2.8}$$

where $U(\phi)$ is the most general, renormalizable potential

$$U(\phi) = \mu^2\phi^2 + \lambda\phi^4 \; ; \; \phi^2 = |\phi|^2 \tag{2.9}$$

Therefore, $\mathcal{L}$ is invariant under

$$\begin{aligned} \phi &\to e^{-i\theta(x)}\phi \\ A_\mu &\to A_\mu - \frac{1}{g}\partial_\mu\theta \end{aligned} \tag{2.10}$$

If $\mu^2 < 0$, the minimum of the potential is at $|\phi|^2 = -\mu^2/2\lambda$, thus the field ϕ has a non-zero vev $v^2 \equiv |\langle\phi\rangle|^2 = -\mu^2/2\lambda$. As written, is not suitable to do perturbation theory since we would be trying to calculate quantum fluctuations around an unstable solution. To do perturbation theory about the stable solution we translate the field as follows

$$\phi(x) = \frac{1}{\sqrt{2}}(\sigma + \eta(x) + i\chi(x)) \tag{2.11}$$

where $\sigma^2 = -\mu^2/\lambda$. This gives

$$\mathcal{L}' = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\eta)^2 - \frac{1}{2}(2\lambda\sigma^2)\eta^2 + \frac{1}{2}(\partial_\mu\chi)^2 + \frac{1}{2}\sigma^2 g^2 A_\mu A^\mu + \text{(interaction terms)} \tag{2.12}$$

L and L' are of course completely equivalent if we can solve the problem exactly; however we cannot do so and while L gives perturbative nonsense (negative squared mass), L' gives a reasonable spectrum.

Should we have only global U(1) invariance i.e. $L \to L_0 = \partial_\mu\phi\partial^\mu\phi^* - U(\phi)$, then $L' \to L'_0 = \frac{1}{2}(\partial_\mu\eta)^2 - \frac{1}{2}(2\lambda\sigma^2)\eta^2 + \frac{1}{2}(\partial_\mu\chi)^2$ + interaction terms, and the system consists of two interacting scalars with masses $m_\eta^2 = 2\lambda\sigma^2$ and $m_\chi^2 = 0$ (it is the spectrum we would have found starting from L_0 if we could have solved the dynamics exactly). The massless scalar is a Goldstone boson and its presence is quite general: if a field theory has a symmetry of the Lagrangian which is not a symmetry of the vacuum, then Goldstone's theorem [13] assures that there will be at least one massless boson. The number of these bosons is the number of generators of the symmetry group of the Lagrangian minus the number of generators of the symmetry group of the vacuum. However, this is not true for local gauge symmetries.

Note that L' seems to describe a massive vector boson and two scalars, thus a total of five degrees of freedom, while the original has only four (two scalars and a massless vector boson). Since a simple change of variables cannot create new degrees of freedom, L' must contain fields which do not correspond to physical particles. To exhibit this we go back to L (in 2.8) and do the following transformation

$$\begin{aligned} \phi(x) &= \frac{1}{\sqrt{2}}(\sigma + \eta(x))\exp(i\chi(x)/\sigma) \\ A_\mu(x) &= B_\mu(x) + \frac{1}{g}\,\partial_\mu\chi(x) \end{aligned} \qquad (2.13)$$

we now get

$$L'' = -\frac{1}{4} B_{\mu\nu}^2 + \frac{g^2\sigma^2}{2} B_\mu^2 + \frac{1}{2}(\partial_\mu\eta)^2 - \frac{1}{2}(2\lambda\sigma^2)\eta^2 \qquad (2.14)$$

$$+ \frac{\lambda}{4}\eta^4 + g^2 B_\mu^2 (2\sigma\eta + \eta^2)$$

$\chi(x)$ is gone! We have used the gauge degree of freedom to gauge $\chi(x)$ away, therefore L'' is not invariant under gauge transformations. Thus, the theory consists of a massive gauge boson and a massive scalar, four degrees of freedom as we started with. As (2.13) suggests, the χ field becomes the longitudinal component of the massive vector field B_μ. This method of acquiring mass due to ssb is called the Higgs mechanism.

To Summarize:

a) L is invariant under the usual gauge transformations, but it contains a negative squared mass and, therefore, it is unsuitable for quantization (since one wants to do perturbation type things).

b) L' is still gauge invariant, but the transformation laws are more complicated. It can be quantized in a space containing unphysical degrees of freedom (L', in a suitable gauge, is used for general proofs of renormalizability as well as practical calculations).

c) L'' is no longer invariant under any kind of transformations, but it exhibits clearly the particle spectrum of the theory. L'' can be obtained from L' by specifying the gauge of the latter (L'' is not renormalizable by power counting, but since it is gauge equivalent to L', it can be used for practical calculations).

We now turn to the construction of the GWS model.

2.3 The GWS Model

The GWS model [4] is a unified description of the electromagnetic and weak interactions in terms of a gauge theory based on two internal symmetries of the Lagrangian: weak isospin and weak hypercharge, described by the groups SU(2) and U(1) respectively. Ssb takes place via the Higgs mechanism in such a way that the only symmetry of the vacuum is electromagnetic gauge invariance. The weak interactions are then identified as the interactions mediated by the massive vector bosons associated with the broken generators.

The group SU(2) has three generators T_i, $i = 1,2,3$ and therefore there are three associated gauge bosons A^i_μ; the coupling constant is g. Y, B_μ and g' are the generator, gauge field and coupling of U(1) respectively. The couplings g and g' are independent thus, in that respect, the model is not truly a unified model of the electromagnetic and weak interactions. Fermions are put into left-handed SU(2) doublets and right-handed SU(2) singlets. For instance the u and d quarks split as

$$\begin{bmatrix} u_L \\ d_L \end{bmatrix},\ u_R,\ d_R \tag{2.15}$$

where $u_{R,L} = (1 \pm \gamma_5)u$, and the electron and electron neutrino are grouped as

$$\begin{pmatrix} (\nu_e)_L \\ e^-_L \end{pmatrix},\ e^-_R \tag{2.16}$$

(ν_R is absent since it is not observed; note that one can equally formulate the theory in terms of left-handed fields (ν_L, e_L^-) and e_L^+). The group (u, d, e^-, ν_e) forms the first family, with two other families known: (c, s, μ, ν_μ) and (t, b, τ, ν_τ). Also, since left- and right-handed fermions are assigned to different representations, parity violation is automatically built into the model. The hypercharge assignments are chosen so that $Q = T_3 + Y/2$ is the electric charge operator. Since the theory is chiral (i.e. left- and right-handed fields couple differently, thus there will be gauge couplings proportional to γ_5 and, therefore, there will be anomalies), one has to make sure that the anomalies of gauge currents vanish, leading to the prediction that there be a quark for each observed lepton.

In the minimal model one introduces a $Y = +1$, SU(2) doublet of Higgs scalars ϕ (extensions of the Higgs sector will be discussed in Section 5.2), for which the most general renormalizable potential is

$$U(\phi) = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 \qquad (2.17)$$

One chooses $\mu^2 > 0$, so that the potential be minimized at $\langle\phi^\dagger\phi\rangle = \mu^2/2\lambda$. Thus, the vacuum will contain a non-zero expectation value $\langle\phi\rangle$, of the Higgs scalar (ssb takes place even for small, negative μ^2, if λ is sufficiently small, due to radiative corrections. This will be discussed in Section 3.1). Since the potential depends only on $\phi^\dagger\phi$, the orientation of $\langle\phi\rangle$ is not determined. By convention one chooses $\langle\phi\rangle =$ $(0, \sigma/\sqrt{2})$; any other orientation can be brought to the conventional one by an appropriate global SU(2) transformation.

Since $T_i\langle\phi\rangle = (\tau_i/2)\langle\phi\rangle \neq 0$ (τ_i are the Pauli matrices), $Y\langle\phi\rangle = \langle\phi\rangle \neq 0$ and $Q\langle\phi\rangle = (T_3 + Y/2)\langle\phi\rangle = 0$, the symmetries associated with the generators T_1, T_2 and $T_3 - Y$ are spontaneously broken. However, the subgroup generated by $Q = T_3 + Y/2$ is unbroken and it is to be identified with the $U(1)_{e.m.}$ gauge group of electromagnetism. Therefore, by choosing $\mu^2 > 0$, SU(2) x U(1) breaks down to U(1) leaving one massless gauge boson (the photon) and three massive vector bosons (that SU(2) x U(1) breaks down to $U(1)_{e.m.}$ for the entire perturbative domain of the scalar interaction is not the case in models with more Higgs scalars. See Section 5.2).

To work out the particle spectrum of the theory we just have to find the effects of $\langle\phi\rangle$ on the various couplings. First note that a mass term for the fermions is not gauge invariant and it is thus forbidden. However, their SU(2) x U(1) invariant Yukawa couplings to ϕ will give them masses due to $\langle\phi\rangle$. For instance, the most general Yukawa interaction of a group of leptons is

$$\mathcal{L}_Y = -g_Y \bar{L}\phi R + \text{h.c.}$$

$$L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L \quad \text{and} \quad R = \ell_R \tag{2.18}$$

As we did in the previous section, we write $\phi = \exp(i\tau_i\xi_i(x)/2\sigma)(0,(\sigma + \eta)/\sqrt{2})$, where $\sigma^2/2 = \langle\phi^\dagger\phi\rangle$. An appropriate SU(2) gauge transformation, $\phi \rightarrow \exp(i\tau_i\alpha^i(x)/2)\phi$ with $\alpha^i(x) = \xi^i(x)/\sigma$, gets rid of the fields $\xi_i(x)$, giving mass to three of the four vector bosons. The Yukawa term will then be

$$L_Y = -g_Y \bar{L}\begin{pmatrix} 0 \\ (\sigma+\eta)/\sqrt{2} \end{pmatrix} R + \text{h.c.} = -\frac{g_Y}{\sqrt{2}} \bar{\ell}(\sigma + \eta)\ell \qquad (2.19)$$

giving mass to the leptons (the neutrinos remain massless). Thus η has scalar couplings (no γ_5's) and $g_Y = \sqrt{2}\, m_\ell/\sigma$; m_ℓ is arbitrary except for upper limits to be discussed in Section 6.1. The scalar Lagrangian

$$L_S = |\partial_\mu \phi - \frac{i}{2} g' B_\mu \phi - \frac{i}{2} g \tau^i A^i_\mu \phi|^2 - U(\phi) \qquad (2.20)$$

becomes

$$\frac{1}{2} (\partial_\mu \eta)^2 + \frac{(\sigma+\eta)^2}{8} (0,1)(g' B_\mu + g \tau^i A^i_\mu)^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$+ \frac{\mu^2}{2} (\sigma + \eta)^2 - \frac{\lambda}{4} (\sigma + \eta)^4 \qquad (2.21)$$

thus, the scalar η has mass $m_H^2 = 2\mu^2$; it is arbitrary except for limits to be discussed in Chapter V. Defining the fields

$$Z_\mu = \frac{-gA^3_\mu + g' B_\mu}{\sqrt{g^2 + g'^2}} \quad ; \quad A_\mu = \frac{gB_\mu + g' A^3_\mu}{\sqrt{g^2 + g'^2}}$$

and $$W^\pm_\mu = \frac{1}{\sqrt{2}} (A^1_\mu \pm i A^2_\mu) \qquad (2.22)$$

one gets $M_W = g\sigma/2$, $M_Z = \sqrt{g^2 + g'^2}(\sigma/2)$ and $M_A = 0$ from (2.21). Thus, A_μ is the photon and from its coupling to the electron one gets that e =

$gg'/\sqrt{g^2 + g'^2}$ is the positron's charge. The W and the Z are the charged and neutral vector bosons responsible for the weak interactions and have been detected recently by experiments at CERN [5]. By comparing the theoretical value for the muon decay rate, as calculated in the GWS model, to the experimental result, one gets $\sigma^2 = (\sqrt{2}\, G_F)^{-1} = (247\ \mathrm{GeV})^2$.

Conventionally, the weak mixing angle is defined by $g'/g = \tan\theta_W$, thus $g\sin\theta_W = g'\cos\theta_W = e$. Using the known experimental value of σ one can show that (including radiative corrections [4]) $M_W = 38.5$ $\mathrm{GeV}/\sin\theta_W(M_W)$ and M_Z is obtained from $M_Z = M_W/\cos\theta_W(M_W)$ ($\rho = M_W/M_Z\cos\theta_W$ $= 1$ at the tree level in the GWS model. See Section 5.2). The extension of the model to hadrons (see ref. [15]) provides a natural explanation for the suppression of strangeness changing neutral currents. A six quark version of the model provides a natural mechanism for CP violation [16]. The renormalizability of the model was proven in ref. [17].

Several features of the GWS model have been confirmed experimentally. A large number of different experiments give very close values of $\sin\theta_W$, the world average being $\sin^2\theta_W(M_W) = 0.215 \pm 0.012$ [14]. The experiments recently done at CERN [5] confirmed the existence of the W and the Z with masses $M_W = 81 \pm 2$ GeV and $M_Z = 93 \pm 2$ GeV, in good agreement with the theoretical values [14] $M_W = 83.0 \pm 2.4$ GeV and $M_Z = 93.8 \pm 2.0$ GeV (the uncertainties are due to the uncertainties in $\sin\theta_W$).

This concludes our review of the GWS model. The standard model (SM) of the strong, weak and electromagnetic interactions is just the combination of the GWS model with QCD. The gauge group is the direct product $G_s = SU(3) \times SU(2) \times U(1)$ with couplings g_s, g and g' respectively. There are several features of the SM that are not understood, such as the absence of ν_R, the number of families, the value of $\sin\theta_W$, the quantization of electric charge and the arbitrary masses of fermions and scalars. Some of these can be understood in the context of grand unified theories, to which we now turn for a few comments.

2.4 Grand Unified Theories

The idea of grand unified theories (GUTS; see [18] for a thorough review) is to embed the gauge group of the SM in a simple gauge group G_U with just one coupling constant g_U. At energies $-q^2 \sim M_W^2$ (where q^2 is some typical momentum transfer squared) we already mentioned that the strong, weak and electromagnetic interactions are described by a SU(3) x SU(2) x U(1) symmetry with very disparate couplings; these couplings are scale dependent, however, and one finds [19] that they do come together at $-q^2 \sim M_X^2 \sim (10^{15}\ \text{GeV})^2$ with $\alpha_s(M_X) = \alpha_w(M_X) = \alpha'(M_X) = \alpha_U \sim 1/40$ ($\alpha_i = g_i^2/4\pi$). The evolution of the couplings is depicted in Fig. (2.1).

Thus, G_U breaks down to SU(3) x SU(2) x U(1) at $-q^2 \sim M_X^2$ and this in turn, breaks down to $SU(3) \times U(1)_{e.m.}$ at $-q^2 \sim M_W^2$. Another reason to expect the GUT symmetry to break at a very large scale is the stability of the proton. In GUTS, the electric charge operator, Q, is a generator of the group, and in most GUTS this means that Q is traceless. Thus, it is not possible to put the quarks alone into a fermion representation of the GUT since the sum of the quark charges is not zero in a given family. Instead, some antiquarks or leptons must be in the same representation as some of the quarks. This means that some of the gauge vector bosons of GUTS must mediate transitions from quarks to antiquarks or leptons (these are sometimes called leptoquarks and are denoted by X and Y in $G_U = SU(5)$ [20]). This generally leads to baryon and lepton number violating processes and, thus, to proton decay (there are models [21] with stable protons however). A typical diagram is given in Fig. 2.2. Treating the proton decay as a four-Fermi interaction in analogy with muon decay [22], one gets a rough estimate $\tau_p \sim M_X^4/\alpha_U^2 m_p^5$ (where m_p is the proton mass). Since the current experimental lower limit on proton decay is $\tau_p(p \to e^+\pi^0) > 10^{32}$ years [23], $M_X > 10^{15}$ GeV.

Another very important effect of the baryon number violating reactions lies in the early universe at temperatures $T \sim M_X$. All observations indicate the absence of antimatter in the universe [24]. Thus, the net baryon number density if non-zero and [25]

$$\frac{\Delta n_B}{n_\gamma} = n_{baryon} - n_{antibaryon}/n_\gamma \sim \frac{n_{baryon}}{n_\gamma} \sim 10^{-10\pm 1} \qquad (2.23)$$

Grand unified theories give baryon number violating reactions, as well as C and CP asymmetries; when these reactions occur out of thermal equilibrium (due to the expansion of the universe), it is possible for them to generate a net baryon number which is at most $n_b/s \sim 10^{-4}$ (but it is most typically $n_b/s \sim 10^{-10}$ [26]); s is the entropy density here and at the present $s = 7.02\, n_\gamma$. As will be discussed in Chapter V, if there are no other processes that can generate baryon number at the electroweak scale (there are models [27] with baryon number violating processes at scales O(1 TeV)), one can set a lower limit on the Higgs mass by requiring that the entropy produced in the $SU(2) \times U(1) \to$

$U(1)_{e.m.}$ phase transition not dilute the baryon to entropy ratio beyond the experimental limits.

Another consequence of having leptons and quarks in the same representation is that there are mass relations among them, like $m_b = m_\tau$ (this is at the GUT scale). When the appropriate Yukawa couplings are scaled down to the low energy sector, one gets the very successful prediction $m_b \sim 3m_\tau$ (for the d and s quarks the masses cannot be reliably calculated because QCD becomes strong.)

GUTS have had a profound impact upon cosmology also. The possibility of phase transitions in the early universe led Guth [28] to suggest a variant of the big bang cosmology, the so-called inflationary universe [8]. In this model, a period of sufficient exponential expansion of the universe solves two major cosmological puzzles. One is the horizon problem: distant structures in the visible universe were not in causal contact at very early times if the standard big bang cosmology is correct, thus the observed homogeneity and isotropy of the entire visible part of the universe is a mystery. The other is the flatness problem: the density of the universe, ρ, is very close to the critical density ρ_c, which separtes an open ($\rho < \rho_c$) and a closed ($\rho > \rho_c$) universe. However, ρ_c is an unstable fixed point in an expanding universe and any deviation will grow in time. This, ρ has to be initially fine-tuned to ρ_c with an incredible precision to account for the small deviation today. During a period of inflation, a small, causally connected region expands exponentially to encompass the entire visible universe and this region will be essentially flat, thus solving both puzzles.

CHAPTER III

FIELD THEORY METHODS IN COSMOLOGY

3.1 The Effective Potential

Our discussion of the Higgs mechanism has been entirely classical so far. However, if λ is small quantum corrections to the classical potential $U(\phi)$ become crucial. One can define a function called the effective potential, $V(\phi_c)$, such that its minima give the true vacuum states of the theory exactly, without any approximation. It can be proved that this effective potential is the expectation value of the energy density in a quantum state for which the expectation value of the field ϕ is ϕ_c [24]. Therefore only the absolute minimum of V corresponds to the true ground state of the theory: the vacuum.

Spontaneous symmetry breaking occurs if the quantum field ϕ develops a non-zero expectation value in the vacuum. One can show that this implies that $dV/d\phi_c = 0$ for $\phi_c = \langle\phi\rangle \neq 0$. Therefore one can study ssb just as in classical field theory, but we replace $U(\phi)$ by $V(\phi_c)$. Thus, the Higgs mechanism discussion goes through just as in the classical field theory case.

Of course, one cannot calculate $V(\phi_c)$ exactly; that would imply solving the quantum field theory exactly. A very useful approximation to V, however, is the so-called loop expansion [30]: in the diagramatic expansion for $V(\phi_c)$ one sums first all diagrams with no loops in them (tree graphs), then those with one loop in them, etc ... One can show that this expansion in the number of loops is equivalent to an expansion in powers of h [31] with the result that $V(\phi_c) = U(\phi_c) + O(h)$, thus the tree-level potential is just the classical potential $U(\phi)$. The usefulness of the loop expansion resides in the fact that it corresponds to an expansion in a parameter that multiplies the total Lagrange density and, thus, it is independent of shifts of fields and of the redefinitions of the division of free and interacting parts of the Lagrangian that are associated with such shifts. This calculational procedure, therefore, allows us to survey all the vacua at once at any given level of approximation.

To illustrate the loop expansion we now calculate the scalar loop contribution to $V(\phi_c)$ as a sample computation [32]. Consider a theory defined by

$$L = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4 \qquad (3.1)$$

so that $U(\phi) = -\mu^2\phi^2/2 + \lambda\phi^4/4$. The loop expansion for $V(\phi_c)$ is depicted in Fig. 3.1. Each external leg carries a factor of ϕ_c. Thus, the first graph will have a factor ϕ_c^4 because of the four external legs, a factor 1/4 because of permutations of external legs and a factor $3!\,\lambda$ for the quartic (self-interaction) term (we are considering the first two terms in 3.1 to be the free Lagrangian). Thus, at the tree level (zeroth order in the loop expansion)

$$V(\phi_c) = V_o(\phi_c) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 = U(\phi_c) \qquad (3.2)$$

Then n^{th} one-loop graph has a factor ϕ_c^{2n} due to external legs, a factor $[(3!\,\lambda)/2\,]^n$ for the n vertices (the 1/2 is due to permutations of internal lines in a vertex), a factor $(k^2 + \mu^2)^{-n}$ for the n propagators and a combinatorial factor 1/2n. The result of summing all these graphs is a logarithm and (rotating to Euclidean space and throwing away an infinite, ϕ_c - independent term)

$$V(\phi_c) = U(\phi_c) + \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\ln(k^2 + M^2(\phi_c)) \tag{3.3}$$

where $M^2(\phi_c) = -\mu^2 + 3\lambda\phi_c^2$ is the squared mass in the shifted Lagrangian $(\phi \to \phi - \phi_c)$. Integrating over k_0 (discarding an infinite piece which does not depend on $M^2(\phi_c)$) we get

$$V(\phi_c) = U(\phi_c) + \frac{1}{(2\pi)^3}\int d^3\vec{k}\,\sqrt{k^2 + M^2(\phi_c)} \tag{3.4}$$

Thus, the one-loop effective potential is the sum of the classical potential and the zero-point oscillations of ϕ_c. Finally, defining the renormalized μ^2 and λ as

$$\mu_R^2 \equiv -\frac{d^2V}{d\phi_c^2}\Big|_{\phi_c^2} \quad ; \quad 6\lambda_R \equiv \frac{d^4V}{d\phi_c^4}\Big|_{\phi_c = M_R} \tag{3.5}$$

we can do the integral and get

$$V(\phi_c) = U(\phi_c) + \frac{M^4(\phi_c)}{64\pi^2}\ln\frac{M^2(\phi_c)}{M_R^2} \tag{3.6}$$

where μ^2 and λ in $U(\phi_c)$ are the renormalized quantities and M_R is the renormalization mass used to define λ; it is arbitrary and different M_R's refer to different λ's.

In the GWS model one gets a similar result for vector and fermion loops but $M^2(\phi_c)$ is now the vector (fermion) squared mass in the shifted Lagrangian $M_W^2(\phi_c) = g\phi_c^2/4$ and $M_Z^2(\phi_c) = (g^2 + g'^2)\phi_c^2/4$ (for a fermion $M_f^2(\phi_c) = g_f^2\phi_c^2/2$, where g_f is the Yukawa coupling to the Higgs scalar). Therefore[1]

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 + B\phi^4 \ln\frac{\phi^2}{M_R^2} \tag{3.7}$$

where ϕ is the classical (not the quantum) field and $\phi^2 = 2\phi^\dagger\phi$ (ϕ is the $Y = 1$ isodoublet of scalars. See Section 2.3). Here

$$64\pi^2 B\phi^4 = 6M_W^4 + 3M_Z^4 + M^4 - 12\sum_{\text{quarks}} M_q^4 - 4\sum_{\text{leptons}} M_\ell^4 \tag{3.8}$$

and the couplings are evaluated at the renormalization point M_R. Minimizing $V(\phi)$, we can write the effective potential in terms of its minimum[2] $\sigma^2 = 2\langle\phi^\dagger\phi\rangle$:

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\left(\frac{\mu^2}{\sigma^2}\right)\phi^4 + B\phi^4\left(\ln\frac{\phi^2}{\sigma^2} - \frac{1}{2}\right) \tag{3.9}$$

The first surprise we encounter is that as $\mu^2 \to 0$, ssb remains in the theory (this possibility was first realized by Coleman and Weinberg [32]). The effective potential of (3.9) is depicted in Fig. 3.2 for various values of $\mu^2 < 0$; we see that there is an asymmetric minimum of V even for negative μ^2. The Higgs mass is calculable in the $\mu^2 \to 0$ limit and (if there are no heavy fermions)

$$m_H^2 = \frac{d^2V}{d\phi^2}\Big|_{\phi=\sigma} = 8B\sigma^2 \equiv m_{CW} = (10.4\ \text{GeV})^2 \qquad (3.10)$$

This mass will turn out to be crucial in the discussion of lower limits to the Higgs mass.

3.2 Finite Temperature Field Theory

At very early times the universe was very hot if we assume the standard hot big bang cosmology and, therefore, finite temperature affects are very important. Finite temperature field theory has been extensively discussed in the literature [33]. To take into account the heat bath at temperature β 1/T, one replaces vacuum averages by thermal averages

$$\langle O \rangle = Tr(e^{-\beta H}\ O)/Tr(e^{-\beta H}) \qquad (3.11)$$

where H is the Hamiltonian and O is any operator. One can show that the finite temperature field theory in Euclidean space-time is identical to the T = 0 theory but will all Green's functions periodic in Euclidean time with period β.[3]

The Feynman rules are thus unchanged and so are the propagators, but the boundary conditions imply that k_o is now discrete, $k_o = 2\pi nT$ (n = 0, ±1, ±2,...). Therefore, one substitutes $\int d^4x \rightarrow \int d\tau \int d^3\vec{x}$ and $\int d^4k \rightarrow \Sigma\ 2\pi\ nT \int d^3\vec{k}$. We can now find the effect of finite T in our calculation of the effective potential by going back to equation (3.3) and redoing the integrals with the appropriate replacements. One then finds that

$$V(\phi,T) = V(\phi) + V_T(\phi,T) \tag{3.12}$$

where $V(\phi)$ is the zero-temperature effective potential and

$$V_T(\phi,T) = \frac{T^4}{(2\pi)^4} \int dx\, x^2 \ln(1-\exp-\sqrt{x^2 + M^2(\phi)/T^2}) \tag{3.13}$$

i.e. $V_T(\phi,T)$ is just the free energy of a noninteracting Bose gas [34]. It should be noted that $V_T(\phi,T)$ is finite without any need of renormalization, thus the couplings that enter $V_T(\phi,T)$ are the renormalized couplings. Adding the contributions from all particles one gets.

$$V_T(\phi,T) = \pm \frac{T^4}{(2\pi)^4} \sum n_i \int x^2\, dx \ln(1 \mp \exp - (x^2 + M_i^2(\phi)/T^2)^{1/2} \tag{3.14}$$

where n_i is the number of spin degrees of freedom of the particle i, M_i is its mass in the background field ϕ and the upper (lower) sign refers to bosons (fermions).

To understand the qualitative effect of the added thermal free energy, one can expand V_T for $M^2(\phi)/T^2 << 1$. One then gets

$$B_T = -\frac{\pi^2}{90}\, \eta T^4 + \frac{T^2}{24} \sum n_i M_i^2(\phi) + \dots \tag{3.15}$$

where $\eta = \sum_{bosons} n_i + 7/8 \sum_{fermions} n_i$. The second term is usually positive [35] and will thus raise the free energy of any asymmetric state relative to the $\phi = 0$ state and, therefore, at high enough temperature the asymmetry is restored since $\phi = 0$ is the only minimum of the effective potential. If $V_o(\phi)$ has an asymmetric minimum at $\phi = \sigma$ ($V_o(0)$

$> V_0(\sigma)$), then this minimum will first appear at a nearby point $\phi < \sigma$ at a temperature $T = T_C \gtrsim T_c$, where $T_c \sim \sigma$ is the temperature at which the asymmetric and symmetric states become degenerate (see Fig. 3.3). However, at this temperature there is a barrier between the two minima and the symmetric vacuum becomes metastable for $T < T_c$. Thermal and quantum fluctuations will then tend to force the false vacuum to decay into the true asymmetric ground state. Since the universe is expanding, it becomes crucial to calculate the decay rate per unit volume, Γ, to determine the temperature at which the transition is completed. Of course, if $d^2V_0/d\phi^2(\phi=0) < 0$ then the symmetric vacuum becomes unstable (i.e. the barrier between the vacua disappears) at $T^2 = T_c^2 \sim -d^2V_0/d\phi^2(\phi=0)$ and the transition must be completed at a near temperature, but if $d^2V_0/d\phi^2(\phi=0)$ is very nearly zero or positive, the universe can supercool enormously before Γ becomes significant to overcome the expansion of the universe.

3.3 Decay of the False Vacuum

As we found in Sections 3.1 and 3.2, the effective potential can sometimes have more than one minimum at $T = 0$ or may do so at high enough temperatures. Although classically both minima are stable configurations, quantum tunneling or thermal fluctuations render stable only the absolute minimum of the potential, the true vacuum.

At zero temperature, the decay of the false vacuum [36] proceeds via quantum tunneling which causes the nucleation of bubbles of true vacuum. After being created, the bubbles expand in the false vacuum at a speed that very rapidly approaches that of light, thus converting metastable phase into stable phase. As shown in [36], the decay rate per unit volume (in the semiclassical approximation) is

$$\Gamma = A \exp - B \qquad (3.16)$$

where $A = \Upsilon\sigma^4$ and Υ is presumed to be O(1). B is the Euclidean action of the least action solution (the so-called "bounce"; it can be proved that the least action solution must be O(4) - symmetric [37]) to the equation

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho}\frac{d\phi}{d\rho} = \frac{dV}{d\phi} \quad ; \quad \rho = \sqrt{t^2 + x^2} \qquad (3.17)$$

subject to the boundary conditions

$$\frac{d\phi}{d\rho}\Big|_{\rho=0} = 0 \quad ; \quad \phi \underset{\rho \to \infty}{\to} 0 \qquad (3.18)$$

The first condition ensures the finiteness of the action (after tunneling the field has zero kinetic energy) and the second that far from the site of nucleation the universe remain in the false vacuum. Here $V(\phi)$ is the effective potential and the action is given by

$$B = 2\pi^2 \int \rho^3 \, d\rho \left(\frac{1}{2}\left(\frac{d\phi}{d\rho}\right)^2 + V(\phi)\right) \qquad (3.19)$$

At finite temperature [38] the solution to (3.17) can be required to be O(3)-symmetric only since the solution must be periodic in Euclidean time with period T^{-1} and the full temperature-dependent effective potential should be used in (3.17) and (3.19) (since the decay rate is given by the imaginary part of the free energy at finite temperature). At high temperatures the action is of the form $B = E(T)/T$, where $E(T)$ is the free energy of a bubble of critical size at temperature T. At very low temperatures on the other hand, quantum tunneling dominates and $B = B(T=0)$. One then finds [38] that unless the barrier disappears, $B(T)$ will reach a minimum (correspondingly, the tunneling rate a maximum) at a temperature T^* and that if the transition is not completed by then it will be completed only at exceedingly small temperatures.

To determine the temperature T_i at which the transition is completed, one calculates the fraction of space remaining in the metastable phase as a function of temperature and T_i is then the temperature at which this fraction becomes negligible. This fraction, as a function of time, is given by [38]

$$p(t) = \exp - \int dt' \, f(t') R^3(t') V(t',t) \qquad (3.20)$$

where $V(t',t) = 4\pi[\int dt'' \, R^{-1}(t'')]^3/3$ is the coordinate volume at time t of a bubble formed at t' and R is the scale factor of the universe. As the universe cools down in the symmetric phase RT = constant. From Einstein's equations then[4]

$$(\dot{R}/R)^2 = (8\pi/3M_P^2)\rho \qquad (3.21)$$

we get

$$(\dot{T}/T) = -\chi g(T) \tag{3.22}$$

where $M_p \sim 1.2 \times 10^{19}$ GeV is the Planck mass, ρ is the energy density of the symmetric phase

$$\rho = \rho_0 + \frac{3\pi^2}{90} \eta T^4 = \rho_0 \, g^2(T) \tag{3.23}$$

ρ_0 is the symmetric vacuum's energy density and $\chi = (8\pi\rho_0/3M_p^2)^{1/2}$. Therefore [38]

$$p(T) = \exp - b \int_T^{T_c} dT' \; \frac{e^{-B(T')}}{g(T')T'^4} \left[\int_T^{T'} \frac{dT''}{g(T'')}\right]^3 \tag{3.24}$$

where $b = 4\pi\gamma\sigma^4/3\chi^4 \approx \exp(170)$.

For $T \lesssim T_c$, $g(T) = 1$ and $B(T)$ reaches a minimum at $T = T^*$, thus the main contributions to $p(T)$ in (3.24) will come from $T = T^*$ and $T = 0$. Approximately the integral by the sum of these contributions gives (for $T < T^*$)

$$p(T) \simeq \exp - b\left(e^{B(T^*)}\left(\frac{T^*-T}{T^*}\right)^3 \frac{T_c}{T^*} + e^{-B(T=0)} \ln \frac{T_c}{T}\right) \tag{3.25}$$

We see that if $B(T^*) > \ln b \approx 170$, thermal fluctuations are never sufficient and it is quantum tunneling what drives the phase transition, but since this effect increases only logarithmically with T, the transition will be completed at exceedingly small temperatures. On the other hand, if $B(T^*) < \ln b$, then $p(T)$ becomes negligible as soon as T drops below T^*, and therefore $T_i = T^*$.

The entropy density right before the transition is

$$S_i = -dV(0,T)/dT(T = T_i) = \frac{4\pi^2}{90} n_i T_i^3 \qquad (3.26)$$

where n_i is the number of massless boson degrees of freedom plus 7/8 of the number of massless fermion degrees of freedom in the symmetric phase. Similarly, the entropy density right after the transition is

$$S_f = 4\pi^2 n_f T_f^3/90 \qquad (3.27)$$

where n_f is as n_i but for the particles with masses $< T_f$. Therefore the decay of the false vacuum increases the entropy density by a factor

$$S_f/S_i = (n_f/n_i)(T_f/T_i)^3 \qquad (3.28)$$

Typically $(n_f/n_i) \sim 1$. T_f is found by requiring the energy density, $\rho(\phi,T) = V(d,T) - TdV/dT$, to be the same before and after the transition (the universe reheats to T_f due to the latent heat released in the transition), thus $\rho(0,T_i) = \rho(\sigma,T_f)$.

As will be discussed in Chapter V, requiring the ratio (3.28) not to be too large in the $SU(2) \times U(1) \to U(1)_{e.m.}$ transition, so that the baryon to entropy ratio not be diluted beyond its currently known value ($n_b/s \sim 10^{-11\pm1}$ [25]), will be a powerful constraint on the Higgs mass in the GWS model.

CHAPTER IV

RENORMALIZATION GROUP EQUATION FOR THE EFFECTIVE POTENTIAL

As we saw in Section 3.1, the explicit expresion for the effective potential involves a renormalization scale M_R which is arbitrary; its only purpose is to define the renormalized couplings through equations like (3.5). Therefore, the full effective potential satisfies the following RGE [32]

$$M_R \frac{dV^{full}}{dM_R} = (M_R \frac{\partial}{\partial M_R} + \beta_i \frac{\partial}{\partial g_i} + \gamma\phi \frac{\partial}{\partial \phi})V^{full} = 0 \tag{4.1}$$

(we have used the chain rule here) which expresses the requirement that the physics be invariant under changes in the renormalization mass M_R. Thus, a small change in M_R can always be compensated for by an appropriate small change in the couplings and a small rescaling of the fields. Here $\beta_i = M_R\, dg_i/dM_R$ and γ is the anomalous dimension of the field ϕ, $\gamma \equiv (M_R/Z)dZ/dM_R$ (Z is the wave function renormalization factor).

To explore the implications of equation (4.1) we get rid of $M_R\, \partial/\partial M_R$ using dimensional analysis. Define $t = \ln \phi/M_R$ and consider, for simplicity, just dimensionless couplings g_i. Then we get

$$[\frac{\partial}{\partial t} - \frac{4\gamma}{1-\gamma} - \frac{1}{1-\gamma} \beta_i \frac{\partial}{\partial g_i}]V = 0 \tag{4.2}$$

As is well known,one can construct the general solution to (4.2) by finding the solutions to

$$\frac{d\bar{g}_i}{dt} = \frac{\beta_i(\bar{g}_i)}{1-\gamma(\bar{g}_i)} \qquad (4.3)$$

subject to the boundary conditions that $\bar{g}_i(0,g_i) = g_i(\bar{g}_i(t,g_i)$ is the usual running coupling). Then, the general solution to (4.2) for a purely quartic potential is

$$V = \phi^4 f(\bar{g}_i(t,g_i))\exp 4 \int^t dt' \frac{\gamma(\bar{g}_i(t',g_i))}{1-\gamma(\bar{g}_i(t',g_i))} \qquad (4.4)$$

where f is an arbitrary function. Note that (4.4) is the exact solution for the full effective potential. The loop expansion occurs when we use the loop expansion for β and γ.

The usefulness of (4.3) is that it may allow us to approximate V over a larger range of t. The usual loop expansion, as in equation (3.7), requires both that t and the couplings be small. However, if the solution to (4.3) remains small over a large range of t, then we can trust the approximation over that entire range of t, thus giving us a good approximation to V over a large range of t.

As a simple example of the usefulness of the RGE-improved potential consider the case of a massless self-interacting scalar, so that

$$U(\phi) = \lambda\phi^4/4 \qquad (4.5)$$

The one-loop potential will be (see Section 3.1)

$$V(\phi) = \lambda\phi^4/4 + \frac{(3\lambda\phi^2)^2}{64\pi^2} \ln \frac{3\lambda\phi^2}{M_R^2} \qquad (4.6)$$

Defining $6\lambda_R$ $d^4V/d\phi^4(\phi = M_R)$ and absorbing ln3λ in the definition of the λ coefficient of the quartic term in (4.5), we get

$$V(\phi) = \lambda \frac{\phi^4}{4} + \frac{(3\lambda\phi^2)^2}{64\pi^2} [\ln \frac{\phi^2}{M_R^2} - \frac{25}{6}] \qquad (4.7)$$

It would thus seem that the minimum at the origin has been turned into a maximum and that a minimum has appeared at $\phi = \phi_{min} \sim M_R \exp 1/\lambda$. However, at both points $\ln\phi/M_R >> 1$ and the approximation cannot be trusted.

At the one-loop level, $\beta_\lambda = 9\lambda^2/8\pi^2$ and $\gamma = 0$, therefore the approximate equation for the running coupling is

$$\frac{d\bar{\lambda}}{dt} = \frac{9\bar{\lambda}^2}{8\pi^2} \quad ; \quad \bar{\lambda} = \frac{\lambda}{1-9\lambda t/8\pi^2} \qquad (4.8)$$

Normalizing the solution of (4.4) so that $V(M_R) = \lambda M_R^4/4$, $V(\phi) = \bar{\lambda}(t)\phi^4/4$. Thus

$$V(\phi) = \lambda\phi^4/4(1 - 9\lambda t/8\pi^2) \qquad (4.9)$$

This agrees with the one-loop potential in the region $\lambda << 1$, $|\lambda t| << 1$. However, for large negative t, $|\bar{\lambda}(t)| << 1$ and the approximation (4.9) can be trusted. Thus, the RGE-improved potential is a good approximation near the origin and predicts a <u>minimum</u> rather than a maximum. We cannot solve the problem of the minimum away from $\phi = 0$ since for large t $\bar{\lambda}$ becomes large and we cannot trust the RGE-improved potential either.

We will use again the RGE-improved potential in Chapter VI to study the CW potential (i.e. the potential in the $\mu^2 \to 0$ limit. See Section (3.1)) when heavy fermions are present.

CHAPTER V

BOUNDS ON HIGGS MASSES

5.1 The Minimal GWS Model

As discussed in Section 2.3, the minimal model contains one $Y = 1$ doublet ϕ of scalars for which the most general potential is

$$U(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \qquad (5.1)$$

and the mass of the physical scalar is $m_H^2 = 2\mu^2$. If the scalar's mass is light (i.e. $\mu^2/\sigma^2 \ll 1$ or, equivalently, $\lambda \ll 1$), quantum corrections to the potential above are important and the effective potential becomes (see Section 3.1)

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\left(\frac{\mu^2}{\sigma^2}\right)\phi^4 + B\phi^4\left(\ln\frac{\phi^2}{\sigma^2} - \frac{1}{2}\right) \qquad (5.2)$$

and the scalar's mass is $m_H^2 = d^2V/d\phi^2(\phi = \sigma) = 2\mu^2 + m_{CW}^2$, where m_{CW}^2 $8B\sigma^2 = (10.4\ \text{GeV})^2$ is the Coleman-Weinberg mass: the scalar's mass in the $\mu^2 \to 0$ limit. This potential is graphed in Fig. 3.2. We see that even for negative μ^2 there can be ssb, but for $\mu^2 < 0$ there will be a barrier between the symmetric and asymmetric minima. As discussed in Section 3.2, at high temperatures the symmetry is restored and only the symmetric vacuum exists. As the temperature lowers, the asymmetric vacuum forms and the transition from the symmetric to the asymmetric vacuum must occur (since we live in the asymmetric vacuum today). The first lower limit on the Higgs mass was due to Linde and Weinberg [39] who noted that if the asymmetric vacuum has a higher energy than the asymmetric vacuum at $T = 0$, then the transition cannot occur. From Fig. 3.2 we see that this occurs for $m_H^2 \lesssim \frac{1}{2} m_{CW}^2$ and, therefore, $m_H \gtrsim m_{CW}/\sqrt{2} \sim 7$ GeV. However, one can rule out even higher masses because for $m_{CW} \gtrsim m_H \gtrsim m_{CW}/\sqrt{2}$ a barrier between the two vacua remains at all temperatures and the only way the transition can go is via tunneling. Linde [40] found that the transition would be unacceptably slow (causing too much inhomogeneities to be compatible with the homogeneity of the microwave background) if $m_H \lesssim 0.99\ m_{CW}$. However, Guth and Weinberg [38] pointed out that one should require that the transition not generate too much entropy, since no process that can generate baryon number during or after the transition is known and, thus, if too much entropy is generated in the transition it would wipe out the baryon to entropy ratio, which is known to be $n_b/s_\gamma \sim 10^{-11\pm1}$ today (no mechanism is known that can produce a baryon excess density of more than 10^{-4} relative to the entropy density, therefore an entropy increase ratio of about 10^{6-7} is about as much as can be tolerated). To study the entropy production in the transition one calculates the effective potential at $T = 0$ (see Section 3.2). With this potential we can calculate the entropy increase in the manner discussed in Section 3.3; the result is plotted in Fig. 5.1. The sensitivity of the entropy production to the Higgs mass is easy to understand. For $m_H \gtrsim m_{CW}$ ($\mu^2 > 0$) the symmetric vacuum becomes unstable at $T^2 = T_c^2 \sim - d^2V(T = 0)/d\phi^2(\phi = 0) = \mu^2 > 0$ and the transition must be completed at a near temperature, therefore $T_i \sim \mu$. The reheating temperature, T_f, is fairly insensitive to T_i and $T_f \sim 10$

GeV, thus $S_f/S_i \sim (T_f/T_i)^3 \sim (10 \text{ GeV}/\mu)^3 < 10^6$ if $\mu > 0.1$ GeV. We can expect the entropy increase not to be excessive then, for $m_H \gtrsim 1.01\, m_{CW}$. However, as m_H approaches m_{CW} ($\mu^2 \to 0^+$) the symmetric vacuum can stay to much lower temperatures and at $m_H = m_{CW}$, Guth and Weinberg found, tunneling is the only way out of the symmetric phase and the nucleation rate overcomes the expansion of the universe only at exceedingly low temperatures, $T_i \sim 1$ keV. The situation for $\mu^2 < 0$ is, of course, much worse since in that case the barrier between the vacua never disappears. This result would indicate, then, a lower limit on m_H slightly greater than m_{CW}.

It was later pointed out by Witten [41] that very near CW ($\mu^2 = 0$) chiral symmetry breaking (χsb) drives the transition much before quantum tunneling can play any significant role. The Lagrangian contains Yukawa terms, $\bar{\psi}\psi\phi$, which add a linear term to the potential when chiral symmetry breaks down (i.e. $\langle\bar{\psi}\psi\rangle \neq 0$). This linear term erases the barrier between the vacua and the transition occurs. Thus, the transition temperature T_i is near the critical temperature T_0 (200 - 400 MeV [42]) of the χsb transition. Therefore the entropy increase due to the $SU(2) \times U(1) \to U(1)_{e.m.}$ transition (the χsb transition does not produce a significant amount of entropy [43]) is $S_f/S_i \sim (T_f/T_0)^3 \sim 10^{4-5}$. The entropy production as a function of the Higgs mass including the effects of χsb is plotted as the full line in Fig. 5.1. It thus seems that $m_H = m_{CW}$ is acceptable, but it would require a very efficient baryogenesis in the early universe. It was later noted in ref. [44] however, that the presence of heavy fermions significantly alter the entropy production in the CW $SU(2) \times U(1) \to U(1)_{e.m.}$

transition. As was shown there (see Section 6.2), the presence of heavy fermions creates a nearly-symmetric state ($\langle\phi\rangle \sim 1$ GeV) from which the transition to our vacuum today (i.e. the vacuum with zero vacuum energy density) would generate an excessive amount of entropy if the mass of, say, the top quark $m_t \gtrsim 65$ GeV, thus ruling out $m_H = m_{CW}$ in that case. We conclude that a lower limit on m_H of approximately m_{CW} exists and that $m_H = m_{CW}$ is acceptable only for a limited range of fermion masses.

In contrast to the case of lower limits, which are phenomenologically necessary, there are no phenomenological upper limits on the Higgs mass. However, the Higgs self-coupling grows with energy (unless there are very heavy fermions present (see Section 6.2)) until it becomes significantly greater than one and perturbation theory breaks down. This fact allows one to define a different type of "upper limit". These limits are all based on the assumption that perturbation theory be valid (either at the electroweak scale or up to the GUT scale); this is of course desirable but at no point necessary (it was shown in ref. [45] that a nonperturbative Higgs sector would have very little impact on current phenomenology) and their violation would imply new physics at the scale of the breakdown. Once the Higgs becomes considerably massive ($m_H \gg m_{CW}$) $m_H = 2\lambda\sigma^2$, thus an upper limit on λ becomes an upper limit on m_H.

The first upper limit on m_H was discussed in ref. [46] where it was noted that the requirement that two body reactions of gauge bosons respect partial wave unitarity places an upper limit of $(8\pi\sqrt{2}/3G_F)^{1/2} \sim 1$ TeV on m_H. If the Higgs mass is slightly below this value, however, higher order corrections may be extremely important.

The one-loop RGE for λ is of the form (see Section 6.2).

$$\mu \frac{d\lambda}{d\mu} = a\lambda^2 + b\lambda + c \tag{5.3}$$

where a, b and c are independent of λ (but may depend on the gauge and Yukawa couplings). As discussed in [47] the solution to this equation grows with energy until it diverges at some scale; if $\lambda(M_W)$ is large enough, this scale will be smaller than the GUT scale. It is not clear whether one should require $\lambda(M_X)$ to be less than 1, 4π, etc..., but the existence of a singularity clearly shows a breakdown of perturbation theory. Thus, the upper limit becomes [47] $m_H \lesssim 175$ GeV if all fermion masses are below 150 GeV.[5] A larger Higgs mass would rule out any numerical predictions from grand unified theories.

There are no experimental constraints, as yet, on the mass of the Higgs meson since its coupling to fermions is $O(m_f/250\text{ GeV})$ and, therefore, so weak that its experimental detection is extraordinarily difficult. As discussed in ref. [49] the best methods of detecting the Higgs are decays of vector states of quarkonium ($V \to H + \gamma$) and Z brehmstralung. However, the branching ratios for the latter are $O(10^{-5} - 10^{-6})$, thus one will have to wait until a Z factory is built at SLC and/or LEP to use this detection method. The branching ratio for $V \to H + \gamma$ is

$$\frac{2\sqrt{2}\, G_F\, m_f^2}{e^2} \quad (1 - \frac{m_H^2}{4m_f^2})B(V \to \mu^+\mu^-) \tag{5.4}$$

The maximum branching ratio for a $\bar{c}c$ or $\bar{b}b$ state to go to $H + \gamma$ is $\sim 10^{-4}$ which might be detectable. Unfortunately, unless there are very heavy fermions, the Higgs is heavier than the upsilon (Υ) and , thus, we must await for $\bar{t}t$ decays; here the branching ratios can be relatively large (~1%), with a very good monochromatic-γ signature. There is also the possibility, for a Coleman-Weinberg Higgs, that it will be in the middle of the Υ spectrum (depending on the value of $\sin^2\theta_W$), in which case there will be substantial mixing (see [50] for details). It thus appears that the detection of the standard Higgs must await abundant production of Z's and $\bar{t}t$'s at least (if it is lighter than the Z or $\bar{t}t$).

As pointed out by Veltman [51], for a Higgs of mass greater than 200 GeV it might be that the only way to detect it would be through radiative corrections. Unfortunately, the effects of heavy scalars only vary as ln m_H, and so very high precision is needed; again, a vector boson factory might be the only way to detect it.

5.2 Two Higgs Models

So far we have only studied the Higgs sector in the simplest case (the minimal model); however, this sector of the theory is experimentally poorly known and, unfortunately, it has a lot of room for theoretical arbitrariness. Experiments only constrain the sector weakly and indirectly; for instance, the effective neutral current interactions due to Z boson exchange are described at low energy by

$$L_Z^{eff} = \frac{G_F}{\sqrt{2}} \rho \, J_Z^{\mu} \, J_{\mu Z} \qquad (5.5)$$

where $J_{\mu Z}$ is the standard weak neutral current and $\rho = M_W^2/M_Z^2\cos^2\theta_W$. The current experimental value of ρ is 0.992 ± 0.050 [52]. In the standard model this parameter (at the tree level) is one, but if we introduce N Higgs multiplets Φ_i that transform like representation R_i

under SU(2), then

$$\rho = \sum_{i=1}^{N}(t_i(t_i + 1) - (t_i^3)^2)\sigma_i^2/2 \sum_{i=1}^{N} \sigma_i^2(t_i^3)^2 \tag{5.6}$$

where $\sigma_i^2 \equiv \langle\phi_i^\dagger\phi_i\rangle$ and t_i^3 $(t_i(t_i+1))$ is the eigenvalue of $T_3(\vec{T}^2)$ in the representation R_i. We see that ρ is "naturally" (i.e. without finetuning the vev's) equal to one if all multiplets transform like doublets under SU(2). There is no restriction on the number of such doublets, however.

In addition to the standard Higgs, other non-standard Higgs scalars are experimentally allowed and they are theoretically welcomed in a variety of contexts. In supersymmetric theories (see [53] for a review), at least two doublets with opposite hypercharges are needed to give masses to the charge 2/3 and -1/3 quarks. Additional doublets have also been considered as a possible source of CP violation in the K-system [54] and, in a different context, as an explanation for the absence of CP violation in QCD [11]. An additional triplet has also been suggested as a way of giving masses to the neutrinos [55] without altering the fermion content of the theory. In this section we discuss bounds on the masses of Higgs scalars in two-Higgs and multi-Higgs models. A more detailed discussion of the constraints on masses in these models as well as supersymmetric models is given in ref. [56].

The simplest extension of the minimal model is the inclusion of an additional Higgs doublet. We introduce two complex doublets of Higgs scalars, ϕ_1 and ϕ_2, with hypercharges $Y_1 = Y_2 = 1$. One can assign them hypercharges $Y_1 = -Y_2 = 1$, but a replacement of ϕ_2 by $\tilde{\phi}_2 = i\,\tau_2\,\phi_2^*$ (where τ_2 is the Pauli matrix) shows the equivalence of the two choices since $\bar{\Phi}_2$ has the same isospin but hypercharge opposite to that of Φ_2. There are a total of eight fields, three of which become the longitudinal components of the W and the Z via the Higgs mechanism, leaving five physical scalars: a charged scalar $\chi^{\pm}$, a pseudoscalar χ_0 and two neutral scalars ϕ and η. If the most general Yukawa couplings are allowed, the scalars will mediate flavor changing neutral currents (FCNC), which are highly suppressed relative to charged current processes (for instance $BR(K^+ \to \pi^+ e^+ e^-) = (2.7 \pm 0.5) \times 10^{-7}$, whereas $BR(K^+ \to \pi^0 e^+ \nu_e) = (4.82 \pm 0.05)\%$). As was shown by Glashow and Weinberg (Phys. Rev. D15, 1958 (1977)), the only way to "naturally" suppress FCNC (i.e. to eliminate them at the tree level for all values of the parameters of the theory) is to have the quarks of a given charge coupling only to a single Higgs field; then, one can simultaneously diagonalize the Yukawa and mass matrices (in supersymmetry this restriction occurs automatically).

One can enforce the Glashow-Weinberg condition by imposing a discrete symmetry

$$\Phi_2 \to -\Phi_2 \; ; \; d_R^a \to -d_R^a \text{ (model I)} \tag{5.7}$$

or

$$\Phi_2 \to -\Phi_2 \qquad \text{(model II)} \tag{5.8}$$

The phenomenology of model I has been extensively discussed in [57] and that of model II in [58]. Either of these symmetries leads to the same potential and to the same limits on scalar masses to be discussed below. The most general renormalizable potential consistent with the symmetries above is $V_0(\phi_1,\phi_2) = V_2 + V_4$, where

$$V_2 = -\mu_1^2\ \phi_1^\dagger\phi_1 - \mu_2^2\ \phi_2^\dagger\phi_2 \tag{5.9}$$

and

$$V_4 = \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2 + \lambda_4|\phi_1^\dagger\phi_2|^2 + \frac{\lambda_5}{2}((\phi_1^\dagger\phi_2)^2 + (\phi_2^\dagger\phi_1)^2) \tag{5.10}$$

As in the minimal model,we take $\mu_1^2 > 0$, $\mu_2^2 > 0$ to ensure ssb (the $\mu_1^2 = \mu_2^2 = 0$ limit will be discussed shortly) and $\lambda_1 > 0$, $\lambda_2 > 0$ to have a bounded potential. Also, we choose $\lambda_5 < 0$ so that the vev's of ϕ_1 and ϕ_2 are relatively real (the last term in (5.10) can be written as $\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \frac{1}{2}\lambda_5^*(\phi_2^\dagger\phi_1)^2$. A phase rotation $\phi_2 \to e^{i\theta}\phi_2$, with a corresponding redefinition of quark phases, can then be used to make λ_5 real with the chosen sign). Unlike the minimal model, however, we have two ways to break SU(2) x U(1). If λ_4 is negative (positive if $Y_2 = -1$), then a parallel alignment of ϕ_1 and ϕ_2 is favored and the remaining symmetry is $U(1)_{e.m.}$; if λ_4 is positive (negative if $Y_2 = -1$), then a perpendicular alignment is favored and the SU(2) x U(1) symmetry is completely broken. Thus, we have to restrict λ_4 to be less than zero and we can write.

$$\phi_1 = \langle\phi_1\rangle + \phi_1' = \frac{1}{\sqrt{2}}\begin{pmatrix} o \\ x \end{pmatrix} + \begin{pmatrix} \alpha^\pm \\ (\phi_0 + i\alpha_0)/\sqrt{2} \end{pmatrix}$$

$$\phi_2 = \langle\phi_2\rangle + \phi_2' = \frac{1}{\sqrt{2}}\begin{pmatrix} o \\ y \end{pmatrix} + \begin{pmatrix} \chi^\pm \\ (\eta_0 + i\chi_0)/\sqrt{2} \end{pmatrix} \tag{5.11}$$

where $x^2 + y^2 = \sigma^2 = (247\ \text{GeV})^2$. The α's disappear via the Higgs mechanism and the masses of the Higgs scalars can be found to be

$$m^2_{\chi_0} = -\lambda_5\sigma^2 \quad ; \quad m^2_{\chi} = -\tfrac{1}{2}(\lambda_4 + \lambda_5)\sigma^2 \tag{5.12}$$

The ϕ_0 and η_0 are not mass eigenstates. The mass matrix is

$$\begin{bmatrix} 2\lambda_1 x^2 & (\lambda_3+\lambda_4+\lambda_5)x^2y \\ (\lambda_3+\lambda_4+\lambda_5)xy & 2\lambda_2 y^2 \end{bmatrix} \tag{5.13}$$

with eigenvalues

$$\frac{1}{2} m^2_{\phi,\eta} = \lambda_1 x^2+\lambda_2 y^2 \pm\sqrt{(\lambda_1 x^2-\lambda_2 y^2)^2 + x^2y^2(\lambda_3+\lambda_4+\lambda_5)^2} \tag{5.14}$$

As in the minimal model, these are free parameters. However, since $\lambda_4 < 0$ and $\lambda_5 < 0$

$$m^2_{\chi} \geq \frac{1}{2} m^2_{\chi_0} \tag{5.15}$$

In models with $N \geq 3$ doublets, there are $N - 1$ charged scalars, $N - 1$ pseudoscalars and N neutral scalars. Although (5.15) is not valid among scalars of each multiplet, one can show [56] that

$$\sum_i m^2_{\chi_i} \geq \frac{1}{2} \sum_i m^2_{\chi_i^0} \quad ; \quad \prod_i m^2_{\chi_i} \geq \frac{1}{2^{N-1}} \prod m^2_{\chi_i^0} \tag{5.16}$$

Note also that

$$m^2_\phi \leq \bar{\lambda}\sqrt{2}/G_F \quad ; \quad \bar{\lambda} \equiv 2\sqrt{\lambda_1\lambda_2} + \lambda_3 + \lambda_4 + \lambda_5 \tag{5.17}$$

This is a particular case of a very general upper bound [59] on the mass of the lightest nonsinglet neutral scalar in the GWS model with arbitrary Higgs sector ((5.17) becomes an upper bound if one requires that no self-coupling be divergent between M_W and M_X. See discussion below).

An interesting special case of the model is $\lambda_5 = 0$ (this is stable under radiative corrections), in which case (5.7) or (5.8) becomes a continuous symmetry $\phi_2 \to e^{i\theta}\phi_2$; such a symmetry is an example of a Peccei-Quinn $U(1)_{PQ}$ symmetry, which was first postulated in [11] as an explanation of the absence of CP violation in QCD. If such a symmetry is present, its spontaneous breakdown by $\langle\phi_2\rangle$ leaves a pseudo-Goldstone boson, namely χ_0, called the axion. Although it is massless at the treelevel ($m^2_{\chi_0} = -\lambda_5\sigma^2 = 0$), χ_0 gets a small mass due to non-perturbative instanton effects [10].

In order to discuss the lower bounds on the Higgs masses, it is necessary to discuss radiative corrections to the effective potential $V_0(\phi_1,\phi_2)$, just as in the minimal model. However, in models with more than one physical Higgs field, the question of quantum corrections are important only along those directions in field space for which V_4 (= $f_{ijk\ell}\phi_i\phi_j\phi_k\phi_\ell$ in general) is very flat. In the minimal model, quantum corrections are important if $\mu^2/\sigma^2 = \lambda << 1$ and the potential is then very flat. In the CW case ($\mu^2 = 0$), one can choose M_R such that $\lambda(M_R) = 0$ (in the $\mu^2 \to 0$ limit, $\lambda \to 0$ (assuming there is ssb, so that σ is held fixed as $\mu^2 \to 0$) and there must be a renormalization mass M_R such that $\lambda(M_R) = 0$) and V_4 (in that case $\lambda\phi^4/4$) vanishes for all values of ϕ. The entire one-loop potential is then $B\phi^4 \ln \phi^2/M_R^2$, which can be minimized to give the usual result, $V(\phi) = B\phi^4(\ln \phi^2/\sigma^2 - \frac{1}{2})$. In the multi-Higgs case (with $\mu^2_{ij} \equiv 0$, where $V_2 \equiv -\mu^2_{ij}\phi_i\phi_j$), one can choose the renormalization point such that

$$V_4(\vec{n},\ \lambda_i(M_R))\Big|_{\vec{n}^2=1} = 0 \qquad (5.18)$$

where $\vec{n}$ is a unit vector in ϕ_i-space. If $\vec{n}_0$ is a solution to (5.18), then V_4 vanishes along the entire range $\phi_i = (\vec{n}_0)_i\phi$. The entire one-loop potential <u>along this ray</u> is then $B\phi^4 \ln \phi^2/M_R^2$, which breaks the symmetry along this direction. In contrast to the minimal model, only the particular combination of couplings occurring in (5.18) needs to be small for quantum corrections to be important.

In the two-Higgs model, if $2\sqrt{\lambda_1\lambda_2}+\lambda_3+\lambda_4+\lambda_5 \equiv \bar{\lambda} \ll 1$, then $m^2_\phi/\sigma^2 \ll 1$ and the potential is very flat along the ϕ-direction. If $\mu_1^2 = \mu_2^2 = 0$, then V_4 vanishes along the ϕ direction if $\bar{\lambda}(M_R) = 0$. The entire one-loop correction is then (neglecting fermions)

$$V_1(\phi) = \frac{1}{64\pi^2\sigma^4}\{2m_\chi^4 + m_\chi^4 + m_\eta^4 + 6M_W^4 + 3M_Z^4\} \times$$

$$\times\ (\ln \phi^2/M_R^2 - 25/6) \qquad (5.19)$$

where the masses are the tree-level masses found by shifting the fields ($\phi \to \phi-\sigma$): $M_W = \frac{1}{2}g\sigma$, $M_Z = \frac{1}{2}(g^2 + g'^2)^{1/2}\sigma$, $m^2_\chi = -\frac{1}{2}(\lambda_4 + \lambda_5)\sigma^2$, $m^2_{\chi_0} = -\lambda_5\sigma^2$, $m^2_\eta = 2\sqrt{\lambda_1\lambda_2}\,\sigma^2$ and $m^2_\phi = 0$. Note that $V_1(\phi)$ is the potential <u>only along the ϕ-direction</u>. One can get rid of M_R by minimizing V_1; this turns the round bracket into $(\ln \phi^2/\sigma^2 - \frac{1}{2})$, where $\sigma = \langle\phi\rangle$.

The masslessness of ϕ at the tree-level (like the masslessness of the Higgs at the tree-level in the minimal CW case) is due to scale invariance – ϕ is a pseudo-Goldstone boson associated with the breaking of scale invariance. Its mass is calculable at the one-loop level and it is the curvature of V_1, along the ϕ direction, at the symmetry breaking point σ. Therefore

$$m_\phi^2 = \left.\frac{d^2V_1}{d\phi^2}\right|_{\phi=\sigma} = \frac{1}{8\pi^2\sigma^2}(2m_\chi^4 + m_\chi^4 + m_\eta^4 + 6M_W^2 + 3M_Z^4) \qquad (5.20)$$

Δe see that in contrast to the minimal model, m_ϕ is not totally calculable because of the arbitrariness of the other scalar masses, but $m_\phi \gtrsim m_{CW} - 10.4$ GeV (since the extra terms in 5.20 are positive). In a model with N doublets, $M_\phi^2 = (6M_W^4 + 3M_Z^4 + \Sigma\,(2m_\chi^4 + m_\chi^4\ \ ) + \Sigma\, m_\phi^4\,)/8\pi^2\sigma^2$.

We can now consider the bounds on Higgs masses in the two- and multi-Higgs models. As reviewed in Section 5.1, there is a lower limit to m_H, $m_H \gtrsim m_{CW} = 10.4$ GeV, in the minimal model. However, in the two-doublet model we just found that CW ssb ($\mu_1^2 = \mu_2^2 = 0$) gives only a single constraint on the mass of one of the neutral scalars (see equation (5.20) above). Since the lower limits discussed in Section 5.1 were very close to m_{CW}, one expects that the only lower limit in the present model will be on the mass of that single scalar. In fact

$$m_H^2 = m_\phi^2 \gtrsim \frac{1}{8\pi^2\sigma^2}\,[6M_W^4 + 3M_Z^4 + 2m_\chi^4 + m_\chi^4 + m_\eta^4] \gtrsim m_{CW}^2 \quad (10.4\text{ GeV})^2 \qquad (5.21)$$

The first inequality is due to the fact that if we let μ_1^2 and μ_2^2 become slightly negative (which would lower m_ϕ) a barrier develops between the symmetric and asymmetric vacua at $T = 0$, thus too much entropy would then be produced in the GWS transition which would wipe out the baryon to entropy ratio. The second inequality is due to the lack of stringent lower limits to the scalar masses m_{χ_0}, m_η and m_χ. The question of whether CW symmetry breaking itself is acceptable was addressed in Ref. [61]. As was shown there (in the absence of heavy fermions), if the scalars other than ϕ are sufficiently heavy, quantum tunneling becomes more efficient and drives the transition earlier than χsb if (roughly; see [61] for details) $(\sum_S m_S^4)^{1/4} \gtrsim 180$ GeV, which significantly suppresses the entropy production. The results in models with more doublets are very similar lower limits (see [56] for details).

"Upper limits", as defined in the previous section, are very similar to the minimal model. If one requires that partial wave unitarity be respected by tree graphs in Higgs + Higgs → Higgs + Higgs scattering at large energies [62] then, for instance, the upper limits to the charged and pseudoscalar Higgs masses are found to be $\sim(32\pi^2 M_W^2/g^2)^{1/2} \sim 1.7$ TeV. Similar limits apply to the neutral scalars [62]. More stringent limits arise, however, if one requires perturbation theory to be valid at the GUT scale. Just as described in the previous section, one can follow the change of the scalar couplings, as the energy scale increases, using the RGE's (the RGE's for the two-doublet model can be found in [63]); the task becomes cumbersome because of the number of couplings. If no heavy fermions are present, these couplings will in general grow, though they will do so at different rates. Therefore, if we want to believe perturbative calculations in GUTS (which give excellent predictions for $\sin^2\theta_W$ and m_b/m_τ), we must consistently restrict the low energy values (initial values for the RGE's) of these couplings, so that none of them blows up before we reach the GUT scale. These upper limits to the λ_i can then be turned into upper limits to the masses, which are summarized in Figs. 5.2 and 5.3 (see [56] for details). It turns out that the entire hypersurface defined by the upper limits to m_{χ}^2, $m_{\chi_0}^2$, m_{η}^2 and m_{ϕ}^2 can be enclosed in a hypersphere, so that one can state that $(\sum_S m_S^2)^{1/2} \lesssim 260$ GeV. We emphasize that these limits are not phenomenologically required, but if they are not satisfied, no quantitative GUT prediction is reliable. The upper limits on the scalar masses in the N-doublet model are very similar to the two doublet case. See [56] for details.

Because of the added existence of charged and pseudoscalar bosons in the two-doublet model, the phenomenology is much richer. The phenomenology of charged Higgs bosons has been discussed extensively [57,64]. $\chi^{\pm}$ can be produced in e^+e^- annihilation, but only with ~1/4 unit of R and without a sharp threshold; it is not clear that they could be unambiguously seen. Unfortunately the Z will not be a good place to look for charged Higgs particles; the coupling of the Z to charged scalars is small: while the hadronic cross section increases by about 10^3 at the Z (relative to e^+e^- point cross section), pair production of charged Higgs particles increases by $\lesssim 10$ [65]. It is now known that $\chi^{\pm}$ must be heavier than the b quark [66], or else the b would decay semiweakly through its Higgs coupling ($b \to \chi^- \bar{u}$, for example). The K_L - K_S mass difference also gives constraints, as noted by Abbott, Sikivie and Wise [67]. It was noted there that $\chi^{\pm}$ will mediate flavor changing neutral currents at one-loop; if one requires that the $\chi^{\pm}$ contribution to $K^0 - \bar{K}^0$ mixing be less than the W contribution (which agrees fairly well with the data), then one finds (in either model I or II)

$$\left(\frac{y}{x}\right)^2 \lesssim 2m_\chi/m_{charm} \qquad (5.22)$$

(the actual bound is slightly stronger). The bound can be much stricter in the six-quark model, but it depends on unknown mixing angles (see the discussion in [56]). They [67] also considered limits due to one-loop charged Higgs contribution to $K \to 2\pi$; here the W contribution dominates the $\chi^\pm$ contribution for $m_\chi \gtrsim 25$ GeV. These limits are primarily of use if $y/x > 1$; if $x/y > 1$, then the only bound comes from the $D_L - D_S$ mass difference and, using current limits, it is found that $m_\chi(\mathrm{GeV}) \gtrsim 1.5 \times 10^{-3}(x/y)^2$. Also, contributions to $(g-2)_\mu$ were considered by Toussaint [68] and can be large if $x \gg y$.

A major difference between the minimal model and the two Higgs model is the strength of the Yukawa couplings of the neutral scalars. In the minimal model, neglecting Kobayashi-Maskawa mixing, the Yukawa coupling is m_f/σ. In model I of the two-doublet model, ϕ_0 couples to charge $^2/_3$ quarks with strength m_f/x and η_0 couples to charge $-^1/_3$ quarks and leptons with strength m_f/y (ϕ_0 and η_0 can then be rotated to find the physical eigenstates; the couplings are listed in [56]). In model II, ϕ_0 couples to all fermions with strength m_f/x. Thus, since $x/\sigma < 1$ and $y/\sigma < 1$, the branching ratio for $V \to H + \gamma$, for example, can be significantly increased in the two-Higgs model; one can distinguish between model I and II by looking at the Higgs decay product. Current limits are not very restrictive; if, for example, $m_\phi \lesssim 3$ GeV, the failure to observe a monochromatic photon in charmed decays only tells us that, neglecting mixing, $x/y \lesssim 10$ [58]. It has been noted in ref. [69] that the Z might be an excellent place to look for neutral (scalar and pseudoscalar) non-standard Higgses (if they are light enough) since their production considerably exceeds that of charged Higgses near the Z-pole.

The effects of a pseudoscalar on FCNC were discussed in [70]. For instance the transition $s \to d\chi_0$ leads to $K^+ \to \pi^+\chi_0$, which (if $m_\chi > 1$ MeV) leads to $K^+ \to \pi^+e^+e^-$, if $m_\chi \lesssim 400$ MeV. In this range, consistency with the current limits require $(y/x) < 0.01$, a very restrictive constraint. The $b \to s\chi_0$ transition is very sensitive to the top quark mass. If, for example, $m_t = 50$ GeV, $m_\chi \sim M_W$ and $x \sim y$, then the branching ratio is ~5% in both models. A pseudoscalar lighter than the Ξ may be observable in $T \to \chi_0\gamma$ (although if $y/z >> 1$, this may be small even if $b \to \chi_0 s$ is fairly large; see [70] for details).

The phenomenology of Higgs bosons is thus much richer in the two higgs model. The limits are not very restrictive today, but will certainly be improved (or Higgs effects will be found) by more results on b decays. The results for the N-doublet model are very similar.

CHAPTER VI

BOUNDS ON FERMION MASSES

6.1 Upper Bounds

As was mentioned earlier, fermion masses are arbitrary in the GWS model. There are, of course, experimental lower limits on the mass of a heavy (charged) fermion from e^+e^- annihilation experiments, $m_f \gtrsim 21$ GeV [11]. There are some phenomenological upper limits to fermion masses based on the ρ parameter [72] (see Section 5.2) or on the $K_L - K_S$ mass difference [73], but they are either very weak or very sensitive to unknown matrix elements. We now consider theoretical upper limits to fermion masses assuming, for the moment, that the top quark is the only undiscovered heavy fermion. We will discuss the generalization of the results at the end.

It was noted in refs. [74-76] that there is a theoretical upper bound to the top quark mass, m_t, obtained by considering one-loop corrections to the Higgs potential. The one-loop potential was discussed in Section 3.1; writing it in terms of its minimum σ and the Higgs mass $m_H^2 = d^2V/d\phi^2(\phi = \sigma)$ (we choose units such that $\sigma = 1$)

$$V = \frac{1}{8} m_H^2[2\Xi\phi^4 \ln\phi^2 - (3\Xi - 1)\phi^4 + (4\Xi - 2)\phi^2] \qquad (6.1)$$

where $2\phi^\dagger\phi = \phi^2$ and $\Xi \equiv 4B/m_H^2 = (6M_W^4 + 3M_Z^4 + M_H^4 - 12m_t^4)/16\pi^2 m_H^2$. Here $M_H^2\phi^2 = -\mu^2 + 3\lambda\phi^2$. The potential is plotted for various values of Ξ in Fig. 6.1. It is easy to see that if m_t is sufficiently large, then $\Xi <$

0 and the potential is unbounded.

In ref. [74] it was argued that the minimum at σ must be an absolute minimum, thus Ξ must be positive. This leads to an upper limit on m_t given by the $\Xi = 0$ line of Fig. 6.2. However, for large values of ϕ the potential becomes negative at $\phi = \phi_1 = \exp(-1/4\Xi)$; in ref [75] it was noted that if this value is outside the region of validity of perturbation theory, then one cannot say the potential is unbounded. It can be shown [78] that perturbation theory is valid up to the Planck scale; $\phi_1 \gtrsim M_p$ corresponds to $\Xi > -0.006$. In ref. [76], it was pointed out that the SU(2) x U(1) potential is valid only up to the unification scale; $\phi_1 \gtrsim 10^{15}$ GeV gives $\Xi > -0.008$. As can be seen in Fig. 6.2, all of these limits give similar upper bounds to m_t, $m_t \lesssim 100$ GeV.

In ref. [77] it was noted that there are two effects which significantly weaken these upper bounds. First, as discussed in Chapter IV, the one-loop potential requires $|\ln\phi^2| \ll 1$ for it to be valid. The RGE-improved potential can then be used to extend the region of validity of the one-loop potential as long as the running couplings remain small. The main effect of using the RGE-improved potential can be taken into account by using running couplings in the one-loop potential, this is the leading two-loop effect [78]. Thus, for large values of ϕ, the scale dependence of the couplings becomes crucial. The β function for the Yukawa coupling g_t is (neglecting mixing).

$$\beta_t = (9g_t^3/2 - 32\pi\alpha_s g_t - 9\pi\alpha_w g_t - 17\pi\alpha' g_t/3)/16\pi^2 \qquad (6.2)$$

If $g_t(\sigma)$ is small enough ($m_t < 250$ GeV), then g_t decreases with scale, and this has the effect of reversing the sign of Ξ at large scales. If Ξ changes sign before the potential turns over, then our vacuum is absolutely stable. Thus, there is a larger range of masses for which our vacuum is absolutely stable. The lower solid curve in Fig. 6.3 gives the upper limit to the region of absolute stability; one can see that the bounds are already much weaker than those of refs. [74-76].

Second, there is no phenomenological requirement that we live in an absolute vacuum. A given potential is acceptable if a) during, say, the GUT phase transition, the universe goes into the correct SU(3) x SU(2) x U(1) vacuum, b) it stays there until the electroweak transition, c) during the electroweak transition, the universe goes into the SU(3) x U(1) vacuum, and d) it stays there for at least 10^{10} years. As was shown in ref. [77], this last constraint gives the strongest bound on m_t.

If m_t is above the upper bound of absolute stability, then the potential does turn over and becomes negative before Ξ changes sign. However, at large enough ϕ, Ξ changes sign and the potential turns around and becomes positive. Our vacuum is thus metastable with a barrier separating it from the stable vacuum. We have to require then, that the lifetime of our universe be greater than 10^{10} years. As discussed in Section 3.3, the nucleation rate of stable phase per unit volume, f, is ~ exp (-B), where B is the action of the bubble solution. The fraction of space filled with new phase at time t is approximately 1 - exp($-ft^4$). Since, in our units, the age of the universe is e^{101}, the fraction of space filled with stable phase today is 1 - exp(-exp(404 - B)). Requiring this fraction to be negligible is the same as requiring that $B \gtrsim 404$; this, in turn, becomes an upper limit to m_t. It turns out that B itself is extremely sensitive to the top quark mass, thus the uncertainties associated with the precise expansion rate, bubble overlap, the prefactor in the nucleation rate, etc ..., are utterly negligible in determining the upper bound. The result is plotted in Fig. 6.3 - the upper solid line corresponds to the $B = 404$ line and is an upper bound on m_t.

Although the limit on m_t for which our vacuum is absolutely stable is sensitive to uncalculated two-loop effects, the upper bound to m_t due to the lifetime of the universe is very insensitive to two-loop effects; dropping the running of couplings entirely changes the limit by $\lesssim 5\%$. Thus $m_t \lesssim 200$ GeV, a much weaker limit. If there are additional quarks, one replaces m_t^4 by $\Sigma\, m_q^4$; the limits on fourth generation masses are then more severe. If there are additional Higgs scalars (see Section 5.2), the abscissa in Fig. 6.3 refers to one of the neutral scalars and the ordinate is replaced by $(m_t^4 - (1/12)\Sigma\, m_S^4)^{1/4}$.

6.2 The CW Transition

As discussed at the end of Section 3.1, the mass of the Higgs scalar is calculable in the CW model, $m_H^2 = 8B\sigma^2$. Since B must be positive, CW symmetry breaking is ruled out if the mass of a colored fermion $m_q \gtrsim 84$ GeV ($\Sigma\, m_q^4 \gtrsim (84\ \text{GeV})^4 + (1/12)\Sigma\, m_S^4$ if there are many scalars and colored fermions). For smaller masses, Witten noted [41] that χsb drives the transition in the CW model. While the universe is trapped in the symmetric phase, quarks are massless and the strong interactions have a $SU(N_f) \times SU(N_f)$ chiral symmetry. As the temperature drops, this chiral symmetry is broken in a presumably first order phase transition at $T_o \sim 200$ MeV and $\bar{\psi}\psi$ gets a vev. As soon as this happens, the Yukawa couplings of the Higgs scalar to the quarks

$$L_Y = - \sum g_i \bar{\psi}_i \psi_i \phi \tag{6.3}$$

add an extra linear term to the effective potential and $V(\phi) = V_{CW}(\phi) - b\phi$, where $b = (\Sigma g_i < \bar{\psi}_i\psi_i>)/\sqrt{2}$ and V_{CW} is the CW potential (see Section 5.1). This immediately destabilizes the symmetric vacuum and although a nearly symmetric metastable vacuum can exist for awhile, the barrier separating it from the true vacuum will disappear at T not much less than T_o and, therefore, $T_1 = T_o = 200$ MeV. Thus $S_f/S_i \sim 10^{4-5}$, which is acceptable but requires very efficient baryogenesis in the early universe.

It was noted later [44] that since for a given quark q, the Yukawa coupling satisfies the RGE of equation (6.2), for $m_q \lesssim 250$ GeV g_q is asymptotically free and thus increases as the scale decreases. For a sufficiently heavy quark, this causes quark loops to dominate at small scales and since they tend to stabilize the symmetric vacuum (because B in (3.8) becomes negative at small scales), but cannot prevent its decay as soon as $<\bar{\psi}\psi> \neq 0$), one finds that a nearly symmetric metastable vacuum forms ($<\phi> = O(1)$ GeV) with a barrier that separates it from the true vacuum ($<\phi> = 247$ GeV) even at T = 0. Thus, one has to worry about the entropy produced in the transition to the true vacuum. To analyse this effect more carefully we use the RGE for the effective potential (see Chapter IV).

The full effective potential satisfies the RGE of equation (4.1). At $T = 0$ (or $T < T_o$), chiral symmetry is broken and $\langle\bar{\psi}\psi\rangle$ gives an effective linear term in $V(\phi)$. The most general solution to (4.2) is then [44]

$$V(\phi) = -h(g_i(t))\langle\bar{\psi}\psi\rangle\phi\Gamma_1(t) + f(g_i(t))\phi^4\Gamma_4(t) \tag{6.4}$$

where $t = \ln(\phi/M_R)$ and M_R is a renormalization mass. f and h are arbitrary functions and the effective couplings $g_i(t)$ are the solutions of the RGE's

$$\frac{dg_i}{dt} = \frac{\beta_i(g_i)}{1-\gamma(g_i)} \quad ; \quad g_i(0) \equiv g_i \tag{6.5}$$

The Γ_n are defined as

$$\Gamma_n(t) = \exp\left[\, n \int_o^t dt' \, \frac{\gamma(g_i(t'))}{1-\gamma(g_i(t'))} \,\right] \tag{6.6}$$

Choosing the normalization condition for λ and g_q as

$$V(M_R) = -\frac{1}{\sqrt{2}}\, g_q \langle\bar{\psi}\psi\rangle M_R + \frac{1}{4}\lambda M_R^4 \tag{6.7}$$

equation (6.4) becomes

$$V(\phi) = -\frac{1}{\sqrt{2}}\, g_q(t)\langle\bar{\psi}\psi\rangle\phi\Gamma_1(t) + \frac{1}{4}\lambda(t)\phi^4\Gamma_4(t) \tag{6.8}$$

This solution to the RGE is an exact result. The loop expansion occurs when we use expressions for $\beta_i(g_i)$ and $\gamma(g_i)$. The relevant g_i's are g_s, g and g', whose β functions are well known, g_q, whose β function is that of equation (6.2), and λ, whose one-loop β function is given by

$$\beta_\lambda = 3\lambda^2/2\pi^2 + 12\lambda(\alpha_q - \frac{1}{4}(3\alpha_W + \alpha')) + 8B \qquad (6.9)$$

where $\alpha_i = g_i^2/4\pi$ and B is the expression of (3.8) ($B = 3(2\alpha_W^2 + (\alpha_W + \alpha')^2 - 16\alpha_q^2)/64\pi^2$). The anomalous dimension is [44]

$$\gamma = -(3g^2/4 + g'^2/4 + 3g_q^2)/16\pi^2 \qquad (6.10)$$

The result for $V(\phi)$, for several values of m_q, is plotted in Fig. 6.4. We see that for a quark of mass greater than 65 GeV, there is a barrier between the two vacua and, as a result, the universe will fall into a nearly symmetric metastable phase when χsb takes place (the Higgs field will not "roll over" the hill since the barrier is larger at $T = T_o$). To show that the existence of this barrier is indeed fatal, it is necessary to calculate the nucleation rate (see Section 3.3) $r \sim \sigma_o^4 \exp(-B(T))$, where $\sigma_o = O(1)$ GeV and B(T) is the Euclidean action of the least action solution. As discussed in Section 3.3, B(T) reaches a minimum, if a barrier exists at zero temperature, at $T = T^*$ and if $B(T^*) > \ln b = \ln(\sigma_o^2 M_p/\sigma^3) = 150$, then far too much entropy is generated. One finds that this is the case if $m_q \gtrsim 65$ GeV, ruling out CW symmetry breaking in that case (for a discussion of the sensitivity of the limit to the various input parameters, definitions of couplings, gauge dependence of Υ, etc ..., see [44]).

Additional fermions make the limit more severe. Including a fourth generation one finds that there is a metastable vacuum if $(\Sigma\, m_q^4)^{1/4} \gtrsim 54$ GeV. Thus, there is a very small window in which one could put the masses of additional quarks. If there are additional scalars one finds that for a wide range of parameters the barrier will exist if $\Sigma\, m_q^4 \gtrsim (65\ \text{GeV})^4 + (65/84)^4\,(1/12)\,\Sigma\, m_s^4$, which is somewhat stronger than the limit required by the positivity of B.

Although it might seem that the barrier vanishes for $m_q \lesssim 65$ GeV, a breakdown of perturbation theory precludes any definitive statement about this range of masses. It might be the case that the barrier still exists in this range of masses; using the fact that eq. (6.8) is an exact result, one can assume different types of behavior for β_q and thus study the potential in the region where the strong interaction are non perturbative (if $m_q \gtrsim 65$ GeV then g_q stops running at $\phi - \langle\phi\rangle = O(1)$ GeV since the quark has a mass ~ 1 GeV in the nearly symmetric vacuum. Thus, the potential can be continued all the way to the origin); many types of behavior do give a metastable vacuum, but some do not (see [44] for details). If such a vacuum existed for $m_g \lesssim 65$ GeV also, CW ssb would be ruled out entirely in the minimal model; however, the breakdown of perturbation theory does not allow us to make any definitive statement for $m_q \lesssim 65$ GeV.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The electroweak theory of Glashow, Salam and Weinberg is based on the principle of local gauge invariance (i.e. invariance under local phase transformations of the matter fields) as the fundamental link of a unified understanding of the weak and electromagnetic interactions. The gauge invariance implies the existence of vector bosons in a number equal to the number of generators of the group of gauge symmetries which, for the GWS model, is four: the photon and the massive vector bosons $W^{\pm}$ and Z. Thus, with the recent detection of the W and the Z, the lonesome photon is now listed as a partner of the W and the Z under the new entry of gauge bosons in the latest Review of Particle Properties [79]. If the full gauge symmetry of the Lagrangian were an exact symmetry of the vacuum, then the W and the Z should be massless, just like the photon; the symmetry must, therefore, be broken. In the GWS model, a set of Higgs scalars is introduced which acquire vev's, breaking the symmetry to the observed U(1) symmetry of electromagnetism. Masses are then generated for the W and the Z in a gauge invariant fashion. Fermion mass terms are not allowed in the Lagrangian by the SU(2) x U(1) gauge symmetry; instead, quark and leptons get their masses via their Yukawa couplings to the Higgs scalars. The gauge sector of the theory is both experimentally well explored and theoretically very constrained. The structure of the self-couplings and couplings to matter of the W and the Z are completely determined by the gauge structure of the theory. Several experiments give similar values of $\sin\theta_W$, thus from $\sin\theta_W$ and the electron's charge one can determine g and g'. The scale of symmetry breakdown, σ, is fixed by the muon lifetime, thus $M_W = g\sigma/2$ and $M_Z = \sqrt{g^2 + g'^2}\,\sigma/2$ are predicted to be $M_W = 83.0 \pm 2.4$ GeV and $M_Z = 93.8 \pm 2.0$ GeV, in good agreement with the experimental values $M_W = 81 \pm 2$ GeV and $M_Z = 93 \pm 2$ GeV. The structure of the coupling of the W to fermions is in good agreement with charged current weak interactions and the SLC will be able to explore in detail the couplings of the Z to fermions. In contrast to this, the mechanism of ssb is a poorly understood aspect of the model. The masses of both fermions and Higgs scalars are completely arbitrary in the model, thus the masses of the known leptons and quarks have to be set to their experimentally known values. Whether the Higgs mechanism is the mechanism of symmetry breakdown has yet to be tested experimentally, thus finding a Higgs scalar or ruling it out is very crucial. For these reasons, Higgs physics will undoubtedly be very important in the near future, thus it is important to have limits on these arbitrary masses in order to test the ssb of the model.

In this thesis we have discussed in detail the constraints on the masses of Higgs bosons and fermions in the GWS model. In order to make the discussion self-contained, we included Sections II-IV. In Section II we briefly described gauge theories and then discussed how, if wanted, the gauge symmetry can be spontaneously broken via the Higgs mechanism, so that masses can be generated for the gauge bosons without breaking gauge invariance. We then described the specific structure of the GWS model of the electroweak interactions. We then discussed the arbitrariness of the SM and how GUTS eliminate some of the arbitrariness and their impact upon cosmology. Finally, in Section III-IV we introduced the effective potential and described how it is used to study symmetry breaking phase transitions in the early universe. Sections V and VI contain the details of how the various constraints arise. We concluded that, in the minimal model, there is a lower limit to the Higgs mass of roughly $m_{CW} \sim 10.4$ GeV, and that $m_H = m_{CW}$ is allowed only for a limited range of quark masses. Also an upper bound of roughly 200 GeV exists on the mass of a heavy quark (or $\sqrt{3} \times 200$ GeV for a heavy lepton). There are no experimental constraints on the Higgs mass as yet and, from e^+e^- annihilation experiments, the current lower limit to the mass of a heavy fermion is 21 GeV. Upper limits to the Higgs mass exist, based on the assumption that perturbation theory is valid at various scales. For two-body reactions of gauge bosons to respect partial wave unitarity, $m_H \lesssim 1$ TeV. However, for perturbation theory to be valid all the way up to the GUT scale M_X, $m_H \lesssim 175$ GeV; otherwise all GUTS' predictions are not reliable. In multi-Higgs models, there is a lower bound on the mass of only one of the scalars (any of the neutral scalars), which is at least m_{CW} if no heavy fermions are present, but it can be higher or lower depending on the masses of the other scalars and the other fermions (see 5.2). The upper bounds on the various Higgs scalars' masses are very similar to those of the minimal model. There are experimental constraints on the masses of some of the scalars; for example, the mass of charged scalar $m_\chi > m_b$ or else the b-quark could decay semiweakly as $b \to \chi^- \bar{u}$. The phenomenology is much richer than in the minimal model, but there are many more parameters.

FOOTNOTES

F1 Absorbing logarithms of couplings in the definition of λ_R.

F2 Note that the scalar loop contribution is $(3\lambda\phi^2 - \mu^2)^4 \ln(3\lambda\phi^2 - \mu^2)/M_R^2$ and, therefore, has an imaginary part for $3\lambda\phi^2 < \mu^2$. However, $3\lambda\sigma^2 - \mu^2 > 2B\sigma^2 > 0$ and, thus, $V(\phi)$ is real at its minimum.

F3 A heuristic argument for this is the following. The thermal average at time t is

$$\mathrm{Tr}(e^{-\beta H})\langle O_t\rangle = \langle e^{-itH}\, O\, e^{itH}\rangle$$

$$= \mathrm{Tr}(e^{-\beta H}e^{-itH}\, O\, e^{itH}) = \mathrm{Tr}(e^{itH}e^{-\beta H}e^{itH}\, O)$$

Thus $\langle O_t\rangle = \langle O\rangle$ at $t = -i\beta$.

F4 We drop the k/R^2 term since RT = constant and, therefore, this term is $O(T^2)$, but at high temperatures $\rho(T) \sim T^4$ and at low temperatures ρ is dominated by the vacuum energy.

F5 A stronger upper bound can be obtained if one assumes that the theory will become trivial unless $y(t) \equiv \lambda(t)/(g'(t))^2$ is driven to an ultraviolet stable fixed point. See ref. [48].

REFERENCES

1. C.N. Yang and R.L. Mills, Phys. Rev. 96, 191 (1954).

2. E.S. Abers and B.W. Lee, Phys. Rep. 9C, 1 (1973).

3. O.W. Greenberg, Phys. Rev. Lett 13, 598 (1964); M.Y. Han and Y. Nambu, Phys. Rev. 139, 1006 (1965); W.A. Bardeen, H. Fritzsch and M. Gell-Mann, In "Scale and Conformal Symmetry in Hadron Physics", ed. by R. Gatto, 139 (Wiley, 1974).

4. S.L.Glashow, Nucl. Phys. 22, 579 (1961); S. Weingerg, Phys. Rev. Lett 19, 1264 (1967); A. Salam, in "Elementary Particle Theory", ed. by N. Svartholm, 367 (Almqvist and Wiksell, Stockholm, 1968); S. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2, 1285 (1970).

5. G. Arnison et. al., Phys. Lett. 122B, 103 (1983); 126B, 398 (1983); M. Banner et. al., Phys. Lett. 122B, 476 (1983); 129B, 130 (1983).

6. P.W. Higgs, Phys. Rev. Lett. 12, 132 (1964); 13, 508 (1964); Phys. Rev. 145, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964); G.S. Guralnik, C.R. Hagen and T.W.B. Kibble, Phys. Rev. Lett 13, 585 (1964); T.W.B. Kibble, Phys. Rev. 155, 1554 (1967).

7. H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B, 365 (1973); D. Gross and F. Wilczek, Phys. Rev. D8, 3633 (1973); S. Weinberg, Phys. Rev. Lett. 31, 494 (1973); Phys. Rev. D8, 4482 (1973).

8. A.Guth, Phys. Rev. D23, 347 (1981); A.D. Linde, Phys. Lett. 108B, 389 (1982); A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982); S.W. Hawking and I.G. Moss, Phys. Lett. 110B, 35

(1982); A. Guth and E. Weinberg, Phys. Rev. D23, 876 (1981); W. Press, in "Cosmology and Particles", Proc. of the Rencontres de Moriond 1981, ed. by J. Andouze et.al. (Frontieres, Gif-sur-Yvette, 1981).

9. R. Cowsik and J. McClelland, Phys. Rev. Lett. 29, 669 (1972).

10. S. Weinberg, Phys. Rev. Lett. 40, 223 (1978); F. Wilczek, ibid., 279 (1978).

11. R. Peccei and H. Quinn, Phys. Rev. Let 30, 140 (1977); Phys. Rev. D16, 1791 (1977).

12. L.F. Abbott and E. Farhi, Phys. Lett. 101B, 69 (1981); Nucl. Phys. B189, 547 (1981); L.F. Abbott, E. Farhi and Schwimmer, Nucl. Phys. B203, 493 (1982).

13. J. Goldstone, Nuovo Cim. 19, 154 (1961); J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. 127, 965 (1962).

14. W.J.Marciano and A. Sirlin, in "The Second Workshop on Grand Unification"; J.P. Leveille, L.R. Sulak and D.G. Unger editors, 151 (Birkhauser, 1981).

15. S. Glashow, et. al. of ref. [4].

16. M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

17. G. 't Hooft, Nucl. Phys. 35, 167 (1971); B.W. Lee and J. Zinn-Justin, Phys. Rev. D5, 3121, 3137, 3155 (1972); G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972); B50, 318 (1972). The absence of anomalies was discussed in D.J. Gross and R. Jackiw, Phys. Rev. D6, 477 (1972) and C. Bouchiat, J. Iliopoulos and Ph. Meyer, Phys. Lett. 38B, 519 (1972).

18. P. Langacker, Phys. Rep. 72, 185 (1981).

19. H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

20. H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974).

21. See ref.[18] and V.A. Kuzmin and M.E. Shaposhnikov, Phys. Lett. 125B, 449 (1983); G.J. Gounaris and K. Tamvakis, Thessaloniki preprint; S.S. Gershtein and Yu F. Pirogov, Yad. Fiz. 37, 552 (1983).

22. A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B135, 66 (1978),

23. R.M. Bionta et.al., Phys. Rev. Lett. 51, 27 (1983).

24. G. Steigman, Ann. Rev. Astron. Astrophys. 14, 339 (1976).

25. J.Yang, M.S. Turner, G. Steigman, D.M. Schramm and K.A. Olive, Bartol preprint BA-83--34 and references therein.

26. E.W. Kolb and M.S. Turner, Ann. Rev. Nucl. and Part. Sci., 33, 645 (1983).

27. M. Claudson, L.J. Hall and Ian Hinchliffe, Berkeley preprint UCB-PTH-83/19 (Oct. 1983).

28. A. Guth of ref. [8].

29. K. Symanzik, Comm. Math. Phys., 16, 48 (1970).

30. R.Jackiw, Phys. Rev. D9, 1686 (1974) and references therein; S. Coleman and E. Weinberg, Phys. Rev. D7, 1888 (1973).

31. Y.Nambu, Phys. Lett. 26B, 626 (1968); Boulware and L.S. Brown, Phys. Rev. 172, 1628 (1968);

32. S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177, 2238 (1969).

33. D.A. Kirzhnits and A.D. Linde, Pisma Zh. Edsp. Tecr. Fiz. 67, 1263 (1974) [JETP Lett. 40, 628 (1975)]; L. Dolan and R. Jackiw, Phys.

Rev. D9, 3320 (1974); C Bernard, Phys. Rev. D9, 3312 (1974); S. Weinberg, Phys. Rev. D9, 3356 (1974). For a review see A.D. Linde, Rep. Prog. Phys. 42, 389 (1979).

34. K. Huang, "Statistical Mechanics", Chapter 9 (Wiley, 1963).

35. P. Langacker and S.Y. Pi, Phys. Rev. Lett. 45, 1 (1980).

36. S. Coleman, Phys. Rev. D15, 2929 (1977); C. Callan and S. Coleman, Phys. Rev. D16, 1762 (1977); I.K. Affleck and F. DeLuccia, Phys. Rev. D20, 3168 (1979). See also R. Brandenberger, 37. S. Coleman, V. Glaser and A. Martin, Comm. Math. Phys. 58, 211 (1978). See also R. Brandenberger, Ph.D. Thesis, Harvard University (1983).

38. A. Guth and E. Weinberg, Phys. Rev. Lett. 45, 1131 (1980).

39. S. Weinberg, Phys. Rev. Lett. 36, 294 (1976); A.D. Linde, Pisma Zh. Eksp. Teor. Fiz., 23, 73 (1976) [JETP Letters 23, 64 (1976)].

40 A.D. Linde, Phys. Lett. 70B, 306 (1977).

41. E. Witten, Nucl. Phys. B177, 477 (1981).

42. J. Kogut et.al., Phys. Rev. Lett. 48, 1140 (1982); Phys. Rev. Lett. 50, 393 (1983); A. Parisi, R. Petronzio and F. Rapuano, Phys. Lett. 128B, 418 (1983).

43. J. Kogut et.al., Phys. Rev. Lett. 51, 869 (1983).

44. R. Flores and M. Sher, Nucl Phys. B238, 702 (1984).

45. W. Appelquist and C. Bernard, Phys. Rev. D22, 200 (1980) and references therein.

46. M.Veltman, Acta Phys. Pol. B8, 475 (1977); Phys. Lett. 870, 253 (1977); B.W. Lee, C. Quigg and H. Thacker, Phys. Rev. Lett. 38, 883 (1977); Phys. Rev. D16, 1519 (1977); D. Dicus and A. Mathur, Phys. Rev. D7, 3111 (1973).

47. N.Cabbibo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B158, (1979).

48. M.A. Beg, C. Panagiotakopoulos and A. Sirlin, Phys. Rev. Lett. 52, 883 (1984).

49. B. Barbiellini et. al., DESY report 79/27 (1979); J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B106, 292 (1976); F. Wilczek, Phys. Rev. Lett. 39, 1304 (1977).

50. J. Ellis, M.K. Gaillard, D.V. Nanopoulos and C.T. Sachrajda, Phys. Lett. 83B, 339 (1979).

51. M. Veltman of ref. [46].

52. J.E. Kim, P. Langacker, M. Levine and H.H. Williams, Rev. Mod. Phys. 53, 211 (1981).

53. H. Haber and G. Kane, Michigan preprint UM HE TH 83-17 (Jan, 1984).

54. T.D. Lee, Phys. Rev.D8, 1226 (1973); S. Weinberg, Phys . Rev. Lett. 37, 657 (1976).

55. See e.g. T.P. Cheng and L.F. Li, Phys. Rev. D22, 2860 (1980); P.B. Pal and L. Wolfenstein, Phys. Rev. D25, 766 (1982); G.B. Gelmini and M. Roncadelli, Phys. Lett. 99B, 411 (1981); H. Georgi, S. Glashow and S. Nussinov, Nucl. Phys. B193, 297 (1981).

56. R. Flores and M. Sher, Ann. Phys. 148, 95 (1983).

57. D. Toussaint, Ph.D. thesis, Princeton University, 1978, unpublished; G. Barbiellini et.al., DESY report 79/27 (1979); J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B106, 292 (1976); F. Wilczek, Phys. Rev. Lett. 39, 1304 (1977); A.I. Vainshtein, V.I. Zakharov and M.A. Shifman, Sov. Phys. USP. 23, 429 (1980); A. Ali, DESY report 81/060 (1981); B.M. Mc. Williams, Ph.D. Thesis,

Carnegie Mellon University, 1980, unpublished.

58. H.E. Haber, G.L. Kane and T. Sterling, Nucl. Phys. B161, 493 (1979).

59. P. Langacker and H.A. Weldon, Phys. Rev. Lett. 52, 1377 (1984).

60. E. Gildener and S. Weinberg, Phys. Rev. D13, 3333 (1976).

61. R. Flores and M. Sher, Phys. Lett. 103B, 445 (1981).

62. H. Huffel and G. Pocsik, Z. Phys. C8, 13 (1981).

63. K. Inoue, A. Kakuto and Y. Nakano, Prog. Theor. Phys. 62, 307 (1979).

64. J.R. Donoghue and L.F. Li, Phys. Rev. D19, 945 (1979); Y. Tomozawa, Phys. Rev. D18, 2556 (1978); B.M. McWilliams and L.F. Li, Nucl. Phys. B179, 62 (1981).

65. J. Dorfan, Proc. of the 10th SLAC Summer Institute, 137 (1982).

66. D.R.T.Jones, G.L. Kane and J.P. Leveille, Phys. Rev. D24, 299 (1981); S. Stone, AIP Conf. Proc. 81, 30 (1981).

67. L.F. Abbott, P. Sikivie and M.B. Wise, Phys. Rev. D21, 1393 (1980).

68. D. Toussaint, Phys. Rev. D18, 1626 (1978).

69. N.G. Deshpande, X. Tata and D.A. Dicus, University of Oregon preprint OITS 230 (December, 1983).

70. M.B. Wise, Phys. Lett. 103B, 121 (1981).

71. E. Lohrmann, Desy report DESY 83-102 (1983).

72. M.S.Chanowitz, M.A. Furman and I. Hinchliffe, Pys. Lett. 78B, 285 (1978); M. Veltman, Acta Phys. Pol B8, 475 (1977).

73. A. Buras, Phys. Rev. Lett. 46, 1354 (1981).

74. P.Q. Hung, Phys. Rev. Lett. 42, 873 (1979).

75. H.D. Politzer and S. Wolfram, Phys. Lett. 82B, 242 (1979).

76. N. Cabibbo, L. Maiani, A. Parisi and R. Petronio, Nucl. Phys. B158, 295 (1979).

77. R. Flores and M. Sher, Phys. Rev. D27, 1679 (1983).

78. K.T. Mahanthappa and M. Sher, Phys. Rev. D22, 1711 (1980).

79. Particle Data Group, Rev. Mod. Phys. 56 (1984).

FIGURE CAPTIONS

Fig. 2.1 Evolution of couplings (alpha) as functions of $Q = \sqrt{-q^2}$, where q^2 is a typical momentum transfer squared. The curve labeled strong (weak, electromagnetic) corresponds to the evolution of $\alpha_3 = g_s^2/4\pi$ ($\alpha_2 = g^2/4\pi$, $\alpha_1 = (5/3)\, g'^2/4\pi$); where g_s (g, g') is the gauge coupling of the group SU(3) (SU(2), U(1)) of the SM (see Section 2.3). The factor 5/3 in the definition of α_1 is included so that $\sqrt{3/5}$ Y is the appropriately normalized generator of the U(1) subgroup of G_U = SU(5).

Fig. 2.2 Typical diagrams that contribute to proton decay due to exchange of superheavy gauge bosons X and Y. The quark-antiquark pair can then form neutral mesons such as π^0, ρ^0, δ, η, etc...

Fig. 3.1 The loop expansion for the effective potential. The first row is the tree-level approximation, the second is the one-loop approximation, etc...

Fig. 3.2 The effective potential, as a function of ϕ, in the minimal model. The potential is graphed for various values of $\mu^2 < 0$ and the corresponding value of the mass of the Higgs scalar, m_S is given in each case.

Fig. 3.3 The temperature-dependent effective potential, as a function of ϕ, for various temperatures. As the temperature falls, an asymmetric vacuum starts to develop (at $T = T_{c_2}$); it becomes degenerate with the symmetric vacuum at $T = T_c$. At $T = T_{c_1}$, the latter vanishes.

Fig. 5.1 Entropy production in the minimal model as a function of m_S/m_{CW}, where m_S is the mass of the Higgs scalar. The dashed line is the entropy produced if tunneling alone drives the transition. The full line takes chiral symmetry breaking into account.

Fig. 5.2 "Upper limits" of Higgs masses plotted in the $(\chi^{\pm}, \chi_o)$ plane for various values of $m_\phi = m_\eta$. The straight line represents the $m_\chi \geq m_{\chi_o}/\sqrt{2}$ limit.

Fig. 5.3 "Upper limits" of Higgs masses plotted in the (ϕ, η) plane for various values of m_χ (m_{χ_o} is assumed to be small). The straight line here merely represents the fact that one scalar is heavier than the other. The error in the masses is about $0.02\sigma^2$, thus the convergence of the three lines is not real.

Fig. 6.1 The one-loop effective potential V (in units in which $\sigma = 1$) for various values of Ξ (see Section 6.1). Here $V = m_H^2 \tilde{V}/8$.

Fig. 6.2 The value of Ξ for given values of m_H and m_t (see Section 6.1).

Fig. 6.3 Upper bound on m_t. The lower (upper) dashed curve is the previous limit of ref. [74] ([75]). Below the lower full curve, the present vacuum is absolutely stable (region A). In region B our vacuum is unstable, but with a lifetime $\tau > 10^{10}$ yr. In region C, $\tau < 10^{10}$ yr; thus region C is disallowed.

Fig. 6.4 The zero-temperature effective potential V, as a function of ϕ, for different values of the top quark mass m_t. m_t is given in GeV's.

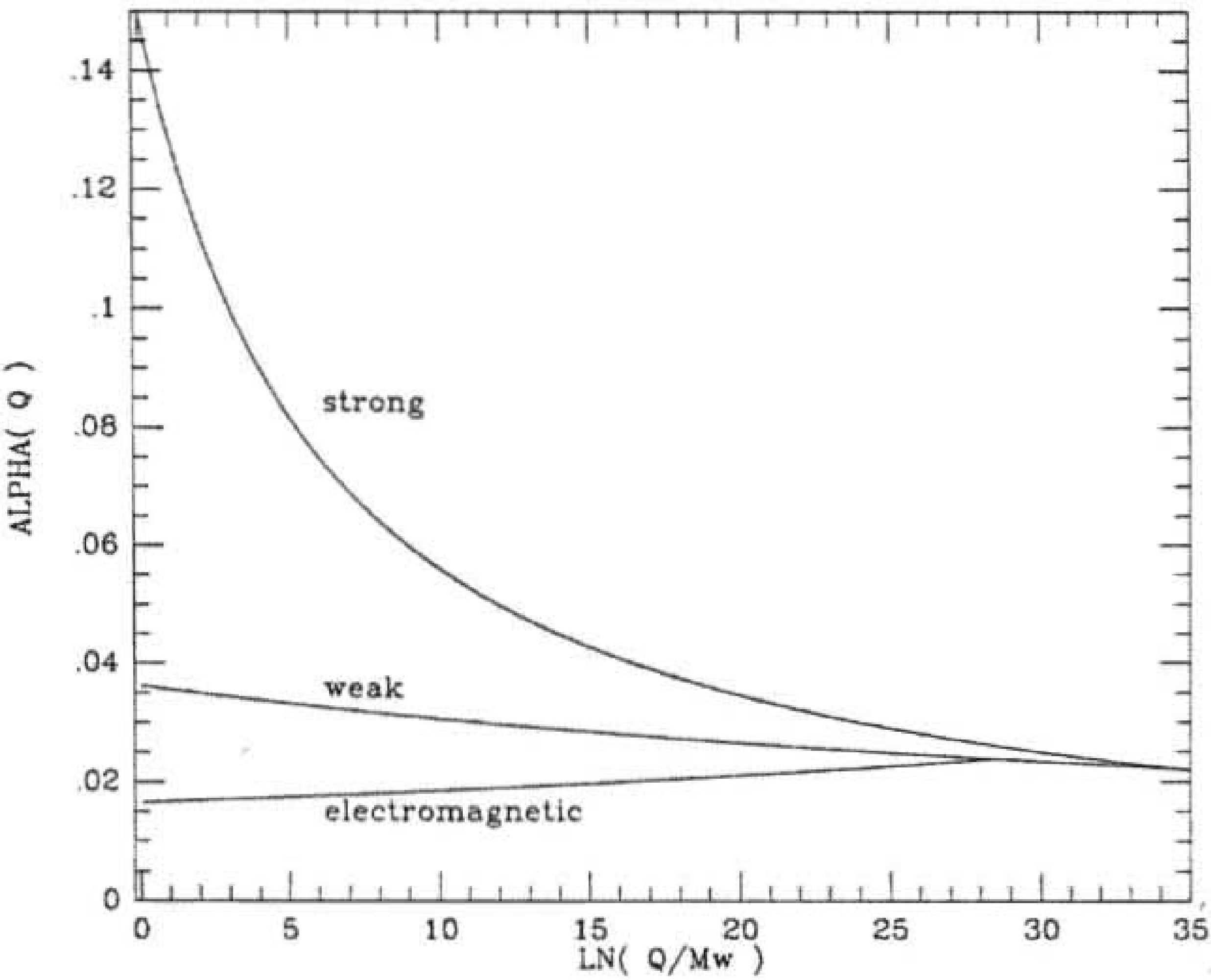

ALPHA(Q)
.14
.12
.1
.08
.06
.04
.02
0
strong
weak
electromagnetic
0
5
10
15
20
25
30
35
LN(Q/Mw)

FIG. 2.1

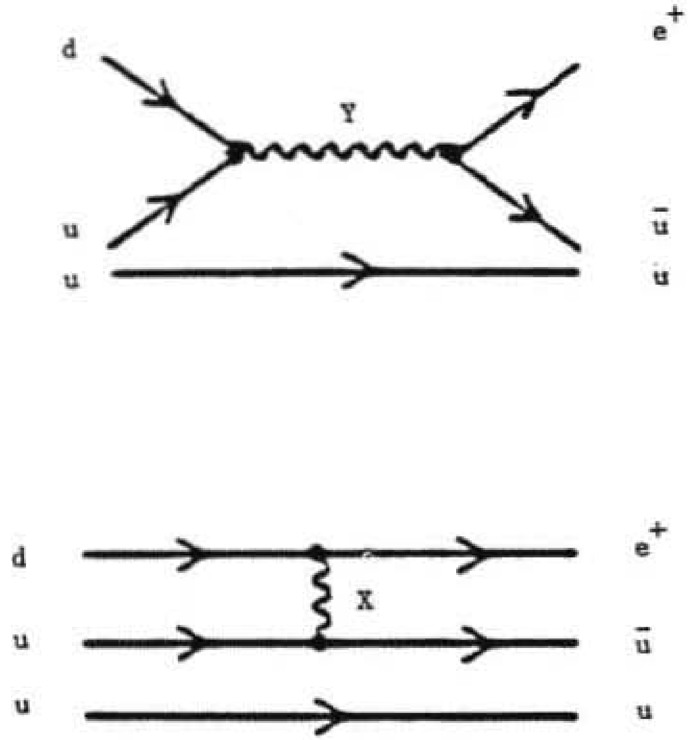


FIG. 2.2

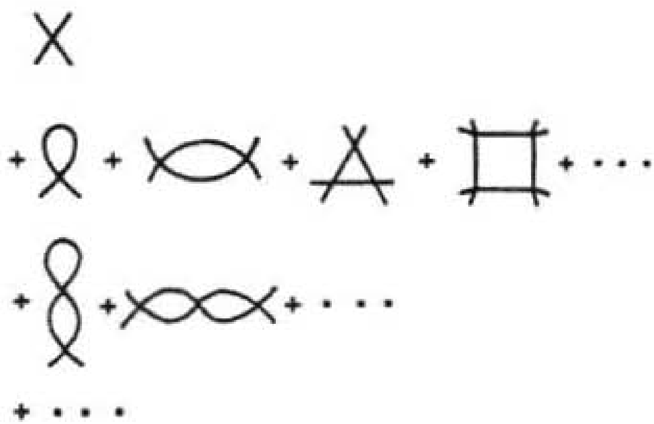


FIG. 3.1

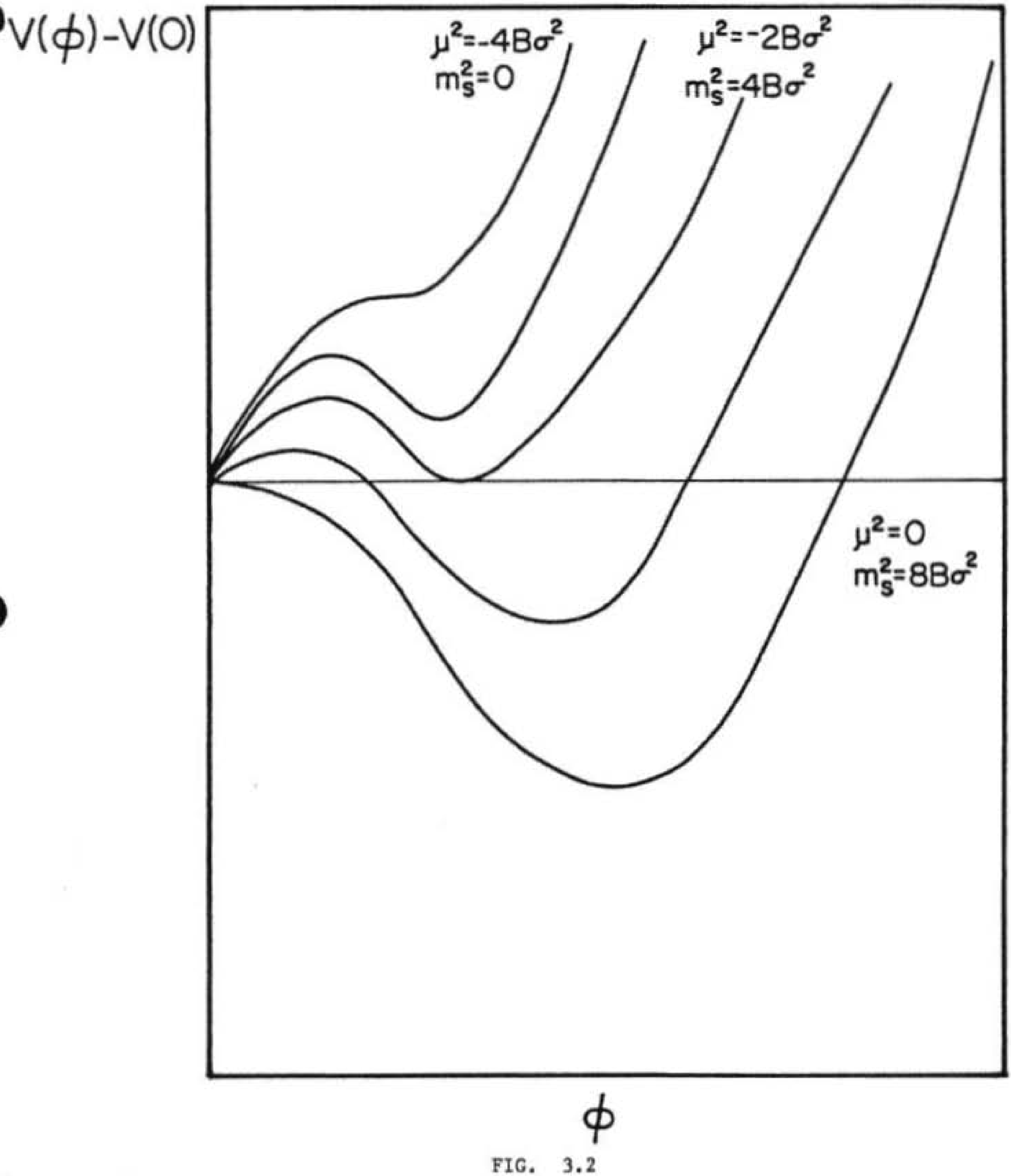


FIG. 3.2

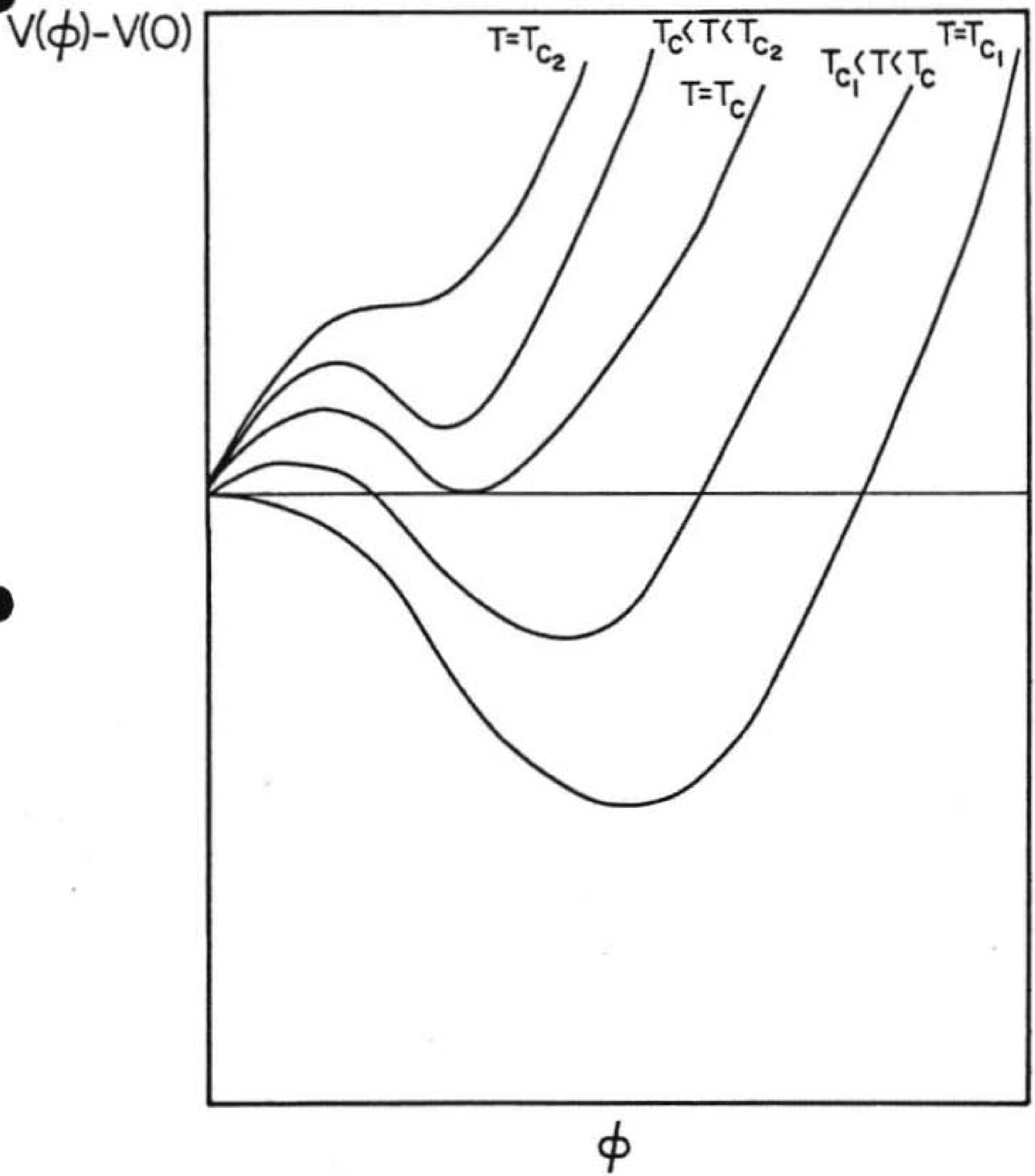


FIG. 3.3

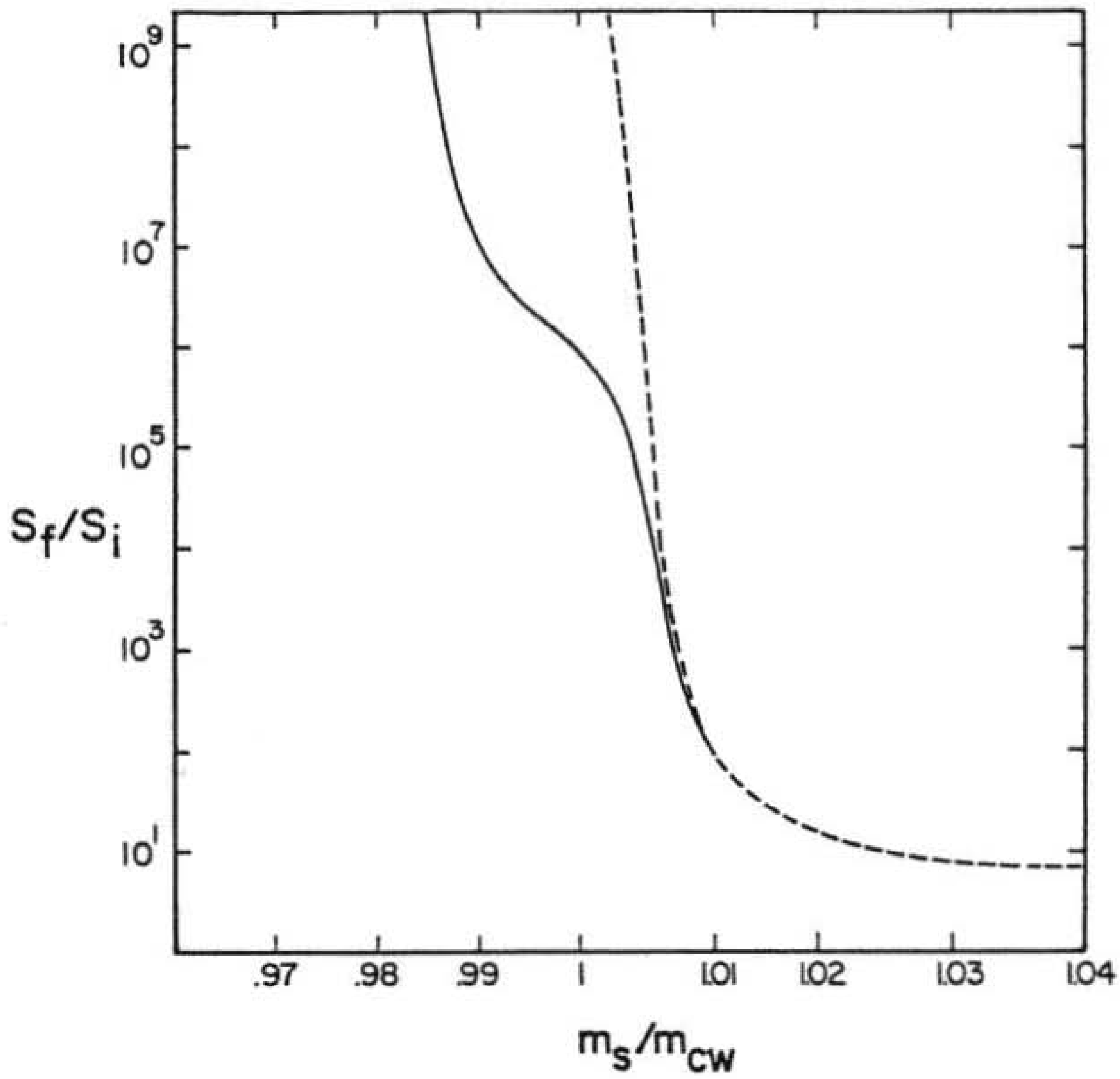


FIG. 5.1

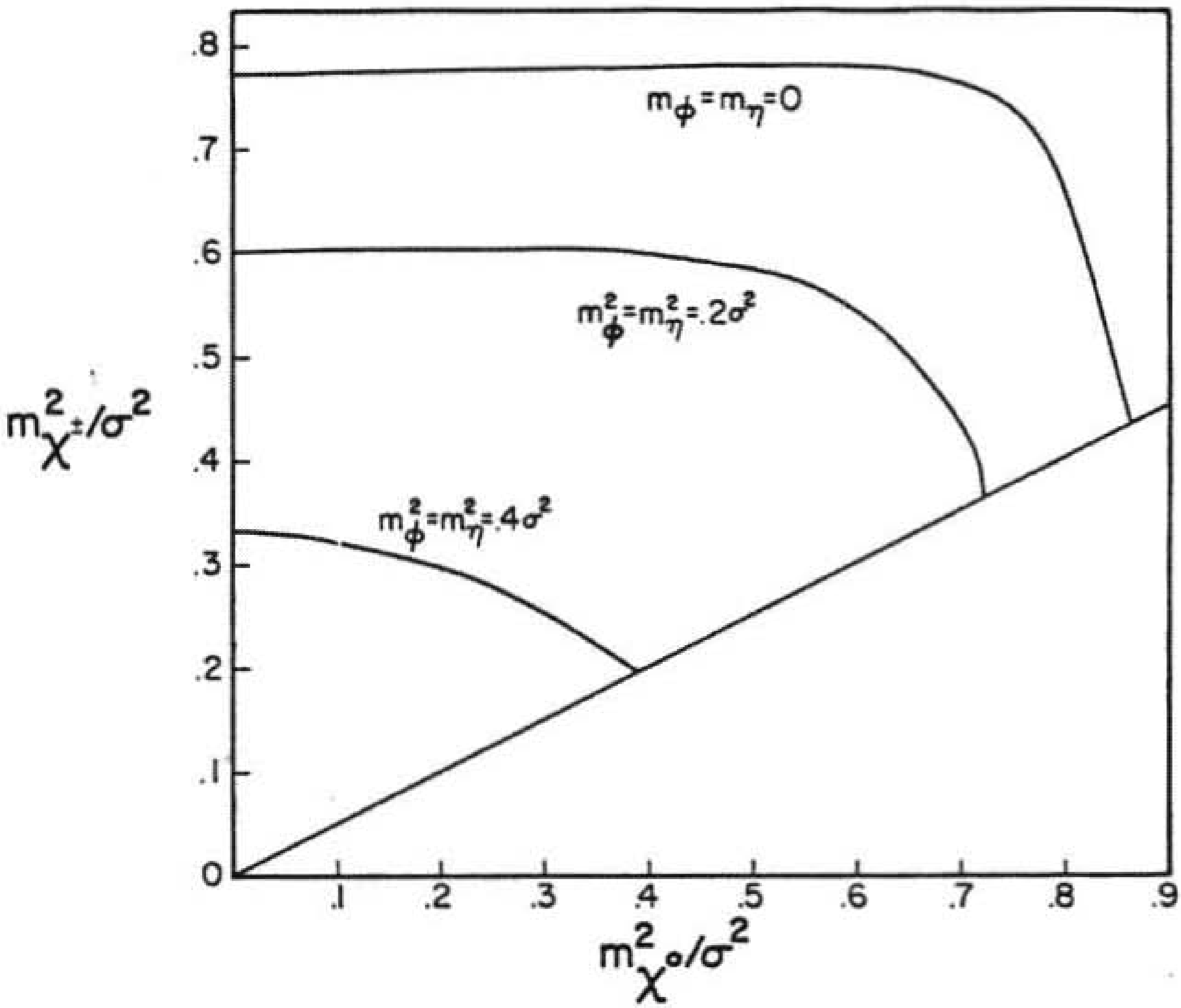
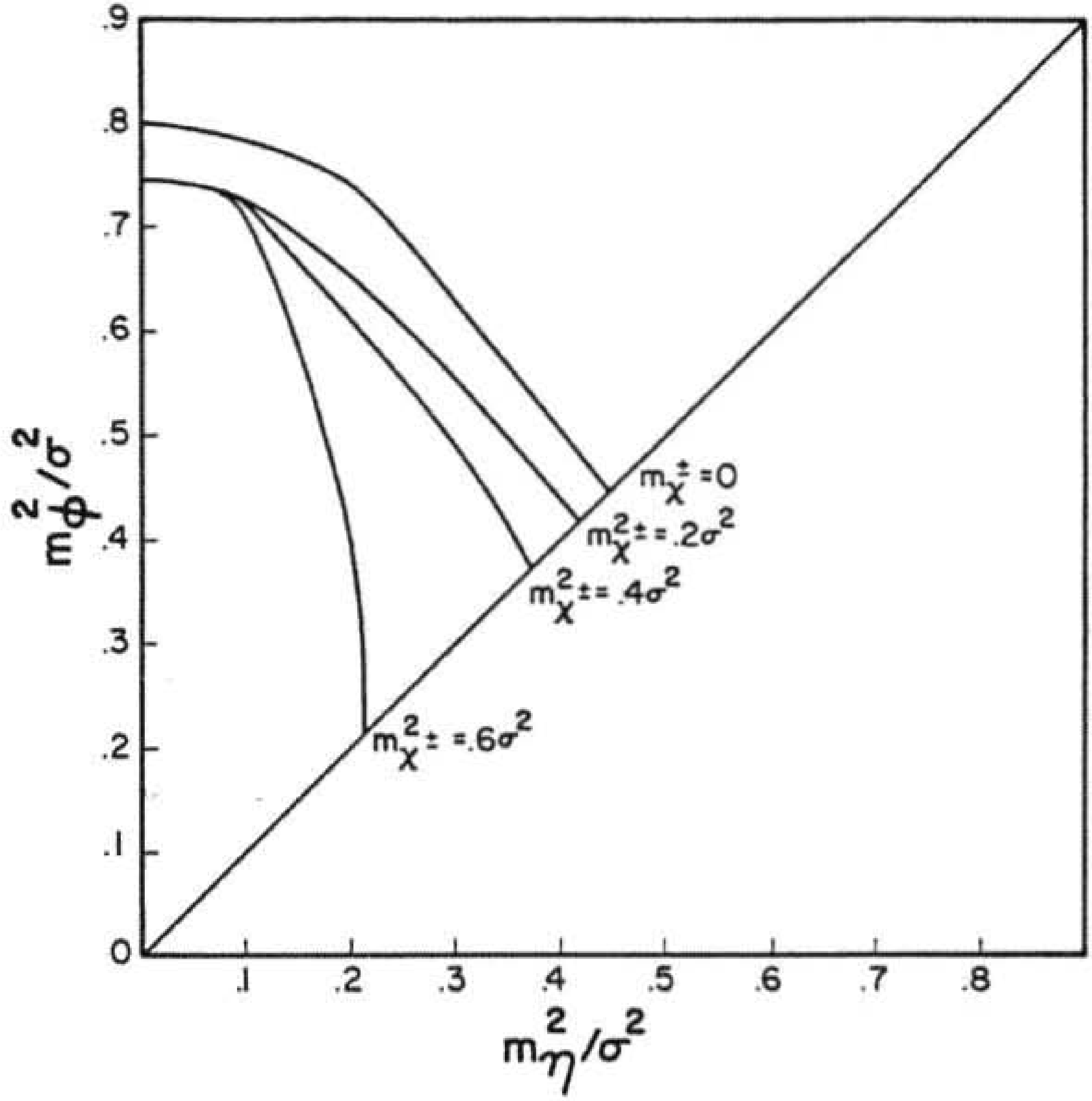

.8
.7
.6
.5
.4
.3
.2
.1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
$m_\phi = m_\eta = 0$
$m_\phi^2 = m_\eta^2 = .2\sigma^2$
$m_\phi^2 = m_\eta^2 = .4\sigma^2$
$m^2_{\chi^\pm}/\sigma^2$
$m^2_{\chi^0}/\sigma^2$

FIG. 5.2



.9
.8
.7
.6
.5
.4
.3
.2
.1
0
.1
.2
.3
.4
.5
.6
.7
.8
m_ϕ^2/σ^2
m_η^2/σ^2
$m_{\chi^\pm} = 0$
$m_{\chi^\pm}^2 = .2\sigma^2$
$m_{\chi^\pm}^2 = .4\sigma^2$
$m_{\chi^\pm}^2 = .6\sigma^2$

FIG. 5.3

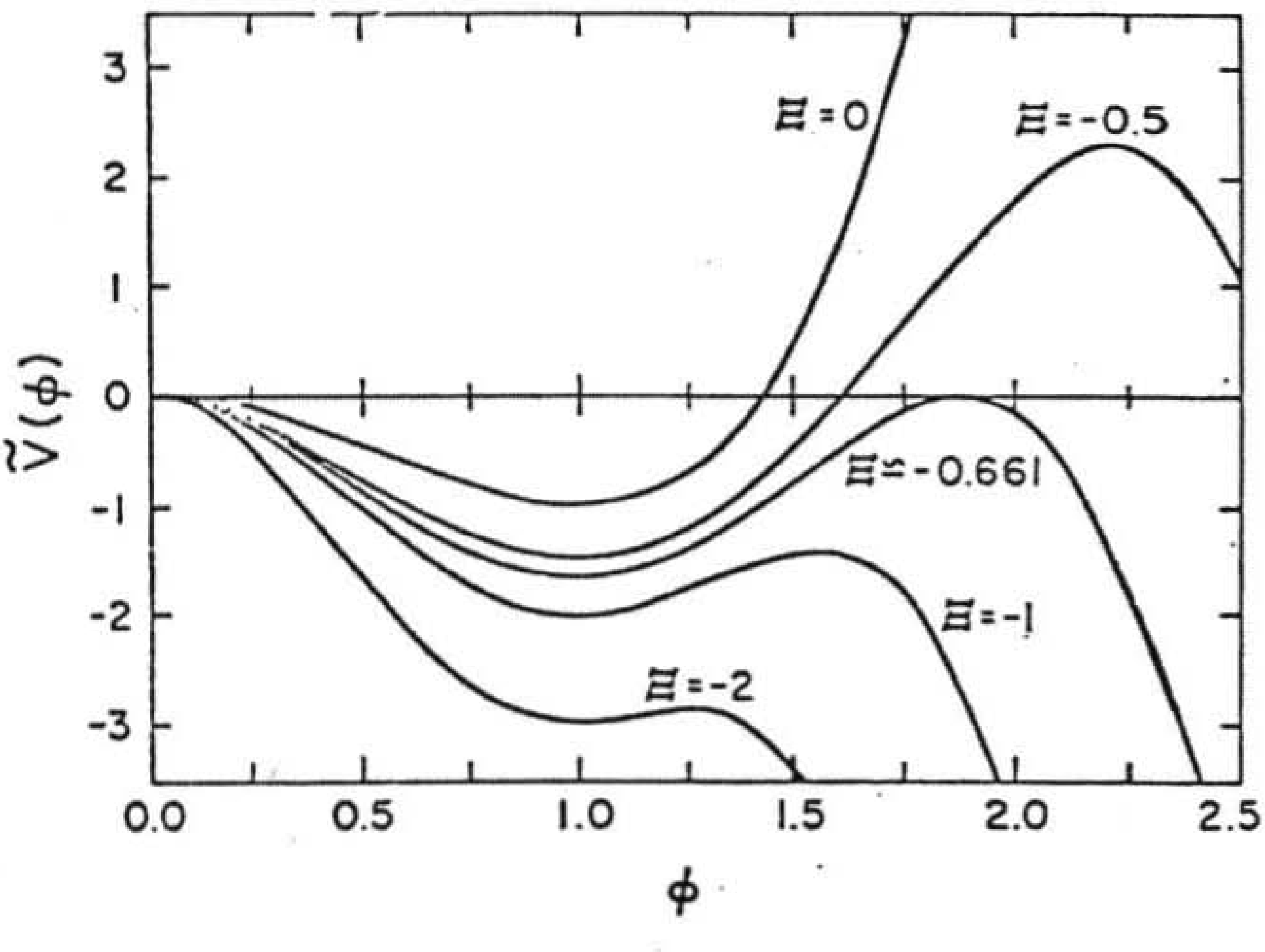


FIG. 6.1

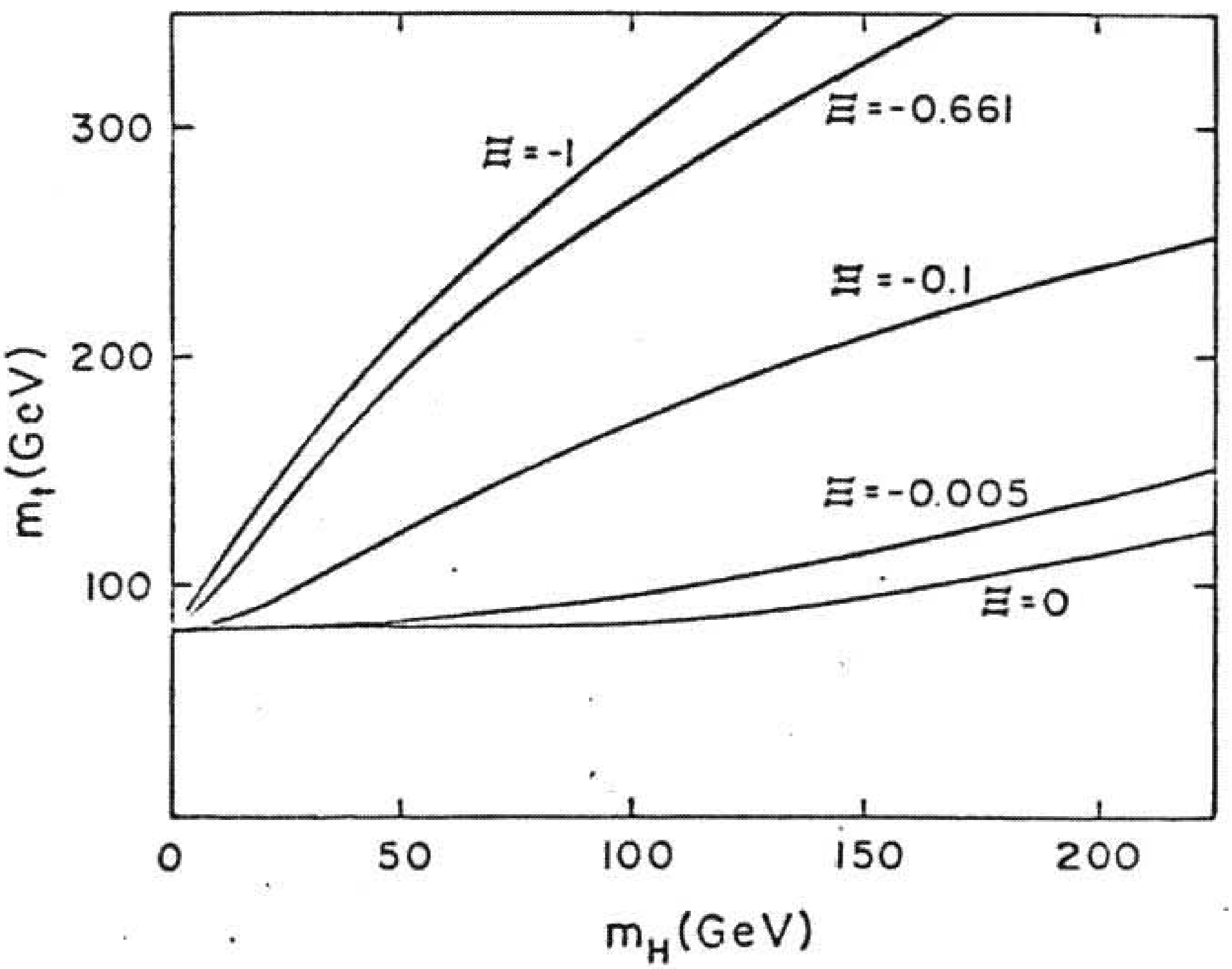


FIG. 6.2

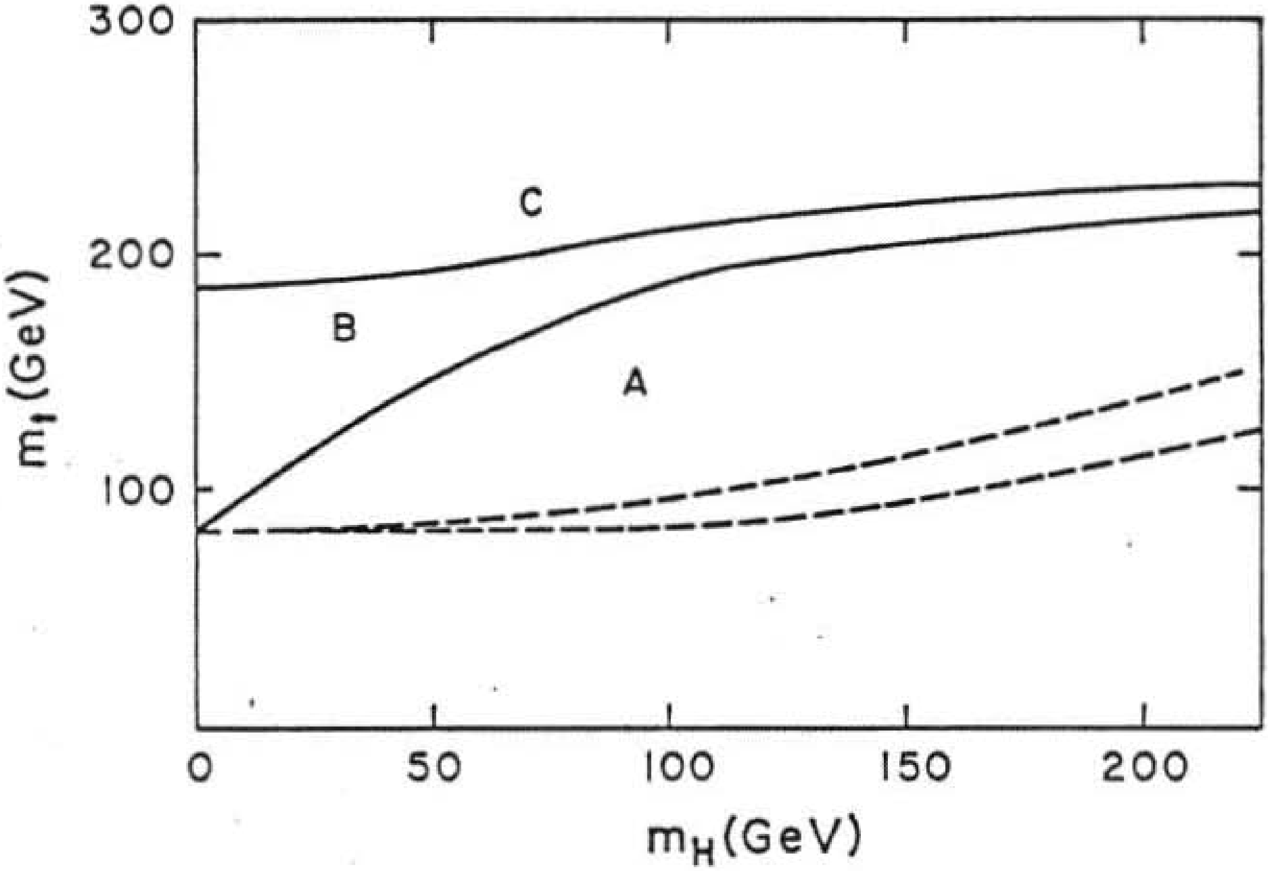
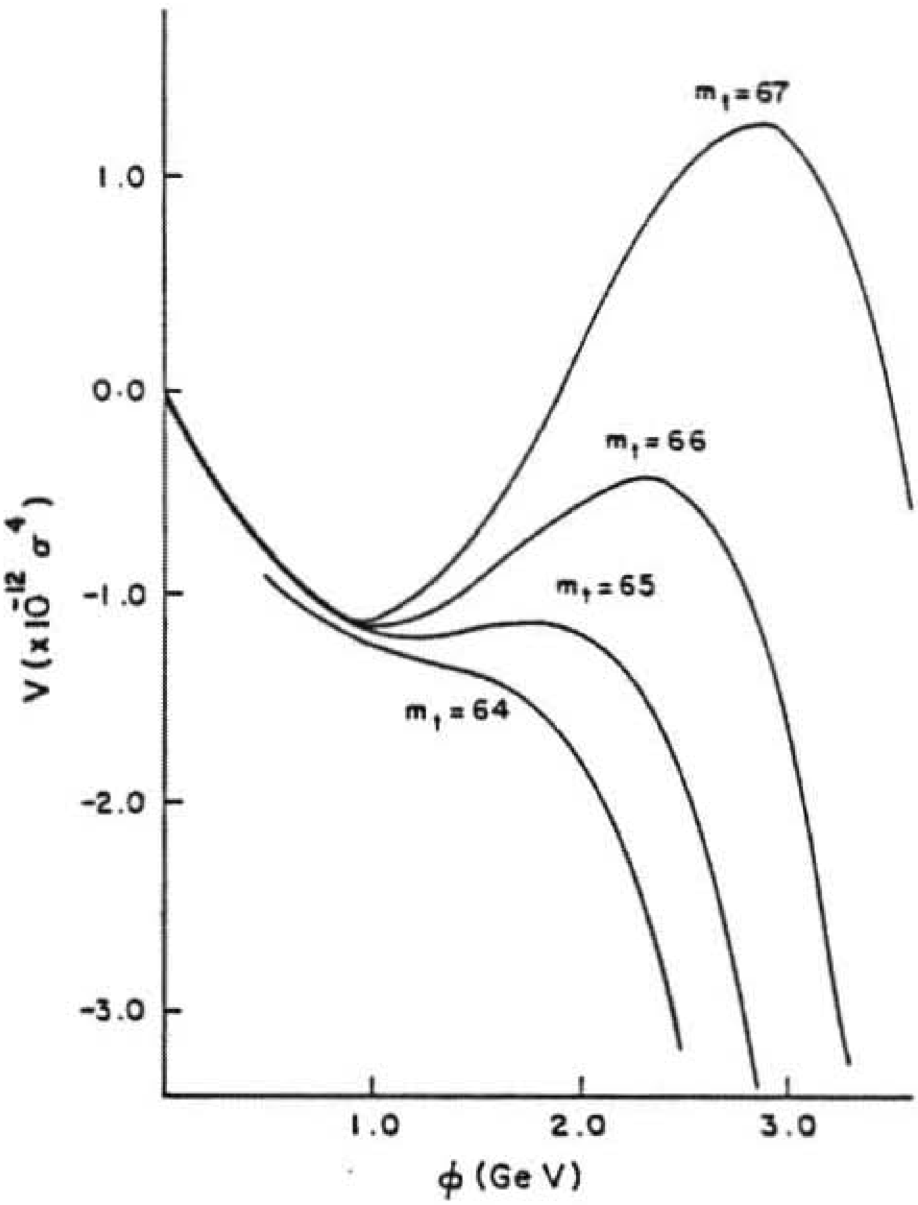


FIG. 6.3



$m_t = 67$
$m_t = 66$
$m_t = 65$
$m_t = 64$
1.0
0.0
-1.0
-2.0
-3.0
$V (\times 10^{-12} \sigma^4)$
1.0
2.0
3.0
ϕ (GeV)

FIG. 6.4