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The Missing Dimension: Two-Dimensional Approaches to Matters Epistemic

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Abstract:

The paper begins with an overview of epistemic two-dimensionalism, as developed by David Chalmers (1996, 2002, 2004, 2006a, forthcoming, manuscript b, c) and Chalmers and Frank Jackson (2001). It then shows that the two-dimensional framework can be extended to account for epistemic possibility, broadly construed. Finally, a solution to the problem of accounting for the epistemic possibility of necessarily false statements is offered.

I. Standard Semantics

According to what we might call ‘standard semantics’ – a theory of meaning which owes much to thinkers such as Gottlob Frege (1892), Bertrand Russell (1902),¹ C. I. Lewis (1943), Rudolf Carnap (1947), Richard Montague (1973), David Kaplan (1973), Saul Kripke (1980) and David Lewis (1980) – sentences express, relative to contexts, so-called Russellian propositions. Different constituents of sentences contribute different entities to these propositions. For example, names such as ‘Hillary Clinton’ and indexicals such as ‘I’ and ‘now’ contribute their referents, predicates such as ‘is human’ contribute properties, and so on. Russellian propositions are structured sets or collections of such worldly entities.

Because the semantic contribution of an expression may depend on who the speaker is, where she is located, when she is speaking, and so forth, Russellian propositions are said to be

¹ See in particular his correspondence with Frege (1980).

expressed by sentences only relative to contexts of use (or speech situations). So, 'I am hungry now' expresses different propositions relative to different contexts. If uttered by me at 3 p.m. on January 8, 2007 in Chicago, the sentence 'I am hungry now' will express a proposition that consists of me, the property of being hungry and the time indicated by '3 p.m. on January 8, 2007 Central time'. If uttered by you at 3 pm. on August 7, 2006 in New York, on the other hand, it will express a proposition that consists of you, the property of being hungry and the time indicated by '3 p.m. August 7, 2006 Eastern time'.

Given standard semantics, the constituents of propositions determine extensions relative to circumstances of evaluation (Kaplan 1973). For simplicity's sake, we shall take circumstances of evaluation to be worlds, but circumstances of evaluation could also, at least in principle, be taken to be pairs of a world and a time, or triples of a world, a time, and a location (or something even more complex).² The extension determined by an individual relative to a world is just the set containing the individual at the world in question, the extension determined by a property relative to a world is the set of the individuals that instantiate the property at the world in question, and the extension determined by a proposition is its truth-value.³ So, the extension determined by the property of being human at the actual world is the set of all humans, the extension determined by the proposition that Brit is sitting at January 25, 2007 8:43 in St. Louis is the truth-value *true*, and so on.

As Lewis (1943), Carnap (1947), and Kaplan (1973) envisaged it, while Russellian propositions are best understood as structured entities, one can think of such Russellian propositions and their constituents as determining intensions. The intension determined by a property is a function from worlds to the set of individuals instantiating the property at these worlds. So, the intension of the property of being human is a function from worlds to the set of humans (or the empty set) at these worlds. The intension determined by a Russellian proposition

² See e.g. Kaplan (1973), Montague (1973), MacFarlane (forthcoming), Recanati (forthcoming a, forthcoming b), and Brogaard (manuscript).

³ Or the actual world, as suggested by C. I. Lewis (1946: 57).

is a function from worlds to truth-values, or equivalently, the set of possible worlds at which the Russellian proposition is true. So, the proposition that Brit is human determines a set of possible worlds in which Brit exists and is human.

Given the standard framework, an intension determined by a proposition can be said to be true at a world iff the world in question is a member of it. Likewise, an intension can be said to be necessary iff every possible world is a member of it, and it can be said to be contingent iff some possible worlds are members of it, and some are not. So, the proposition expressed by my utterance of the sentence 'Clark Kent (if he exists) is Superman' determines the set of possible worlds in which Clark Kent either does not exist or else has the property of being (identical to) Superman. As every possible world will be a member of this set, the intension of 'Clark Kent (if he exists) is Superman' is necessary. The proposition expressed by my utterance of the sentence 'Brit is hungry now', on the other hand, determines the set of possible worlds in which Brit is hungry at the time of speech. Since not every possible world is a member of this set, and the intension is consistent, the intension in question is contingent.

II. The Missing Dimension

Since the mid 20th century it has been widely assumed that there is close connection between meaning and intension (Lewis 1943, Carnap 1947).⁴ The meaning of a sentence, it was thought, could simply be assimilated to the intension determined by the proposition expressed by the relevant sentence (perhaps relative to context). On this approach, the meaning of an utterance of the sentence 'Brit is hungry now' is the set of possible worlds at which (Brit and the time of speech exist and) Brit instantiates the property of being hungry at the time of speech.

But, as recent work by David Chalmers (1996, 2002, 2004, 2006a, forthcoming, manuscript b, c) and Chalmers and Frank Jackson (2001) has made vivid, the classic picture is

⁴ There is also lexical meaning or character, in Kaplan's (1973) sense. 'Meaning' should here be understood in the non-lexical sense.

unduly simplistic. Chalmers and Jackson suggest, quite plausibly, that at least one aspect of meaning should line up with the cognitive abilities of speakers to pick out the referents of the relevant expressions in hypothetical scenarios. But, as standard semantics has it, meaning is not anchored in the abilities of speakers to identify referents. Suppose we are back in ancient Greece. Athena learns, as everyone does at the time, that Hesperus is the brightest object in the evening sky, and that Phosphorus is the brightest object in the morning sky. As we now all know, Hesperus just is Phosphorus (i.e., the planet Venus). But Athena does not have an ability to pick out Venus as the referent of 'Hesperus' and 'Phosphorus' in hypothetical scenarios. If Mars were the brightest object in the evening sky, and Jupiter were the brightest object in the morning sky, Athena would quite reasonably pick out Mars as the referent of 'Hesperus' and Jupiter as the referent of 'Phosphorus'. It seems plausible that Athena's ability to identify the referents of 'Hesperus' and 'Phosphorus' in hypothetical scenarios should have some bearing on meaning. But if standard semantics with its one dimension of meaning were correct, Athena's cognitive abilities to pick out referents in hypothetical scenarios would have no relation to meaning. And it is this assumption epistemic two-dimensionalism seeks to challenge.

Following Chalmers and Jackson, let us call the intension, as traditionally construed, the 'subjunctive intension' and the extension, as traditionally construed, the 'subjunctive extension'.⁵ The subjunctive intension (extension) of an expression, then, is the intension (extension) standard semantics assigns to the expression. So, the subjunctive intension of 'Hesperus' is a function from worlds to Venus or the null extension, the subjunctive intension of 'water' is a function from worlds to H₂O or the null extension, and the subjunctive intension of 'Hesperus (if it exists) is Phosphorus' is a function from worlds to the truth-value *true*.

Two-dimensionalists add to the standard picture what they call the 'epistemic intension'. Unlike the subjunctive intension, the epistemic intension of a use of an expression is firmly

⁵ The subjunctive intension of an expression is also known in the literature as the '2-intension' or the 'secondary intension'. The epistemic intension of an expression (see below) is also known as the '1-intension' or the 'primary intension'.

grounded in the speaker's abilities to pick out the extension of the expression in hypothetical scenarios. As Chalmers and Jackson (2001) emphasize, a speaker may be in possession of such an ability to make a judgment concerning an expression's referent or application even if the expression does not admit of philosophical analysis. As the post-Gettier literature indicates, it is unlikely that the concept of knowledge will ever admit of 'counterexample-free analysis'. Even so, competent language-users have no difficulty making judgments concerning the concept's application in hypothetical scenarios.⁶ To offer a counterexample to an attempted analysis of the concept of knowledge just is to make a judgment concerning its application. Of course, due to vagueness and indeterminacy, competent language-users may not be able to make a judgment vis-à-vis the concept's application in all hypothetical scenarios but in the core cases the claim that competent language-users have the ability seems indisputable.

Speakers acquire such abilities through exposure to the language. Repeated encounters with applications of the concept of knowledge put them in a position to apply the concept. So, the ability to apply the concept is grounded in a posteriori (perhaps implicit) knowledge concerning the concept's application. But the further application of the concept in hypothetical scenarios requires no additional knowledge of actual empirical facts.⁷ So, there is a sense in which the ability (if properly exercised) yields a priori evaluations. The ability is acquired by way of exposure to others' uses of the concept and to others' evaluations of uses of the concept, but exercising the ability hypothetically requires no further empirical information about the actual world.⁸

Chalmers and Jackson (2001) think the concept of knowledge is a confirming instance of a more general hypothesis to the effect that a speaker's use of an expression is associated with an ability to make judgments concerning the expression's application in hypothetical scenarios. The

⁶ For discussion, see also Garcia-Carpintero (2006).

⁷ As we will see below, the ability in question is an ability to determine the epistemic extension in semantically neutral descriptions of hypothetical scenarios.

⁸ There is an interesting parallel between Chalmers and Jackson's notion of apriority so construed and the notion of apriority expounded in the work of Jenkins. See e.g. Jenkins (2005).

epistemic extension of an expression is in each case the extension determined by the reference-fixing abilities of the speaker, given her particular use of the expression. For example, a speaker like Athena may use 'Hesperus' to refer in the actual world to the brightest object in the evening sky. If the speaker's particular use of the expression reveals that she takes 'Hesperus' to pick out the brightest object in the evening sky, then in all scenarios in which some unique object is the brightest in the evening sky, 'Hesperus' refers (in the epistemic sense) to it. And this is so, even if the object that is the brightest in the evening sky is the subjunctive referent of 'Jupiter'. Likewise, if a speaker's use of 'water' reveals that she uses 'water' to refer in the actual world to the potable liquid that flows in rivers and lakes, then in all scenarios in which some potable liquid flows in rivers and lakes, 'water' refers (in the epistemic sense) to it. And this is so, even if the potable liquid that flows in rivers and lakes is not the subjunctive referent of 'H₂O'. Purely qualitative expressions (e.g. 'house', 'red', and 'hot') will have the same subjunctive and epistemic intension. Further, as the epistemic intension of an expression is determined by the speaker's actual use of the expression, the epistemic and subjunctive intensions of an expression coincide at the actual scenario/world pair. So a given expression's epistemic extension at the actual scenario is identical to the expression's subjunctive extension at the actual world.

III. Enriched Propositions and Cognitive Significance

In the preceding sections we took as our starting point standard semantics with its Russellian propositions. Epistemic two-dimensionalists proceed slightly differently (Chalmers manuscript b, c). Instead of taking Russellian propositions to determine intensions, epistemic two-dimensionalists, in particular David Chalmers, take sentences to express, relative to contexts, enriched propositions. Enriched propositions are structured entities. But they are built up slightly differently from Russellian propositions. An enriched proposition has two components: a structured subjunctive intension and a structured epistemic intension. A structured subjunctive intension is just the set of the subjunctive intensions of the constituents of the sentence token.

Likewise, a structured epistemic intension is the set of the epistemic intensions of the constituents of the sentence token. As mentioned above, the epistemic intension of an expression is grounded in the speaker's ability to pick out the expression's epistemic referent in hypothetical scenarios.

A hypothetical scenario is a centered *metaphysically possible* world,⁹ viz. a world in which a particular time and individual are marked. This centering is introduced in order to account for the epistemic intension of indexicals. Utterances of 'John and Mary are getting married today' and 'John and Mary are getting married on March 7, 2007' on March 7, 2007 have exactly the same subjunctive intension. This is because at each world the referent of 'today', as uttered in the actual world on March 7, 2007, just is March 7, 2007. It is quite obvious, however, that a competent speaker may not be able to pick out March 7, 2007 as the referent of 'today' at hypothetical scenarios construed as worlds. Indexicals thus have different epistemic intensions. The epistemic intension of 'I' is a function from scenarios to the individual in the center. And the epistemic intension of 'now' is a function from scenarios to the time in the center.

Note that in order for a speaker to have the ability to pick out the epistemic referent of an expression in a hypothetical scenario, it is not required that the speaker associates with the expression a description (Chalmers 2002). Nor is it required that the speaker acquired a grasp of the expression on a priori grounds. Through repeated encounters with applications of the term 'water', for example, a speaker may acquire an ability to judge whether or not 'water' applies to a given substance in a given hypothetical scenarios. As her ability to make such judgments is determined by her use of the expression at the time in question, her possession of this ability does not require that she have in her possession a description whose intension approximates that of the expression. Nor does it require that there be a description whose intension approximates that of the term. If, for example, the speaker's use of the term 'water' is deferential, she will be inclined to look to others' use of the term in the given scenario in order to identify the epistemic

⁹ This is what Chalmers calls 'Metaphysical Plenitude': For all S, if S is epistemically possible, there is a centered metaphysically possible world that verifies S (2006a: 82). For criticism of this thesis, see e.g. Yablo (1999, 2002), and for replies see Chalmers (1999, 2004).

extension. If the speaker's use of the expression is anchored in perceptual acquaintance with the substance in her environment, then she will be more inclined to identify as the epistemic referent of the term 'water' a substance with superficial properties similar to those of the substance in her actual environment.

As far as Chalmers (2002, 2006a) and Chalmers and Jackson (2001) are concerned, a speaker's ability to make judgments concerning a given expression's application in hypothetical scenarios is idealized. Since real speakers are cognitively limited, they may not have access to their own use of a given expression. I may think that I use 'Gödel' to refer to the discoverer of the incompleteness of arithmetic, when it happens to be the case that I use it to refer to the person called 'Gödel' by those from whom I acquired that name. If so, then the epistemic referent of 'Gödel' need not be the individual I would take to be the epistemic referent, if we keep actual cognitive limitations fixed. Rather, it is the individual I would take to be the epistemic referent if I were logically omniscient and had epistemic access to my own use of the expression.

Moreover, to avoid triviality Chalmers (2002, 2006a, manuscript b, c) and Chalmers and Jackson (2001) stipulate that the judgments in question concern minimal yet complete canonical descriptions of scenarios, where a canonical description is one that contains no terms or kind-terms, and no broadly indexical expressions. Suppose I gave you the following partial description of a scenario, 'The moon is the brightest object in the evening sky', and asked you to tell me whether Hesperus is the brightest object in the evening sky in that scenario. Regardless of how you acquired the name 'Hesperus' you would be inclined to answer 'no'. The problem here is that the description contains an expression whose subjunctive and epistemic referents come apart. Relative to the epistemic intension of the description, it may indeed be true that Hesperus is the brightest object in the evening sky in the described scenario. But relative to the subjunctive intension of the description, it is false that Hesperus is the brightest object in the evening sky at the described scenario. To avoid the "ambiguity" inherent in non-canonical descriptions,

Chalmers and Jackson thus assume that descriptions of scenarios contain no expressions whose subjunctive intensions deviate from their epistemic intensions.

The two-dimensional framework can now be used to account for the sort of cognitive significance we alluded to earlier. For Athena, it is not cognitively significant that Hesperus (if it exists) is Hesperus. But it is cognitively significant or informative that Hesperus (if it exists) is Phosphorus. Athena learns something new when she discovers that the latter is indeed the case. On Chalmers' (manuscript b, c) framework, an enriched proposition is necessary (in the subjunctive sense) iff the subjunctive intension determined by that proposition is true at every possible world. An enriched proposition is cognitively insignificant or necessary (in the epistemic sense) iff the epistemic intension determined by that proposition is true at every hypothetical scenario.¹⁰ When an enriched proposition is cognitively insignificant or necessary in the epistemic sense it is said to be a priori. So a proposition is cognitively insignificant or a priori iff its epistemic intension is necessary, and a proposition is necessary iff its subjunctive intension is necessary. 'Hesperus (if it exists) is Phosphorus' and 'water (if it exists) is H₂O' are then cognitively significant or a posteriori just when one would not use the terms flanking the identity sign interchangeably. Because the notion of an ability to make judgments in hypothetical scenarios is idealized, a statement may thus be a priori or cognitively significant even if one is not aware of it being a priori or cognitively significant.

Chalmers' and Jackson's approach to cognitive significance sidesteps some obvious problems for description theories of names, i.e., theories of names modeled on Frege's philosophy. As Kripke argued, description theories run into modal and epistemic difficulties. The modal difficulty is this. If 'Hesperus' is equivalent to a non-rigidified description D_1 (e.g.,

¹⁰ Strictly speaking, an enriched proposition is cognitively insignificant or necessary (in the epistemic sense) at a world w iff the epistemic intension determined by that proposition is necessary at w , and the proposition is entertainable at w (Chalmers manuscript c, note 18). An enriched intension is live at a scenario v iff the subjunctive component has the same extension as the epistemic component at v . An enriched proposition is entertainable at a world w iff each of its components is live at a scenario corresponding to w . As the entertainability clause plays a role only when a sentence is embedded in mixed modal environments, I shall ignore this complication here.

‘the brightest object in the evening sky’) and ‘Phosphorus’ is equivalent to a distinct non-rigidified description D_2 (e.g. ‘the brightest object in the morning sky’), then ‘Hesperus (if it exists) is Phosphorus’ is not necessary, contrary to our modal intuitions. As the two-dimensional framework outlined by Chalmers and Jackson grants that there is a Kripkean component of meaning, which is the bearer of modal properties such as *being necessary*, it does not run into this difficulty.

The epistemic difficulty is this. One may be competent with a name, even if the information one associates with the name does not suffice to pick out a unique individual. As Kripke puts it, ‘most people, when they think of Cicero, just think of *a famous Roman orator*, without pretension to think that either there was only one famous Roman orator, or that one must know something else about Cicero to have a referent for the name’ (1980: 81). Likewise, one may be competent with a name but associate with the name a description that denotes a unique individual but, alas, the wrong individual. For example, one may use ‘Gödel’ to refer to Gödel while thinking Gödel is the discoverer of the incompleteness of arithmetic, when the fact is that ‘the discoverer of the incompleteness of arithmetic’ picks out Schmidt from whom Gödel stole the proof.

As Chalmers and Jackson’s two-dimensional framework does not require that there is a description whose intension is equivalent to that of the original term, it does not run into this difficulty either. If a speaker is able to use ‘Cicero’ to refer to Cicero in the actual world, in spite of the fact that she possesses insufficient information regarding Cicero, then that shows that the epistemic intension of her use of ‘Cicero’ is not equivalent to that of the descriptive information she possesses regarding Cicero.

Likewise, if a speaker is able to use ‘Gödel’ to refer in the actual world to Gödel but Gödel is not the discoverer of the incompleteness of arithmetic, then that shows that ‘the discoverer of the incompleteness of arithmetic’ and ‘Gödel’, as the speaker is using these expressions, have different epistemic intensions. For the epistemic intension of ‘Gödel’ is

determined by the speaker's use of the term. As our speaker is able to use 'Gödel' to refer to Gödel, and Gödel is not the discoverer of the incompleteness of arithmetic, she does not use 'Gödel' to refer to the discoverer of the incompleteness of arithmetic. If, on the other hand, she really did use 'Gödel' interchangeably with 'the discoverer of the incompleteness of arithmetic', then it could hardly be complained that her use of 'Gödel' yields the wrong referent.

So, Chalmers' and Jackson's two-dimensional framework does not run into the problems which Kripke outlined for description theories of names.¹¹ Moreover, their framework can be seen to yield an account of what deep epistemic operators such as 'it is a priori that' and 'it is a posteriori that' operate on:¹² unlike alethic operators, which operate on subjunctive intension, 'it is a priori that' and 'it is a posteriori that' operate on epistemic intension. So, if a speaker uses 'Hesperus' and 'the brightest object in the evening sky' interchangeably, then 'Hesperus (if it exists) is the brightest object in the evening sky' will be a priori. If not, not.

IV. Attitude Ascriptions

Besides offering an account of the second dimension of meaning, epistemic two-dimensionalism has also been utilized in developing a new account of belief reports (Chalmers 2006b, manuscript b, c). The problem posed by belief reports for standard semantics is that intersubstitutability of co-referring terms seems illegitimate in belief context (Frege 1892). Consider, for instance:

- (1) Lois Lane believes Superman can fly
- (2) Lois Lane disbelieves that Clark Kent can fly

¹¹ Contrary to what Soames (2005) and Byrne and Pryor (2006) argue and Stalnaker (2006) suggests. See also Chalmers (2006b) for a reply.

¹² The expression 'deep epistemic possibility' is from Chalmers (manuscript, a). According to Chalmers, a proposition is deeply epistemically possible for S iff it is compatible with what a priori for S.

Intuitively, (1) and (2) could both be true. But this is puzzling, as ‘Superman’ and ‘Clark Kent’ ought to be intersubstitutable (given standard semantics). Saul Kripke (1979, §3) introduced a variation on Frege’s puzzle which does not rest on the assumption that names refer to the same entities in all linguistic contexts. Consider:

- (3) Peter believes that Paderewski has musical talent
- (4) Peter disbelieves that Paderewski has musical talent

As Kripke set up the puzzle, (3) and (4) are both true because Peter fails to realize that Paderewski, the statesman, is identical to Paderewski, the composer. But on a standard semantic account, the assumption that (3) and (4) could both be true is puzzling, for the embedded clauses express the very same proposition.

Numerous authors have offered solutions to these puzzles.¹³ Many agree that in order to give correct truth-conditions of belief reports we need to introduce modes of presentation (see e.g. Schiffer 1990). Lois Lane disbelieves that Clark Kent can fly relative to a certain way of presenting Clark Kent. Likewise, Peter believes that Paderewski has musical talent relative to one way of presenting Paderewski but not relative to another.

Modes of presentation carry notable similarity to epistemic intensions (Chalmers manuscript c). It is thus very plausible that epistemic intensions can play the role of modes of presentation. Chalmers offers the following truth-conditions for ‘S believes that *p*’: an utterance of ‘S believes that *p*’ is true iff S has a belief with the subjunctive intension of the ‘that’ clause, as used by the ascriber, and an epistemic intension which is coordinate with (i.e. appropriately connected to) the epistemic intension of the ‘that’ clause, as used by the ascriber.

On this account, (1) and (2) may both be true because Lois Lane associates different epistemic intensions with ‘Superman’ and ‘Clark Kent’. Likewise, (3) and (4) may both be true

¹³ Representative works include Richard (1990), Schiffer (1990), Bach (1997), and Soames (2005).

because Peter associates different epistemic intensions with ‘Paderewski’ in different settings. Note here that the subject and the ascriber may associate different epistemic intensions with ‘Clark Kent’ or ‘Paderewski’. What is important is that the epistemic intension of the ‘that’ clause is coordinate with that of the subject’s belief. Chalmers admits that there may not be a regimented account of coordination. Rather, it seems clear that the notion of coordination will depend to a large extent on the interests and purposes of the conversationalists. In contexts where the speakers care less about the precise choice of words, it may even be that (2) comes out false because it is appropriate in the relevant context to say that Lois Lane believes that Clark Kent can fly.¹⁴

It might perhaps be thought that we could make do with epistemic intensions and just take belief operators to operate on them. But we can’t. For consider an utterance in the actual world of (Chalmers 2006b, manuscript c):

(5) Twin Oscar believes that water is tasty

Suppose the watery substance in Twin Oscar’s environment is composed of XYZ rather than H₂O. Even if Twin Oscar believes that the watery substance in his environment is tasty, there is a strong feeling to the effect that (5) is false. But if belief operators operated on epistemic intensions, (5) would be true. For the epistemic intension of Twin Oscar’s belief that water is tasty is the same as that of the ‘that’ clause in (5) as uttered in the actual world, hence the need for subjunctive intensions in the analysis of belief ascriptions. As Twin Oscar is surrounded by XYZ whereas the utterer of (5) is surrounded by H₂O, the subjunctive intension of Twin Oscar’s belief will differ from that of the ‘that’ clause. So, (5) comes out false.

Chalmers notes that the framework used to account for attitude ascriptions can be extended to account for a priori knowledge ascriptions. Consider for instance:

¹⁴ See also Richard (1990).

(6) It is not a priori knowable by Athena that Hesperus (if it exists) is Phosphorus.

Suppose the speaker of (6) uses the names 'Hesperus' and 'Phosphorus' interchangeably. Then it is clearly not the case that the epistemic intension of 'Hesperus (if it exists) is Phosphorus' is contingent for the speaker. Nonetheless, (6) may well be true. What is required for (6) to be true is that Athena fails to use Hesperus and Phosphorus interchangeably. Likewise, consider an utterance of the following sentence in the actual world:

(7) It is a priori knowable by Twin Oscar that water (if it exists) is the clear liquid in rivers and lakes.

If 'it is a priori knowable' operated on epistemic intension in the same way as 'it is a priori that', (7) ought to be true, but it seems clearly false, as Twin Oscar has never been exposed to water but only to T-water.

The lesson is that there is a significant difference between operators of the form 'it is a priori that' and 'it is cognitively significant that', on the one hand, and 'it is a priori knowable by Athena' and 'it is a priori knowable by Twin Oscar that', on the other. Unlike the former, the latter sort of operator operates on both epistemic and subjunctive intension.

V. Strict Epistemic Possibility

Thus far we have focused on epistemic intensions. When an enriched proposition has a contingent epistemic intension, it can be said to be a posteriori or cognitively significant. In an unpublished paper (manuscript a) Chalmers calls epistemic possibility of this sort 'deep epistemic possibility' and distinguishes it from what he calls 'strict epistemic possibility'. As a rough characterization, a proposition is strictly epistemically possible iff its truth is not ruled out by

what we know. Many propositions are known to be true, even though they are not known to be true on a priori grounds. Suppose I know on a posteriori grounds that Kripke is wearing a green shirt today. In spite of my having this knowledge, my use of ‘Kripke’ is unlikely to give reference-fixing priority to the property of wearing a green shirt. So, ‘Kripke is not wearing a green shirt today’ is deeply epistemically possible for me. That is, it is compatible with my use of the term ‘Kripke’ that Kripke is not wearing a green shirt today, and so it is compatible with what I am in a position to know on a priori grounds that Kripke is not wearing a green shirt today. But it is not compatible with what I know on *a priori* or *a posteriori* grounds that Kripke is not wearing a green shirt today. So, the proposition that Kripke is not wearing a green shirt today is deeply but *not* strictly epistemically possible for me.

For empirical statements it is a straightforward affair to extend epistemic two-dimensionalism to account for strict epistemic possibility. We can simply model strict epistemic possibility on deep epistemic possibility. ‘For all S knows, it might be that *p*’ is true iff the epistemic intension of ‘*p*’ is true at some hypothetical scenario compatible with what S knows, where a hypothetical scenario is compatible with what S knows iff the canonical descriptions of the scenario have intensions that are compatible with what S knows.¹⁵ For example, ‘for all Athena knows, Hesperus (if it exists) might not be identical to Phosphorus’ is true iff the epistemic intension of the ‘that’ clause is true at some hypothetical scenario compatible with what Athena knows. Since there is a hypothetical scenario compatible with what Athena knows at which Mars is the epistemic referent of ‘Hesperus’ and Jupiter is the epistemic referent of ‘Phosphorus’, ‘for all Athena knows, Hesperus (if it exists) might not be identical to Phosphorus’ is true.

However, there are obstacles for this account. Arithmetical statements present one such obstacle. Suppose Goldbach’s conjecture is true. For all we know, it might be false, as we are

¹⁵ We take ‘S knows that’ to be an unanalyzable mental state operator. See Williamson (2000, introduction).

not yet in a position to prove it. It is no exaggeration to say that there exists no very good account of the epistemic possibility of false arithmetical statements. Existing accounts tend to idealize on knowers (Hintikka 1962). For instance, they tend to assume that speakers know all mathematical theorems, which clearly cannot be the case if false mathematical statements may be epistemically possible for the speakers in question.¹⁶

To account for the epistemic possibility of false mathematical statements, some authors have suggested that epistemic possibilities correspond to impossible worlds.¹⁷ However, this assumption is not entirely happy. Suppose, for the sake of illustration, that numbers are places in mathematical structures, as assumed by mathematical structuralists (Benacerraf 1965, Resnik 1997, Shapiro 1997). According to structuralism, numbers have properties only in virtue of standing in relations to other places in the structure of which they are part. Strictly speaking, the number two in the real number structure is not the same as the number two in the natural number structure (unless we stipulate otherwise).

Suppose now that relative to the actual number structure, Goldbach's conjecture is true: every even number greater than two is the sum of two primes. Since mathematical statements, if true, are necessary, it is true in every possible world that relative to the actual number structure Goldbach's conjecture is true. So, there is no possible world where the Goldbach conjecture is false relative to the actual number structure. Worse: there is no impossible world where this is the case either. For the actual number structure is what the actual number structure is. Relative to it, Goldbach's conjecture is true (*ex hypothesi*). So, if mathematical structuralism is true, it won't help to introduce impossible worlds in order to account for the epistemic possibility that Goldbach's conjecture is false.

At first glance, it seems that epistemic two-dimensionalism will do no better in this respect. For while there are deeply epistemically possible hypothetical scenarios in which the

¹⁶ For discussion of Hintikka's framework, see e.g. Hendricks (2003) and Hendricks and Symons (2006).

¹⁷ For paraconsistent approaches, see e.g. Hintikka (1975), Restall (1995) and Priest (2002, 2003).

mathematical theory conventionally employed is different from the theory conventionally employed in the actual world, the epistemic extensions of ‘primality’ and ‘evenness’ at these scenarios will concern the properties actually ascribed by the predicates. This is because mathematical expressions have the same epistemic and subjunctive intensions. So, there is no deeply epistemically possible scenario in which the epistemic extension of ‘is an even number greater than two’ is not a subset of ‘is the sum of two primes’.

Nor is it obvious that there are deeply epistemically (non-deductively closed) *impossible* scenarios in which the epistemic extension of ‘is an even number greater than two’ is not a subset of the epistemic extension of ‘is the sum of two primes’. For, for any scenario, if we had a complete canonical description of the language of arithmetic with all its definitions, it is highly plausible that we would be able to determine that the mathematical theory conventionally employed in the scenario is not the theory conventionally employed in the actual world. And this is so, even if the description contains only semantically neutral words. Suppose, for instance, that we are given the following fragments as part of a canonical description of a scenario in which mathematicians claim to have proven that Goldbach conjecture is false:

n satisfies ‘is prime’ $=_{\text{def}}$ if n satisfies ‘is smaller than 1×10^{300} ’, then n satisfies ‘has exactly two factors, one being 1 and the other being n ’. If not, then n satisfies ‘is square’.

n satisfies ‘is square’ iff n satisfies ‘is an x such that there is some m such that $x = m^m$ ’,

n satisfies ‘is even’ iff n satisfies ‘is divisible by 2’.

Given these descriptions, we are clearly not tempted to say that the mathematicians in the scenario proved that there is some large even number n which cannot be written as the sum of two

primes. They didn't prove *that*. They proved that there is some large even number n that cannot be written as the sum of two primes*.

The solution to the problem is to substitute partial descriptions of scenarios for complete ones. Given that our grasp of the concepts of arithmetic is less than perfect, partial descriptions best capture how we use the language of arithmetic. If we had available a *full* description of our earlier scenario, it would not be compatible with our use of the language of arithmetic that Goldbach's conjecture is false. For while we do not grasp arithmetical concepts perfectly, we are in a position to know whether or not a given language employed in the given scenario concerns our mathematical theory.¹⁸ If, on the other hand, we have available only a partial description, then we may not be in a position to determine whether or not the language spoken in the scenario concerns the mathematical theory conventionally employed in the actual world. Since in reality we are not in a position to tell, for any even number n , whether n is the sum of two primes, our grasp of mathematical concepts is best represented as an ability to make judgments concerning the concepts' application in partially described scenarios. But there are centered metaphysically possible worlds in which 'even number greater than two' and 'can be written as the sum of two primes' are such that the epistemic extension of the former is a subset of that of the latter, and so there are centered metaphysically possible worlds relative to which Goldbach's conjecture has as its epistemic extension the truth-value: false.

The two-dimensional account of the strict epistemic possibility of false arithmetical statements can be extended to give a more general account of the strict epistemic possibility of false necessary statements. To illustrate, suppose God exists necessarily, and suppose that 'God' was introduced into the language as an abbreviation for 'the omniscient being none of whose properties are contrary'. It is then necessary and a priori *knowable* that God is omniscient. But for all I know, God might exist without being omniscient. How do we account for the strict epistemic contingency of 'God exists but is non-omniscient'? If God exists and is necessarily

¹⁸ Some of us will, of course, lack this ability.

omniscient, then there is no metaphysically possible (centered) world in which ‘God exists and is non-omniscient’ has as its epistemic referent the truth-value: true. Moreover, there is no metaphysically impossible (centered) world in which ‘God exists and is non-omniscient’ has as its epistemic referent the truth-value: true. For if God exists, then he is omniscient. So, if God exists and is non-omniscient, then God exists and is omniscient and non-omniscient. But, ex hypothesi, God (if he exists) does not instantiate contrary properties.

Since God (if he exists) does not instantiate contrary properties, a scenario in which the epistemic referent of ‘God’ is non-omniscient must be a centered metaphysically possible world. Yet ex hypothesi, there is no metaphysically possible world where God exists and is non-omniscient. Of course, if we use ‘God’ deferentially, then there are metaphysically possible worlds in which ‘God’ is used to refer to a non-omniscient being by the language-users in those worlds. However, ‘God’ need not be used deferentially.¹⁹

Epistemic two-dimensionalism has the resources to deal with this problem. Suppose for reductio that canonical descriptions of hypothetical scenarios are complete. Every canonical description will then a priori imply that God exists and is omniscient.²⁰ But then the strict epistemic intension of ‘God exists and is omniscient’ is necessary. However, as I don’t know that God exists and is omniscient, the strict epistemic intension of ‘God exists and is omniscient’ is contingent. By reductio it follows that at least some canonical descriptions of hypothetical scenarios are incomplete. For example, they might simply be descriptions of physical facts of reality. Some such partial descriptions may a priori imply that God exists and is omniscient. For instance, a description of a scenario in which God has taken the shape of a material being may a priori imply that God exists and is omniscient. But other partial descriptions do not a priori

¹⁹ Note that it may be strictly epistemically possible for me that God exists and is non-omniscient, even if the epistemic intension of my use of the term ‘God’ is that of the description ‘the omniscient being all of whose properties can be co-instantiated’. For I may not have epistemic access to my own use of the term.

²⁰ To say that the description a priori implies that God exists and is omniscient is to say that a speaker with excellent logic skills and epistemic access to her own use of the word ‘God’ would be able to infer (on a priori grounds) that God exists and is omniscient from the description in question.

imply that God exists and is omniscient, as it does not follow on a priori grounds from purely physical facts that some immaterial being exists and is omniscient. The strict epistemic intension of ‘God exist and is omniscient’ will thus be contingent.

In short, it seems that we can offer a two-dimensional account of the strict epistemic possibility of false necessary statements if we take strict epistemic possibility to be analyzable in terms of partial canonical descriptions of hypothetical scenarios.

VI. Conclusion

Epistemic two-dimensionalism challenges the assumption made by standard semantics that there is just a single dimension to (non-lexical) meaning. According to epistemic two-dimensionalists, all expressions are associated with a subjunctive and an epistemic intension. Whereas the subjunctive intension of an expression is grounded in the essence of the actual referent of the expression, the epistemic intension of an expression is grounded in the speaker’s abilities to pick out the extension of the expression in canonical descriptions of hypothetical scenarios. Epistemic two-dimensionalism is motivated by its ability to yield realistic accounts of such phenomena as cognitive significance, belief ascriptions, and deep and strict epistemic possibility.

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