

Knowability and a Modal Closure Principle

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Does a factive conception of knowability figure in ordinary use? There is some reason to think so. ‘Knowable’ and related terms such as ‘discoverable’, ‘observable’, and ‘verifiable’ all seem to operate factively in ordinary discourse. Consider the following example, a dialog between colleagues A and B:

A: We could be discovered.

B: Discovered doing what?

A: Someone might discover that we're having an affair.

B: But we are not having an affair!

A: I didn't say that we were.

A's remarks sound contradictory. In this context the factivity of ‘someone might discover that’ explains this fact. So there is some reason to believe that knowability and related modalities are factive in ordinary use. For factive treatments of knowability in the context of epistemic theories of truth, compare Tennant (2000: 829) and Wright (2001: 59-60, n. 17).

The factivity of knowability will not be questioned herein, since the paper is concerned to examine some problems that arise in connection with factive knowability. In particular, the paper examines a clutch of issues concerning principles of modal epistemic logic and the knowability of truth. It begins with a puzzle, a closure paradox of knowability, that threatens to show that a factive interpretation of knowability entails the invalidity of a modest modal closure principle. The negative argument is that the puzzle in its original form does not tell against the joint validity of closure and factivity. That is because the puzzle rests on contingent assumptions whose compossibility is doubtful. The positive argument is that there is a formulation of the puzzle that does prescribe

revision of normal modal principles. The result may be taken as a new and improved paradox of knowability and as data for the future analysis of factive concepts of *possible* knowledge.

The puzzle was introduced by Sven Rosenkranz (2004) and threatens to show that a factive interpretation of knowability entails the invalidity of a familiar closure principle.

The closure principle says that possibility is closed over necessary implication:

$\Diamond\varphi, \Box(\varphi \rightarrow \psi) \vdash \Diamond\psi$. The discussion focuses on instances of the principle where our propositional variables take epistemic propositions of the form ' $K\varphi$ ', to be read 'somebody at some time knows that φ '. Accordingly the focus is on instances of the closure principle (CL) that have the following form:

$$(CL) \quad \Diamond K\varphi, \Box(K\varphi \rightarrow K\psi) \vdash \Diamond K\psi.$$

The factivity of knowability (FA) may be formulated as follows:

$$(FA) \quad \Box(\Diamond K\varphi \rightarrow \varphi).$$

(FA) says that, necessarily, knowability implies truth. Rosenkranz's puzzle promises to demonstrate the incompatibility of these two principles, (CL) and (FA).

If a factive conception of knowability is inconsistent with the minimal modal logic that is avowed by the closure principle, we will have learned something very interesting about ordinary and semantic anti-realist conceptions of knowability (and related conceptions of discoverability, verifiability, observability, etc.). We will have learned that the notion of possibility that figures in the concept cannot be understood with our most familiar modal machinery. After all, that machinery employs a notion of possibility that is closed over necessary implication. Combining our best account of knowledge with a normal account of possibility will not explain the logical behavior of

'knowable'. We will likely have to begin, instead, with a semantically indecomposable treatment of the concept.

The thesis that (CL) and (FA) cannot both be valid is connected in important ways to the feasibility of semantic anti-realism (AR)—i.e., the view that truth and knowability necessarily coincide:

$$(AR) \quad \Box(\varphi \leftrightarrow \Diamond K\varphi)$$

The right-to-left formulation of (AR) is to be understood as a consequence of endorsing (FA). Thus the anti-realist might be asked to reject (CL).

A second consequence of the puzzle emerges in connection with a semantic paradox developed by Berit Brogaard and Joe Salerno (2002). There it is argued that to avert a Fitch-like paradox of knowability, i.e., the KK-paradox, an anti-realist who embraces (FA) and (CL) must reject the KK-principle—viz.,

$$(KK) \quad K\varphi \vdash KK\varphi.$$

One might contend, of course, that if (CL) and (FA) cannot both be valid, then (KK) is not responsible for the paradox.

A negative thesis and a positive thesis will be defended here. The negative thesis is that, as formulated, the initial puzzle does not show what it was intended to show. It does not provide us with a good reason to reject the joint validity of closure and factivity, and that is because the argument rests upon contingent assumptions, the compossibility of which is doubtful. Consequently, it is all but clear that the semantic anti-realist should reject closure and all but clear that the KK-principle is not responsible for the KK-paradox. The positive thesis is that there is a version of the closure paradox that is much more effective. That puzzle is developed here and its significance is evaluated.

The Knowability Paradoxes

What has come to be called the *knowability paradox* was first published by Frederic Fitch as Theorem 5 (1963: 139). It is equivalent to the claim that if each truth is knowable (by somebody at some time), then each truth will be known (by somebody at some time):

$$(T5) \quad \forall\varphi(\varphi \rightarrow \Diamond K\varphi) \vdash \forall\varphi(\varphi \rightarrow K\varphi)$$

Let us call the premise the *knowability principle*, or (KP), and the conclusion the *omniscience principle*, or (OP).¹ Here is a way of formulating Fitch's proof of (T5).

Suppose (KP) is true.

Then the following instance of it is true: $(p \ \& \ \sim Kp) \rightarrow \Diamond K(p \ \& \ \sim Kp)$.

But the consequent is false: it is not possible to know both p and $\sim Kp$.

That is because the supposition that both p and $\sim Kp$ are known is provably inconsistent.²

Thus one may conclude $\sim(p \ \& \ \sim Kp)$.

This (classically) implies $p \rightarrow Kp$.

Since p occurs in none of our undischarged assumptions, we may generalize to get (OP),

$\forall\varphi(\varphi \rightarrow K\varphi)$. QED.

The anti-realist is of course committed to the knowability principle, and so, must contend with the knowability paradox. Otherwise, she is absurdly committed to the principle that every truth will be known. If we restrict the class of knowable truths, as either Neil Tennant (1997) or Michael Dummett (2001) does, then Fitch's result is blocked. Dummett restricts the knowability principle to what he calls "basic" propositions. Tennant restricts it to so-called "Cartesian propositions". A Cartesian proposition is one that can be consistently known. If properly motivated, either restriction blocks Fitch's

proof, because Fitch's substitution of the conjunction, $p \ \& \ \sim Kp$, for φ in $\forall\varphi(\varphi \rightarrow \Diamond K\varphi)$ is neither basic nor Cartesian.

Brogaard and Salerno (2002: 146), to derive the same conclusion as Fitch, employ different modal and epistemic resources. Importantly, use is made of (CL), (FA) and (KK), whereas Fitch applies none of these. Unlike Fitch's paradox, the result is not blocked by the restrictions proposed by Tennant and Dummett.

This KK-paradox makes use of the principle (KP) in either of its restricted forms, (KK), (CL) and (FA). It proceeds this way.

Assume p , where p is basic and Cartesian.

Then, by either of the two aforementioned restricted knowability principles, $\Diamond Kp$.

Applications of (KK) and of Necessitation yield $\Box(Kp \rightarrow KKp)$.

So, by (CL) one may infer $\Diamond KKp$.

Therefore, by (FA), Kp .

Hence, by Conditional Proof, $p \rightarrow Kp$.

Now we may generalize:

for any basic Cartesian proposition φ , if φ is true then φ is known; i.e.,

$\forall\varphi(\varphi \rightarrow K\varphi)$.

It is concluded that the anti-realist who embraces a restriction strategy and a factive conception of knowability has two options.

- (i) Pending further analysis of the modal operator \Diamond , hold on to (FA), which has some plausibility; and reject (KK), the most controversial of the modal epistemic resources.

- (ii) Hold on to (KK) and (FA) and reject the minimal normal logic that commits us to (CL).

It is here assumed that (FA) is secure. Thus (i) and (ii) are the only ways to avoid a paradox of knowability. The restriction strategies in their current forms do nothing to safeguard against this KK-result.

The Pharaonic Paradoxes

Two ‘pharaoh arguments’ can be advanced to show that (FA) and (CL) cannot both be valid. The first (Rosenkranz 2004: 70) makes use of the principles (CL) and (FA). Let p be ‘the pharaoh is in his tomb’ and q be ‘the door of the tomb is open (at some time)’.

Suppose that the world would allow us to recognize that the pharaoh is in the tomb:

1. $\Diamond Kp$

Moreover, suppose that the door of the tomb is never open:

2. $\sim q$

These are intuitively plausible assumptions.³ The argument requires the further assumption that “necessarily, coming to know that the pharaoh is in the tomb [implies coming] to know that the door of the tomb has been opened” (Rosenkranz 2004: 70):

3. $\Box(Kp \rightarrow Kq)$

By (CL) it follows from 1 and 3 that $\Diamond Kq$.

But then, by (FA), it follows that q .

This contradicts assumption 2.

It might now be concluded that we are to reject the validity of (CL), given a factive reading of $\Diamond K$.⁴

As it stands, the argument is unsound because Premise 3 is false. The mistake is to think that there is a necessary, rather than a material, connection between knowing that the pharaoh is in his tomb and knowing that the door of the tomb is open. If this is correct, then the foregoing reductio, which rests on the necessity of the aforementioned conditional, does nothing to raise doubts about the plausibility of (CL). If the point is incorrect, and the case for Premise 3 can be made convincing, then there is still the option of rejecting the initial assumption that $\Diamond Kp$ and $\sim q$ are compossible. This option amounts to rejecting the commitment to both the factive knowability of the pharaoh's being in the tomb and the door's never being open.

Indeed, if truly the only possible way to know that the pharaoh is in the tomb is to open the door, then we would not want to accept that the actual world permits recognizing the pharaoh's entombment when the door is actually never open. Admittedly, whether this kind of reply ultimately succeeds depends on a richer explanation of factive knowability---an explanation which is yet to be developed. The point here is that it seems premature to invite the anti-realist to give up on the minimal, normal modal logic avowed by (CL), when she has the option of denying the compossibility of the starting assumptions.

The second Pharaoh argument is more interesting, since it does not raise questions about the necessity of the conditional, $Kp \rightarrow Kq$. As before, let p be 'the pharaoh is in his tomb' and q be 'the door of the tomb is open (at some time)'. Note that this proof too uses the principles (CL) and (FA).

Suppose that, as a matter of fact, the pharaoh is in his tomb, and that if one comes to know that the pharaoh is in his tomb then one comes to know that the door is open:

$$p \ \& \ (Kp \rightarrow Kq).$$

Suppose further that this conjunction is factively knowable:

$$(i) \quad \Diamond K(p \ \& \ (Kp \rightarrow Kq)).$$

Note that it is logically necessary that if one knows the conjunction $p \ \& \ (Kp \rightarrow Kq)$, then one knows q :

$$(ii) \quad \Box(K(p \ \& \ (Kp \rightarrow Kq)) \rightarrow Kq)$$

By the closure principle (CL), premises i. and ii. entail $\Diamond Kq$. By the factivity principle (FA), it follows that q —the door of the tomb is open.

But “the door may well not have been opened” (Rosenkranz 2004: 71). This is meant to present us with an inconsistency.

Apart from the usual logical rules governing the truth-functional connectives, the proof of line ii. requires only the following principles, each of which is unimpeachable: first, the Rule of Necessitation (i.e., that theorems are necessary):

$$(Nec) \quad \text{If } \vdash \varphi, \text{ then } \vdash \Box \varphi ;$$

secondly, the principle that *knowledge* entails *truth*:

$$(KT) \quad K\varphi \vdash \varphi ;$$

and thirdly, the principle that knowledge distributes over a conjunction:

$$(K \text{ Dis}) \quad K(\varphi \ \& \ \psi) \vdash K\varphi \ \& \ K\psi.$$

The proof of line ii. is straightforward and left to the reader. The inconsistency that results from the above argument arises, because we have made use of the less secure principles (CL) and (FA).

If there is an inconsistency, then it must be that we are assuming that the door is never open. Thus, a reconstruction of the philosophical moral of the second argument is this:

If $\sim q$ and $\Diamond K(p \ \& \ (Kp \rightarrow Kq))$ are compossible, then either (FA) or (CL) is invalid.

One might interpret this result as showing, first, that “(FA) and (CL) cannot both be valid” and, second, that “all [the relevant anti-realist has] to do is reject (CL)” (Rosenkranz 2004: 71).

These interpretations of the result are suspect. It is not apparent that (FA) and (CL) cannot both be valid, but rather that they cannot both be valid *if* the claims

$\sim q$, and $\Diamond K(p \ \& \ (Kp \rightarrow Kq))$

are compossible. Consequently, in the face of the result, an anti-realist (or anyone else) who embraces (FA) will not inevitably be moved to give up (CL). She has the option of denying the compossibility of the initial assumptions. She can deny the knowability of the conjunctive proposition (i.e., that the pharaoh is in the tomb and knowing it implies knowing that the door is open) when the door of the tomb is never open. The truth of this factive knowability assumption is contingent, after all, on whether the door is ever open.

The discontented reader may suggest that, intuitively, the two initial assumptions are jointly satisfiable. But this is not obvious. Whether or not they are jointly satisfiable can be determined only after we have better understood the operative concept of knowability and its constituent modality. *Knowability* in the context of anti-realist discussion is a term of art, and our ordinary use is under-explored. It will not suffice simply to push our pre-theoretic intuitions about what can or cannot be known.

One exciting feature of Rosenkranz's puzzle is that it does not presuppose semantic anti-realism or (KP). That is because a generalized knowability principle is not required, but only the commitment to the knowability of some choice propositions. As

such the puzzle is meant to draw conclusions about some *pre-theoretic*, factive conceptions of knowability. But with calls to reject modest modal principles there comes a need to develop our semantic resources. What is the operant, pretheoretic conception of knowability? Without some development on this score, a revision of our minimal normal modal logic is not convincingly supported, especially not if there is a more conservative way out. In the case at hand, the conservative way out is to reject the compossibility of the contingent assumptions driving the puzzle.

The next section, however, develops a version of the puzzle that does not allow for this defense.

New Closure Paradoxes of Knowability

The puzzles examined above emerge in the context of a debate about the adequacy of Tennant's proposal for resolving Fitch's knowability paradox. As discussed, Fitch's proof does not work against Tennant's restricted knowability principle. However, the KK-knowability paradox was devised to show that Tennant's Cartesian restriction (at least, in its original form) does not guard the anti-realist against paradox. After all, it is a knowability paradox that respects the Cartesian restriction. Of course, unlike Fitch's reasoning, the KK-paradox utilizes the KK-principle (KK), the closure principle (CL), and a factive conception of knowability (even when this is expressed in restricted form—viz., $\Diamond K\varphi \rightarrow \varphi$, where φ is Cartesian). Among Rosenkranz's conclusions was that (KK) is not the source of the problem because there a version of the paradox that utilizes all of these resources minus (KK). But this remains to be demonstrated. In the context of

Tennant's knowability principle, we now argue, there is a closure paradox that does not rest on (KK).

If one captures the factivity of Tennantian knowability by restricting the thesis

$$\Diamond K\varphi \rightarrow \varphi$$

to Cartesian propositions, then one best captures Tennant's theory of truth with the following *Cartesian knowability principle* (CKP):

$$(CKP) \varphi \leftrightarrow \Diamond K\varphi, \text{ where } \varphi \text{ can be consistently known.}$$

The other important principles are (CL) and the following theorem:

$$(T) \Box(K(\varphi \ \& \ (K\varphi \rightarrow \psi)) \rightarrow \psi)$$

(T) is the general formulation of a principle employed in the second Pharaoh argument above and in a knowability paradox developed by Williamson (2000: 111). Indeed the following formulation of the knowability paradox borrows from both this and the pharaoh puzzles. In addition to the usual principles governing the connectives, the argument makes use of only the principle (CKP) of Cartesian knowability, the principle (T) just stated, and the closure principle (CL).

Let p and q be atomic, contingent formulas. Then each of the following four propositions is Cartesian:

$$q, \sim q, p \ \& \ (Kp \rightarrow Kq), \text{ and } p \ \& \ (Kp \rightarrow K\sim q).^5$$

That is, none of

$$Kq, K\sim q, K(p \ \& \ (Kp \rightarrow Kq)), \text{ or } K(p \ \& \ (Kp \rightarrow K\sim q))$$

is provably inconsistent.

Now suppose (CKP):

$$1. \varphi \leftrightarrow \Diamond K\varphi, \text{ where } \varphi \text{ is Cartesian.}$$

And, for reductio, suppose

$$2. p \ \& \ \sim Kp.$$

for some contingent, atomic formula p . Then, since conditionals with false antecedents are true, 2 gives us

$$3. p \ \& \ (Kp \rightarrow Kq).$$

Since $p \ \& \ (Kp \rightarrow Kq)$ is Cartesian, 1 gives us

$$4. (p \ \& \ (Kp \rightarrow Kq)) \rightarrow \Diamond(K(p \ \& \ (Kp \rightarrow Kq))).$$

Trivially, 3 and 4 yield

$$5. \Diamond(K(p \ \& \ (Kp \rightarrow Kq))).$$

An instance of (T) is

$$6. \Box(K(p \ \& \ (Kp \rightarrow Kq)) \rightarrow Kq).$$

So, by (CL), 5 and 6 entail

$$7. \Diamond Kq.$$

Since q is Cartesian, it follows from 1 that

$$8. q \leftrightarrow \Diamond Kq.$$

7 and 8 entail

$$9. q.$$

In the proof thus far, the logical demands on q are only that it be Cartesian. So it is permissible to repeat lines 3 through 9, replacing all occurrences of q with the Cartesian formula $\sim q$, ultimately deriving

$$10. \sim q.$$

Lines 9 and 10 contradict, and this forces us to discharge and negate the assumption at line 2, giving $\sim(p \ \& \ \sim Kp)$. This (classically) entails $p \rightarrow Kp$, which may be generalized to $\forall\varphi(\varphi \rightarrow K\varphi)$. QED.

What has been shown is that if all and only Cartesian truths are knowable, then all contingent, atomic truths are known. A knowability paradox is up and running again.

Since the Cartesian restriction is not violated, it should be clear that it does little to protect anti-realism against paradoxes of knowability. The new paradox is more powerful than the Pharaonic ones, in that it rests on no contingent assumptions. It also has an advantage over the KK-paradox in that it does not require use of the controversial KK-principle. It relies, as already noted, only on (CKP), (T) and (CL). Finally, it has an advantage in strength over Fitch's result in that the required substitutions into the knowability principle are all Cartesian. So to the extent that the Cartesian restriction is thought to be *the* key to solving Fitch's paradox, the new closure paradox is an improved paradox of knowability.

An even more unsettling result may be inspired by Williamson (1992: 68). He shows that an anti-realist embracing (CL) and a factive conception of knowability is committed to $\Diamond\varphi \dashv\vdash \varphi$, a collapse of the logical distinction between possible and actual truth. However, Williamson's formulation of the result violates Tennant's restriction on knowable truth, and that is because his result depends on Fitch's proof, which violates the restriction. Our above closure-result, however, can be converted into a proof of

$$\Diamond\varphi \dashv\vdash \varphi$$

without the violation. The proof uses only (CKP), (CL) and presupposes that necessity entails truth. The proof goes like this.

Tennant's (CKP) is meant to be a theory of truth, and as such is a necessary thesis:

1. $\Box(\varphi \leftrightarrow \Diamond K\varphi)$, where φ is Cartesian.

Moreover, the above closure-argument, recall, shows that " $\varphi \leftrightarrow \Diamond K\varphi$, where φ is Cartesian" entails " $\varphi \rightarrow K\varphi$, where φ is contingent and atomic". Since the premise of this entailment is necessary, we may infer the necessity of the conclusion:

2. $\Box(\varphi \rightarrow K\varphi)$, where φ is contingent and atomic.

Suppose p is a contingent, atomic proposition. It follows that

3. $\Diamond p$.

Since p is contingent and atomic, one may take the following instance of 2:

4. $\Box(p \rightarrow Kp)$.

By (CL), 3 and 4 entail

5. $\Diamond Kp$.

Since p is contingent and atomic, it is Cartesian. Hence, we may take the following instance of 1:

6. $p \leftrightarrow \Diamond Kp$.

5 and 6 entail

7. p .

Our proposition p was an arbitrarily chosen contingent atom.

So what has been proven is this: for all contingent atoms φ , if φ is possible then φ .

That is, (CKP) entails $\Diamond\varphi \vdash \varphi$, for all contingent atoms φ .

The right-to-left direction, $\varphi \vdash \Diamond\varphi$, is trivial if \Diamond is read in accordance with a reflexive accessibility relation.

So, $\Diamond\varphi \dashv\vdash \varphi$, where φ is any contingent atom.

Thus, if the Cartesian knowability principle is true, then---with respect to the class of contingent atoms---there is no significant distinction between possibility and actuality. And so, the Cartesian knowability principle faces new and improved paradoxes of knowability.

It may be objected that the semantic anti-realist tends to be either an intuitionist or a paraconsistentist and that our results, being classical, do not threaten, and so, do not constitute improvements to the original paradox.⁶ There is a sense in which this is right.

The intuitionistic anti-realist who does not favor a Cartesian restriction is threatened by the new paradoxes only to the extent that she is threatened by Fitch's original paradox. That is because the untoward intuitionistic consequences of the new puzzle are intuitionistic consequences of the old puzzle as well. Moreover, those anti-realists favoring paraconsistent revision may not be threatened at all.⁷ In these respects the new paradoxes are no better and no worse than Fitch's original formulation of the paradox.

However, this is not the sense in which we take the new closure paradoxes to be "improved". Reconsider the dialectic that frames our discussion. Fitch's paradox employs exclusively classical principles, although the heart of the result---showing $\sim(p \ \& \ \sim Kp)$ ---is intuitionistically, although perhaps not paraconsistently, valid. With developments of semantic anti-realism that defend intuitionistic logic, the full-blown paradox is avoided. But, many argue that the intuitionistically valid consequences, including $\sim(p \ \& \ \sim Kp)$, are almost as troubling.⁸ Tennant's Cartesian restriction is then implemented to block these troubling intuitionistic consequences. The anti-realist would not need a Cartesian restriction, if she thought that intuitionistic consequences of Fitch's paradox were harmless. So we take the Cartesian restriction to play the role of blocking some intuitionistically troublesome consequences. The new closure paradoxes are developed here to demonstrate the futility of this approach. Among other things, they provide us with an intuitionistically acceptable proof of the troubling conclusion, $\sim(p \ \& \ \sim Kp)$, and they do this without violating the Cartesian restriction. So even though the ultimate conclusion of the above argument, $p \rightarrow Kp$, is derived using exclusively classical principles, the intuitionistically acceptable chunk of the reasoning still marks trouble for the intuitionist disposed to employ the Cartesian strategy. So with respect to

the intuitionistic revisionist who clutches the Cartesian restriction strategy, the new closure paradoxes do constitute an improvement on Fitch's paradox. Like Rosenkranz's original puzzle, the results here are also meant to be a problem for *realists* who happen to embrace a factive conception of knowability. If they take the relevant propositions---e.g., $p \ \& \ (Kp \rightarrow Kq)$ ---to be knowable (for whatever reason), then they have a knowability paradox with which to contend.

A Note on Dummett's Restriction Strategy

The new closure paradoxes are aimed at the Cartesian strategy. They do not violate Dummett's restriction on the class of knowable truths for the obvious reason that they apply the knowability principle to non-basic sentences. For this reason we continue to take the KK-paradox to be the important objection to Dummett's theory. After all, the KK-paradox does not violate Dummett's restriction.

One objection to this line is that we apply (FA) to non-basic sentences and that Dummett may wish to restrict (FA) as well as (KP) to basic sentences.⁹ This would block our substitution of 'Kp' for the propositional variable in the knowability principle.

Dummett's inductive account of truth, however, says nothing about statements of this form. If they are basic, then the KK-proof goes through. If they are not basic, then the KK-proof fails but a semantic clause for K is owed and we shall leave that task to Dummett's anti-realist.

A second worry about the effectiveness of the KK-paradox is that (KK) is highly contentious. Therefore, the paradox is only as effective as one's willingness to accept the principle. We do not wish to defend (KK) or the basicity of 'Kp'. If Dummett's anti-

realist is not committed to them, then we would direct the reader to other versions of the paradox that presuppose neither (KK) nor the basicity of 'Kp'. See for instance Brogaard and Salerno's S4-paradox (2002: 144-145), which turns on a transitive conception of possibility. Whether or not Dummett is amenable to this conception of possibility is another question underdetermined by Dummett's theory of truth. We do not wish to resolve the issue on Dummett's behalf. Our point is that whether or not Dummett's conception of truth treats 'Kp' as basic, there are versions of the knowability paradox that Dummett's strategy does nothing to inoculate against.

Conclusion

The paper examined a clutch of issues concerning principles of epistemic logic and the knowability of truth. It began with a puzzle that threatens to show that a factive interpretation of knowability entails the invalidity of a familiar modal closure principle. It was argued that the puzzle in its original form does not effectively demonstrate this. That is because the argument rests upon contingent assumptions whose compossibility is questionable. It was argued however that there is a versions of the puzzle that do cause troubles for the anti-realist who is inclined to restrict the class of knowable propositions along Tennantian lines. The result may be taken as a new and improved paradox of knowability and as data for the future analysis of factive concepts of possible knowledge.

How are we to conclude? Is it true that a factive conception of knowability enjoins a reject of the modal the closure principle? The answer is yes, but whether the rejection is ad hoc or independently motivated by semantics is yet to be investigated. We need to know how to read the anti-realist's modal terminology and why one should read it that way. Hopefully that will put us in a position to determine the validity of the

resources at work in the paradoxes, and in particular the validity of the closure principle, which is at the heart of the paradoxes discussed in this paper. Our conclusion is simply this: no leading restriction strategy works to protect the knowability principle against the paradoxes of knowability, and this is brought out by versions of Fitch's result that utilize the modal closure principle.¹⁰ Sixty years after its initial formulation, the knowability paradox continues to resist adequate resolution.¹¹

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¹ ‘Omniscience’ is somewhat of an unfortunate label, since the principle does not say that there is a time at which every truth will be known. Rather it says that, for every truth, there is a time at which it is known. We retain the label for convenience, but thank Michael Hand for urging the clarification.

² If $p \ \& \ \sim Kp$ is known then it is true, and so, $\sim Kp$. Also, if $p \ \& \ \sim Kp$ is known then the left conjunct is known. Hence, Kp .

³ We diverge from Rosenkranz by building in reference to times. In so doing the argument is strengthened and a number of red herrings are circumvented. Thanks are owed to Bryan Frances (personal correspondence) and José-Luis Bermúdez (personal correspondence) for independently offering the strengthened case of the pharaoh.

⁴ Rosenkranz credits the case of the pharaoh to Duncan Pritchard.

⁵ The contingency of the formulas is stipulated to bypass a debate about whether necessarily false mathematical propositions, like ‘3 is even’, are Cartesian. Williamson (2000: 110) argues that they are. Tennant (2001: 271-272) disagrees.

⁶ We thank a referee for raising this concern.

⁷ See for instance, Beall (2000) and Wansing (2002).

⁸ Work in this direction is inspired most notably by Percival (1990).

⁹ We thank a referee for raising this and other concerns discussed in this section.

¹⁰ There is a different sort of restriction strategy that holds some promise. The approach recognizes the context sensitivity of the quantifiers implicit in ‘knows that’. Brogaard and Salerno (*forthcoming*) offer a version of this insight.

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