Learning Curves (T.P.Wright, 1936)

Learning curves were first described by T.P. Wright in 1936, studying time required to make airplane parts. Here is a simplified version of data he might have collected. He observed that, as the workers gained more experience, less time was required to manufacture these parts. This effect was not linear, but seemed to have a constant decrease with every doubling of experience.
The Learning percent was described as a ratio of the time to do it the (2*n)th time divided by the time to do it the nth time. What sometimes confuses people is that this results in a **higher** "learning percent" when the rate of learning is **slower**. So a person with a 70% curve has a 30% reduction in effort with every doubling of experience, but an 80% curve corresponds to only a 20% reduction.

Time required to make airplane parts

N repetitions	Time / Unit	Ratio T_n/T_1	Ratio $T_n/T_{n/2}$
1	10		
2	8	0.8	0.8
3	7	0.7	
4	6.4	0.64	0.8
5	6	0.6	
6	5.6	0.56	0.8
7	5.3	0.53	
8	5.1	0.51	0.8

Being an engineer, Wright plotted the data, and when the plot wasn't linear, took the log of both sides to result in the following as linear relationships:

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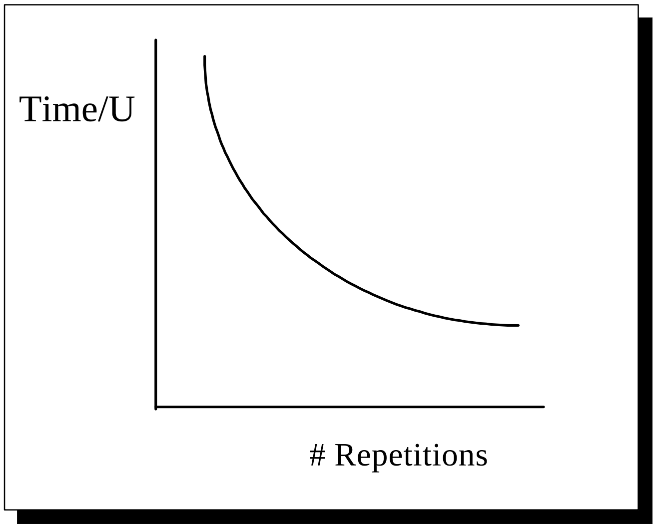
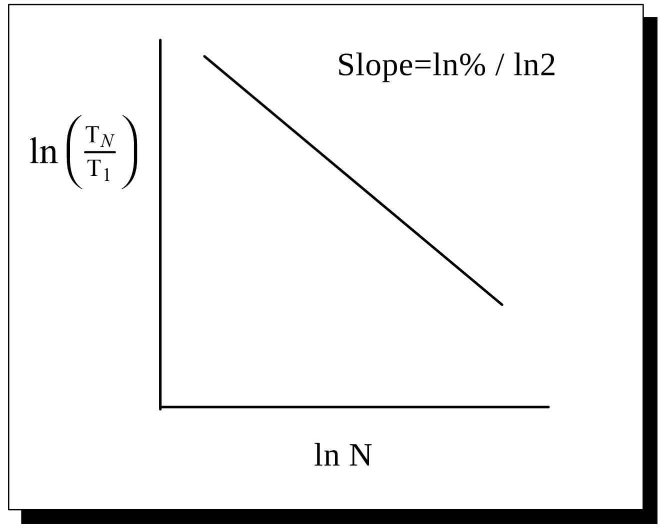




Antilog

$$\ln\left(\frac{T_N}{T_1}\right) = \ln N * \left(\frac{\ln \%}{\ln 2}\right) \quad \text{Antilog}$$

$$\frac{T_N}{T_1} = N^{\left(\frac{\ln \%}{\ln 2}\right)}$$

$$T_N = T_1 * N^{\left(\frac{\ln \%}{\ln 2}\right)}$$

Time to do it the nth time is a function of time to do it the 1st time and the learning %, The natural log of 2 gets into it because the slope involves doublings of experience.

$$\ln \% = \frac{\ln\left(\frac{T_N}{T_1}\right) * \ln 2}{\ln N}$$

$$\% = e^{\ln 2 * \left(\frac{\ln\left(\frac{T_N}{T_1}\right)}{\ln N}\right)}$$

This relationship allows statistical estimation of learning % by regression or curve fitting.

RJB, October 16, 1999

For our purposes, it's easier to use a table.

	70%		75%		80%		85%		90%	
Unit No.	Unit Time	Total Time	Unit Time	Total Time	Unit Time	Total Time	Unit Time	Total Time	Unit Time	Total Time
1	1	1	1	1	1	1	1	1	1	1
2	0.7	1.7	0.75	1.75	0.8	1.8	0.85	1.85	0.9	1.9
3	0.57	2.27	0.63	2.38	0.7	2.5	0.77	2.62	0.85	2.75
4	0.49	2.76	0.56	2.95	0.64	3.14	0.72	3.35	0.81	3.56
5	0.44	3.2	0.51	3.46	0.6	3.74	0.69	4.03	0.78	4.34
6	0.4	3.59	0.48	3.93	0.56	4.3	0.66	4.69	0.76	5.1
7	0.37	3.96	0.45	4.38	0.53	4.83	0.63	5.32	0.74	5.85
8	0.34	4.3	0.42	4.8	0.51	5.35	0.61	5.94	0.73	6.57
9	0.32	4.63	0.4	5.2	0.49	5.84	0.6	6.53	0.72	7.29
10	0.31	4.93	0.39	5.59	0.48	6.32	0.58	7.12	0.71	7.99

This part of the table, from n=1 to n=10 is actually five tables, for different learning percents. Unit time is the ratio of time to do it the nth time to time to do it the first time.

Forecast unit time for any future unit by using the appropriate learning % table and N. Multiply T1 * the Unit Time Factor.
The total time is the cumulative factor, which, when multiplied times T1, gives the total time required to make N units.

Estimate labor cost of 20 US Navy fighter Jets. The prototype took 400 hours to build. Typical Learning Curve is 80%

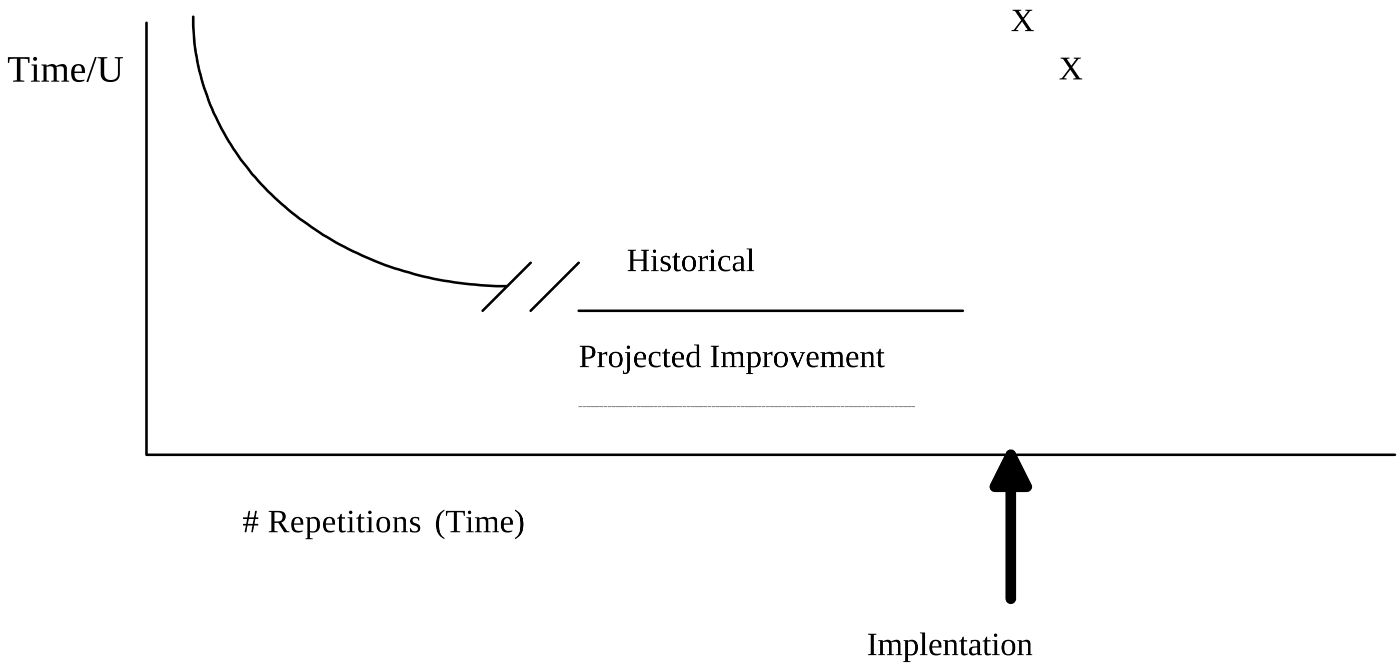
Jet #	Factor	Tn	total time
1	1	400	400
2	0.8	320	720
3	0.7	280	1,000
4	0.64	256	1,256
•			
•			
•			
20			

There is a hard way and an easy way to do this. One way is to fill in the table and add it up. The easy way is to find the cumulative factor on the table and multiply that times the T1:

400 * 10.49 = 4,196 hours.

A company that estimates 400 * 20 =8000 hours is unlikely to bid successfully. On the other hand, you don't want to underestimate, or you will win many bids that you can't fulfill without going bankrupt.

Improvements to an Established Process



There are a few important implications that people tend to neglect.
First, there is a significant "first-attempt" effect. The technical term is "screwed up." Thus, our simple expedient of relating everything to T1 is unworkable in practice. A better way is to arrive at forecasts through regression or curve-fitting models that make use of all the data.
The second consideration is that processes that have been on-stream for a while tend to get worse rather than better because the energy required to improve them gives diminished returns, and boredom leads to inattention. The second law of thermodynamics implies that energy is required just to overcome the pressure of entropy and maintain an orderly process. ("Left to themselves, things go to hell.")
The third thing is that process changes ("improvements") initially require learning, and the first few attempts are likely to be unimpressive. People never seem to expect this. "The only thing we learn from history is that nobody ever learns anything from history."
If people on the third shift don't like a process, it won't work. However, a process that is convenient will be improved beyond your expectations. People on the third shift are there because they don't want to be encumbered by management, and they will find the easiest way to accomplish the job.

	70%		75%		80%		85%		90%	
Unit	Unit	Total	Unit	Total	Unit	Total	Unit	Total	Unit	Total
No.	Time	Time	Time	Time	Time	Time	Time	Time	Time	Time
1	1	1	1	1	1	1	1	1	1	1
2	0.7	1.7	0.75	1.75	0.8	1.8	0.85	1.85	0.9	1.9
3	0.57	2.27	0.63	2.38	0.7	2.5	0.77	2.62	0.85	2.75
4	0.49	2.76	0.56	2.95	0.64	3.14	0.72	3.35	0.81	3.56
5	0.44	3.2	0.51	3.46	0.6	3.74	0.69	4.03	0.78	4.34
6	0.4	3.59	0.48	3.93	0.56	4.3	0.66	4.69	0.76	5.1
7	0.37	3.96	0.45	4.38	0.53	4.83	0.63	5.32	0.74	5.85
8	0.34	4.3	0.42	4.8	0.51	5.35	0.61	5.94	0.73	6.57
9	0.32	4.63	0.4	5.2	0.49	5.84	0.6	6.53	0.72	7.29
10	0.31	4.93	0.39	5.59	0.48	6.32	0.58	7.12	0.71	7.99
11	0.29	5.22	0.37	5.96	0.46	6.78	0.57	7.69	0.7	8.69
12	0.28	5.5	0.36	6.32	0.45	7.23	0.56	8.24	0.69	9.37
13	0.27	5.77	0.35	6.66	0.44	7.67	0.55	8.79	0.68	10.05
14	0.26	6.03	0.33	6.99	0.43	8.09	0.54	9.33	0.67	10.72
15	0.25	6.27	0.33	7.32	0.42	8.51	0.53	9.86	0.66	11.38
16	0.24	6.51	0.32	7.64	0.41	8.92	0.52	10.38	0.66	12.04
17	0.23	6.75	0.31	7.94	0.4	9.32	0.52	10.9	0.65	12.69
18	0.23	6.97	0.3	8.24	0.39	9.72	0.51	11.41	0.64	13.33
19	0.22	7.19	0.3	8.54	0.39	10.1	0.5	11.91	0.64	13.97
20	0.21	7.41	0.29	8.83	0.38	10.49	0.5	12.4	0.63	14.61
21	0.21	7.62	0.28	9.11	0.38	10.86	0.49	12.89	0.63	15.24
22	0.2	7.82	0.28	9.39	0.37	11.23	0.48	13.38	0.63	15.86
23	0.2	8.02	0.27	9.66	0.36	11.59	0.48	13.86	0.62	16.48
24	0.2	8.21	0.27	9.93	0.36	11.95	0.48	14.33	0.62	17.1
25	0.19	8.4	0.26	10.19	0.36	12.31	0.47	14.8	0.61	17.71
26	0.19	8.59	0.26	10.45	0.35	12.66	0.47	15.27	0.61	18.32
27	0.18	8.77	0.26	10.7	0.35	13.01	0.46	15.73	0.61	19.82
28	0.18	8.95	0.25	10.96	0.34	13.35	0.46	16.19	0.6	19.53
29	0.18	9.13	0.25	11.2	0.34	13.69	0.45	16.64	0.6	20.13
30	0.17	9.31	0.24	11.45	0.34	14.02	0.45	17.09	0.6	20.73
31	0.17	9.48	0.24	11.69	0.33	14.35	0.45	17.54	0.59	21.32
32	0.17	9.64	0.24	11.92	0.33	14.68	0.44	17.98	0.59	21.91
33	0.17	9.81	0.23	12.16	0.32	15	0.44	18.42	0.59	22.5
34	0.16	9.97	0.23	12.39	0.32	15.32	0.44	18.86	0.59	23.08
35	0.16	10.13	0.23	12.62	0.32	15.64	0.43	19.29	0.58	23.67
36	0.16	10.29	0.23	12.84	0.32	15.96	0.43	19.73	0.58	24.25
37	0.16	10.45	0.22	13.07	0.31	16.27	0.43	20.15	0.58	24.82
38	0.15	10.6	0.22	13.29	0.31	16.58	0.43	20.58	0.58	25.4
39	0.15	10.75	0.22	13.51	0.31	16.89	0.42	21	0.57	25.97
40	0.15	10.9	0.22	13.72	0.31	17.19	0.42	21.43	0.57	26.54
41	0.15	11.05	0.21	13.94	0.3	17.5	0.42	21.84	0.57	27.11
42	0.15	11.2	0.21	14.15	0.3	17.8	0.42	22.26	0.57	27.68
43	0.14	11.34	0.21	14.36	0.3	18.09	0.41	22.67	0.56	28.24
44	0.14	11.48	0.21	14.57	0.3	18.39	0.41	23.09	0.56	28.81
45	0.14	11.63	0.21	14.77	0.29	18.68	0.41	23.5	0.56	29.37
46	0.14	11.76	0.2	14.98	0.29	18.98	0.41	23.9	0.56	29.93
47	0.14	11.9	0.2	15.18	0.29	19.27	0.41	24.31	0.56	30.48
48	0.14	12.04	0.2	15.38	0.29	19.55	0.4	24.71	0.56	31.04
49	0.14	12.17	0.2	15.58	0.29	19.84	0.4	25.11	0.55	31.59
50	0.13	12.31	0.2	15.78	0.28	20.12	0.4	25.51	0.55	32.14
51	0.13	12.44	0.2	15.97	0.28	20.4	0.4	25.91	0.55	32.69
52	0.13	12.57	0.19	16.17	0.28	20.68	0.4	26.31	0.55	33.24
53	0.13	12.7	0.19	16.36	0.28	20.96	0.39	26.7	0.55	33.79
54	0.13	12.83	0.19	16.55	0.28	21.24	0.39	27.09	0.55	34.33
55	0.13	12.96	0.19	16.74	0.28	21.52	0.39	27.48	0.54	34.88
56	0.13	13.08	0.19	16.93	0.27	21.79	0.39	27.87	0.54	35.42
57	0.13	13.21	0.19	17.14	0.27	22.06	0.39	28.26	0.54	35.96
58	0.12	13.33	0.19	18.19	0.27	22.33	0.39	28.65	0.54	36.5
59	0.12	13.45	0.18	17.48	0.27	22.6	0.38	29.03	0.54	37.04
60	0.12	13.57	0.18	17.67	0.27	22.87	0.38	29.41	0.54	37.57