A Model of Price, Volume and Sequential Information

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Abstract

This paper models the relationship between price and volume by tracking their adjustment path and speed in a world with heterogeneous investors. Motivated by widely-observed information leakage in the stock market and fast-growing electronic communication networks (ECNs), the model features sequential information and direct order matching. I show that both the content and the dissemination speed of information are incorporated in price changes and volume accumulations simultaneously. A convergence trading strategy is proposed based on a joint statistic of price and volume, which should help to improve the timing of market entry and exit. The model offers an explanation for the mixed evidence on the relationship between price change and volume and provides several testable hypotheses.

Key Words: Price, Volume, Sequential Information, Convergence Trading Strategy, Event Study

Key word: G12, G14

Introduction

It's a widely-exploited fact in the investment community that the price and volume are jointly determined in the market. The use of price changes per se undoubtedly loses another dimension of information. Both academic and practitioners have long been interested in the information content of the trading volume and its role in predicting returns. Important studies include those of Chordia, Huh, and Subrahmanyam (2007), Chordia and Swaminathan (2000), Gallant, Rossi and Tauchen (1992), Lamoureux and Lastrapes (1991), Lo and Wang (2000), Lo and Wang (2006), among others. A technical analysis based on the combination of price and volume is widely used by investment firms. This paper models the relationship between price and volume by tracking their adjustment path and speed in a world where trades occur by directly matching order flows submitted by investors with heterogeneous beliefs in response to sequential information arrivals.

The direct order matching setup is motivated by the introduction of the electronic communication networks (ECNs), which are computer-mediated markets that widely disseminate, buy, and sell limit orders from its subscribers and execute trades by directly matching the order flow. The Securities and Exchange Commission (SEC) implemented the Order Handling Rules (OHR) in 1997, including the electronic communication networks (ECNs) rule. The distinctive features of ECNs are the absence of designated market makers and the ability to trade anonymously. A majority of stock markets around the world are organized in a similar way. Among the reasons advanced for their popularity is the greater transparency offered by these systems compared to dealer market settings. In the model herein, both trading price and quantity are observable and the market maker is not necessary, in sharp contrast to adverse selection models, such as Kyle's (1985), which assume the presence of the market maker and that the market maker has an infinite power to absorb the net demand of all traders. Price is determined by net demand since the market makers stand ready to clear the market. The model developed in this paper does not have such constraint.

In this model, information is gradually disseminated to the market. The sequential information assumption is based on frequent observations of gradual information release (pre-announcement abnormal stock returns and abnormal volume) in the stock market around important corporate announcements, such as earnings news, dividend changes, mergers and acquisitions, and bankruptcy filings. For example, Advance Micro Devices (AMD) announced on March 5, 2007 that it was unlikely to meet its previously estimated revenue guidance of $1.6 to $1.7 billion for the first quarter of 2007. Partial information, however, had been gradually leaked to the market for a week before the announcement. As shown in Figure 1, the stock price of AMD fell to $13.95 from $15.68, or a drop of 12.4%, from February 26 to March 5, 2007. Accompanying price changes, volume also exhibited abnormal hikes during this period. The price continued its downward trend and closed at $12.71 on April 5, and the volume continued to accumulate during this period. Then the price started to rise despite the absence of positive news. This example shows that as information is gradually released into the market, the mean reversion will occur once price and volume have accumulated to a certain degree. The model developed herein attempts to propose a convergence trading strategy based on a joint statistic of price and volume changes as a predicator of price convergence following information shocks.

Symbols:
V (March 5, 2007): Advance Micro Devices, Inc. Sees Q1 Revenues below Prior Outlook: it was unlikely to meet its previously estimated revenue guidance of $1.6 to $1.7 billion for the first quarter of 2007.
U (March 5, 2007): Intel Corporation, AMD's competitor, May Have Lost E-mails.
T (April 9, 2007): Advanced Micro Devices, Inc. Lowers Q1 2007 Revenue Guidance: it expects to report revenue of approximately $1.225bn, substantially lower than the average analyst expectation of $1.537bn)

Figure 1: Stock Price and Volume of Advance Micro Devices (AMD) around Earnings Warning in March 2007
Source: http://finance.google.com/finance?client=ob&q=AMD

The current model features two transaction camps, the Buyer Camp and the Seller Camp. Investors have heterogeneous beliefs on the final payoff of the asset and their identities are dynamically determined based on the comparison between their beliefs and the prevailing market price. Note that members in each camp are not fixed. Investors then decide to sell or buy, and trade occurs. Due to change in prices and stock holdings in each camp, its power (the average belief level weighted by holding shares) has changed. I propose that when the two camps reach a state of balance, an equilibrium price obtains. In this model, price and volume are jointly determined as a result of information shocks that affect all market investors.

The main conclusion from this model is that the information content and dissemination speed are incorporated in price change and volume accumulation simultaneously. Due to the confrontation of the Buyer Camp and the Seller Camp, the stock price shows reversion once the accumulated price-volume change hits the limit determined by the information content. The magnitude of reversion is positively correlated with the divergence in investor's initial belief and the volume accumulation in the adjustment path, and is negatively correlated with the aggregate supply (total number of shares outstanding). Additionally, the number of auction rounds (persistence) that the price takes to shift from the old equilibrium price to a new one is negatively correlated with price change multiplied by volume at the transaction level.

Thus, we propose that a joint statistic of price and volume can serve as a better predicator of price convergence, which might be useful to improve the timing of market entry and exit. This can be empirically tested by calculating abnormal returns for a convergence trading strategy based on past price and volume around the arrival of new information. The main results can also be exploited to improve event study statistics. In most prior studies, return alone is used to test the market efficiency, but as frequently observed, both price and volume exhibit abnormal and related movements around the event window of information shock. The model proposed in this paper suggests one possible way to join the volume and return into a single statistic.

This model is also motivated by seemingly inconsistent empirical findings on the relationship between price and volume. Early empirical examination of the volume-price relation conducted by Granger and Morgenstern (1963) finds no correlation between prices or absolute values of price changes and volume using weekly or daily transaction data for stock market price index and for individual stocks. However, a handful of empirical research finds a positive volume-absolute price change correlation at different frequency levels. Morgan found that the variance of price change was positively related to trading volume using monthly and four-day interval data. Crouch (1970a, 1970b) found positive correlations between the absolute values of daily price changes and daily volumes. Epps and Epps (1976) provide empirical support for the contention that the positive correlation between volume and price change occurs at the transaction level. In an event study context, Richardson, Sefcik, and Thompson (1986) found that trading volume increases with the square of abnormal return around announcements of dividend changes. Despite the finding of a positive correlation, some of these tests indicate that the correlation is weak.

The model developed here attempts to explain the underlying reason for mixed findings on the relationship between the absolute stock price change and volume change. The pure empirical observation may conceal the underlying fact that both changes in these two variables are due to the change in the third factor. In particular, the model suggests that simultaneous large volumes and large price changes — either positive or negative — can be traced to their common ties to information shocks. Information shock not only translates into the price change (return), but is also incorporated into the volume accumulation. While price and volume are more likely to move in the same direction in response to news, such is not necessarily the case. Price and volume can even move in opposite directions. Thus, a researcher may find weak or no support for the positive relationship between price and volume. Our model predicts that the price-volume relation should depend on total number of shares outstanding and the divergence in initial beliefs.

Our model is different from the sequential information arrival models originally proposed by Copeland (1976) and extended by Jennings, Starks and Fellingham (1981), Morse (1980), etc. In Copeland (1976), information is disseminated to only one trader at a time, which causes a one-time upward shift in each optimist's demand curve by a fixed amount and a downward shift in each pessimist's demand curve. In our model, information is gradually released to *all* market investors. The model is also related to models of technical analysis, such as that of Brown and

Jennings (1989), that ascribe an informational role to past market statistics — price movements and order flows, and to models in which the process of trading facilitates price discovery, such as Madhavan's (1992).

This paper is organized as follows: Section I summarizes the assumptions of the price-volume model in the paper. Section II shows the existence of equilibrium and provides a characterization of this equilibrium by a set of difference equations. Section III presents findings concerning volume and price change following information shock. The implications and empirically testable hypothesis are presented in section IV, followed by the concluding remarks.

I. Assumptions

The fundamental value of firm F is $\overline{F}$, and the expected equity payoff is $v = \overline{F} + \delta$, where $\delta \propto N(0, \sigma_\delta^2)$. Investors are aggregated into two groups, Camp S (seller) and Camp B (buyer), depending on whether they are willing to sell or buy the equity of firm F at the prevailing market price P_0. The identities of investors are dynamically determined based on the comparison between their beliefs and the prevailing market price. Investors shift between these two groups following their expectation and the realized stock price. For example, at current stock price P_0, an investor belongs to Camp B if his expected value of equity is higher. After he makes the purchase at P_0, and the stock price rises to P', he may become a member of Camp S if he would like to sell the stock at the new price, which is higher than his expected value of equity. Consequently, the composition of each Camp is constantly changing. The short sale is prohibited. The total shares outstanding of the firm F are V. Camp S owns V_S and Camp B owns V_B at t=0.

$$V_S + V_B = V \tag{1}$$

Assume Camp S is initially composed of J investors with heterogeneous beliefs on the final payoff of the asset based on their private information. Each of them has a different valuation P_{S_j} of the asset payoff. Each owns V_{S_j} (j=1,2…J) shares of stock. For an investor to belong to Camp S, the only condition is that $P_{S_j} < P_0$. I define P_S as the aggregate valuation of the asset payoff of Camp S. Taking $\frac{V_{S_j}}{V_S}$ as the pricing kernel (the sum from j=1 to J is one), the expected stock price of Camp S, P_S, can be expressed as the weighted sum of the individual valuation across all investors belong to Camp S. Formally,

$$P_S = \sum_{j=1}^{J} \frac{V_{S_j}}{V_S} P_{S_j} \qquad (2)$$

where $\sum_{j=1}^{J} V_{S_j} = V_S$

Likewise, for an investor k to belong to Camp B, the only condition is $P_{B_k} > P_0$, where P_{B_k} is the aggregate valuation of the asset payoff of Camp B. The expected stock price of Camp B, P_B, can be expressed as:

$$P_B = \sum_{k=1}^{K} \frac{V_{B_k}}{V_B} P_{B_k} \qquad (3)$$

where $\sum_{k=1}^{K} V_{B_k} = V_B$

It can be easily verified that $E(P_B) > P_0 > E(P_S)$.

II. The Equilibrium

A. Initial Equilibrium between Camp B and Camp S

Proposition 1

There exists an equilibrium price P^* for any set of { P_S, P_B }, such that Camp S and Camp B reach the following equilibrium:

$$(P^* - P_S)V_S^* = (P_B - P^*)V_B^* \qquad (4)$$

where P_S and P_B are the expected stock prices of Camp S and Camp B; V_S^* and V_B^* are the aggregate equilibrium stock holdings by Camp S and B, respectively; and $V_S^* + V_B^* = V$.

Proof: See Appendix A.

Proposition 1 proposes that a state of balance between Camp B and Camp S always exists in the form of the multiplication of (a) the difference between the expected value for each Camp and the equilibrium price, and (b) the shares owned by each group. Once there is an imbalance, shares originally owned by Camp B will be transferred to Camp S or vice versa as a result of trading, which is accompanied by the price adjustment until a new balance between two groups restores. Any departure from equation (4) can be absorbed by price changes and/or volume changes.

B. Information shock

Assume trading takes place over an event window, which begins at time t=0 and ends at time t=1 with a single information shock $\Delta\delta$. The fundamental value of the asset changes to v=$\overline{F}+\delta+\Delta\delta$. The information is gradually disseminated to the public over the event window. $\Delta\delta$ is known ex post. Camp S and Camp B will revise their valuations upward (downward) if the information content is positive (negative). However, investors have heterogeneous revisions. The initial equilibrium at P_0 between Camp B and Camp S is destroyed. Price P_0 and the positions of the two groups will adjust until the information shock is completely absorbed and a new equilibrium price obtains.

Assume a stock is traded in R rounds of sequential batch trading. At each round, investors from both groups submit limit orders. The rounds of auctions R are determined by the information content $\Delta\delta$ and the adjustment speed of price and volume. Let t_r denote the time at which the rth auction takes place. I assume $0=t_0<t_1<...<t_R=1$, so the sequence of auction dates < t_r> partitions the interval [0,1]. The R auctions take place sequentially. Let ΔV_r denote the quantity (trading volume) traded at the rth auction, so that $V_{S,r}=V_{S,r-1}-\Delta V_{S,r}$ denotes the aggregate position of Camp S after the rth auction, $V_{B,r} = V_{B,r-1} + \Delta V_{B,r}$ denotes the aggregate position of Camp B after the rth auction, and $\Delta V_{S,r} = \Delta V_{B,r} = \Delta V_r$. The filtration $F_t^{\delta} = \sigma(\delta_{s,} 0 \le s \le t)$ is generated by random variable δ_t, the information absorbed by market participants as to time t, where the information revealed at Round I, $\delta_I = \sum_{i=1}^{I} \alpha_i \Delta\delta$, and $\{\alpha_i\}<1$ for i=1,2,…R is a set of parameters describing the diffusion speed of information. The higher α_i, the faster the information is perceived and absorbed at the ith round. To illustrate, at the first round, $\alpha_1 \Delta\delta$ is incorporated in the price. At each round I, an additional $\alpha_I \Delta\delta$ is incorporated in the price. At the end of Rth round, all information shock is completely absorbed, so I have $\sum_{i=1}^{R} \alpha_i \Delta\delta = \Delta\delta$. Let ΔP_r denote the price change from (r-1)th auction to the rth auction, and $P_r = P_{r-1} - \Delta P_r$ is the market clearing price at rth auction.

Proposition 2:

Suppose the initial equilibrium price P_0 satisfies $(P_0 - P_S)V_S = (P_B - P_0)V_B$. The new equilibrium price after a negative information shock $\Delta\delta$<0 is:

$$P_R = P_0 + \Delta\delta + \frac{\sum_{i=1}^{R} \Delta V_i}{V}(P_B - P_S) \qquad (5\text{-}1)$$

The new equilibrium price after a positive information shock $\Delta\delta$>0 is:

$$P_R = P_0 + \Delta\delta - \frac{\sum_{i=1}^{R} \Delta V_i}{V}(P_B - P_S) \qquad (5\text{-}2)$$

Proof:

See Appendix B.

Interpretations and Implications:

(1) A couple of interesting results emerge from Proposition 2. My model suggests that a better predicator of price adjustment after an event is based on a joint statistic of price and volume, which can be used to improve the timing of entry and exit of the market.

From $P_R = P_0 + \Delta\delta + \frac{\sum_{i=1}^{R} \Delta V_i}{V}(P_B - P_S)$, I can see that the expected price change is determined by both information content and a joint statistic of price and volume. The special case is when there is no divergence of opinions between Camp S and Camp B ($P_B = P_S$), and the expected price change caused by the information shock is equal to the information content $\Delta\delta$. However, price convergence arises from divergent expected beliefs of investors in Camp B and Camp S. Due to the confrontation between two camps, trading will occur if the balance between two camps is not restored. Consequently, price will not fully adjust to the information content. Instead, the final price after the information shock is determined jointly with volume when the information content is completely absorbed by price change and volume accumulation.

(2) The magnitude of price reversion is positively correlated with the initial opinion divergence and the trading volume, and negatively correlated with the total number of shares outstanding. The stock price is more likely to be affected by the competition between two camps if the initial opinion divergence is higher, and if the volume accumulation is greater. The higher trading volume implies that more investors initially belonging to Camp S have sold their shares and shifted to Camp B. Hence there is less downward pressure on the stock price, which will reverse at a relatively higher price. The price of a firm with more shares outstanding is less likely to be affected by the confrontation of two camps. For instance, a price drop following bad news will fully adjust to the information content if there is no opinion divergence. This is because

there is no need for price to compromise to the confrontation between two camps. However, the price drop will be reversed if the initial opinion divergence is high.

In practice, due to market sentiment and irrational behavior such as herding, the actual price usually exceeds the limit specified by the RHS of Equation (5-1) and may show price overreaction and mean reversion. Although it is hard to capture the *actual turning point* of the stock price after an information shock, my model suggests a way to find the *mean reversion point* following the shock, based on the price and volume observed in the event window. Capturing this point is valuable because it allows an investor to enter or exit the market at a 'second-best' opportunity.

(3) Note Equation (5-1) can be rewritten as:

$$\Delta\delta = (P_R - P_0) - \frac{\sum_{i=1}^{R} \Delta V_i}{V} (P_B - P_S) \tag{6}$$

Equation (6) suggests that the information content during the event window is simultaneously reflected in two components: the price change and the volume change. The first part of RHS of Equation (6) represents the price drop induced by the negative information shock. The second part, the ratio of total trading volume over total shares outstanding during the information release period, multiplied by the difference in Camp B and Camp S's expected valuations, reflects volume accumulation accompanying the price drop. Hence the ex post price following the information flow is not only determined by the severity of the negative information shock, but also by the accumulation of trading volume, which reflects the competition of the two camps' forces.

This result on the price-volume relation can be exploited to improve event study statistics. Around the event window when a shock occurs, it is often observed that both price and volume show abnormal movements. However, in most prior studies, return alone is used to test the market efficiency. Even if some papers study abnormal volume movement before and after the event, the abnormal return and abnormal volume are investigated separately. For example, Morse (1980) separately studies the volume and return, but does not combine these two factors. Equation (6) suggests one possible way to join the volume and return into one statistic.

(4) For fixed aggregate information shock $\Delta\delta$, the number of transactions R that take the price from the old equilibrium price to a new one is endogenously determined by ΔP_i and ΔV_i at each round of auction. R is negatively correlated with both ΔP_i and ΔV_i, which means that if the price drop and/or volume change is high for each round of transaction, the speed of adjustment is faster, and the stock price drop will not persist long.

III. Comparative Static Analysis

Proposition 3

For negative information shock, both the magnitude of the price change $|\Delta P_i|$ and the trading volume ΔV_i are positively correlated with the magnitude of information content, characterized by filtration parameter α_I. For positive information shock, the price change *per se* and the trading volume ΔV_i are positively correlated with the magnitude of information content. The sensitivity of the magnitude of price change to volume is negatively correlated with total shares outstanding and positively correlated with the divergence in the beliefs of Camp B and Camp S.

Proof:

See Appendix C.

Implications:

(1) If filtration parameter α_I is higher in Ith round than α_J in Jth round, both the magnitude of price change and trading volume in Ith round will be accordingly higher. It appears that there is a positive correlation between $|\Delta P_i|$ and ΔV_i, in line with empirical evidence documented by Karpoff (1987). However, the pure observation may cover the underlying fact that both changes in these two variables are due to the change in the third factor. Simultaneous large volumes and large price changes, either positive or negative, can be traced to their common ties to information flows. For example, suppose the information shock is released slowly initially and more rapidly later on. The price change will be greater for a round of auction with more information flow, as will be the volume. In short, the positive correlation relationship between $|\Delta P_i|$ and ΔV_i might hold across the rounds of auctions, although this is not a necessary condition.

Take positive information innovation as an example. Compare two rounds of auctions with information dissemination parameters α_I and α_J, with $\alpha_I > \alpha_J$. We have $\Delta P_I + \frac{\Delta V_I}{V}(P_B - P_S) > \Delta P_J + \frac{\Delta V_J}{V}(P_B - P_S)$. To make this inequality hold, we can let (i) $\Delta P_I > \Delta P_J$, $\Delta V_I \le \Delta V_J$, (ii) $\Delta V_I > \Delta V_J$, $\Delta P_I \le \Delta P_J$, (iii) $\Delta P_I \ge \Delta P_J$, $\Delta V_I > \Delta V_J$, or (iv) $\Delta P_I > \Delta P_J$, $\Delta V_I \le \Delta V_J$. Conditions (iii) and (iv) correspond to the positive correlation between volume and price change documented in numerous empirical studies. The existence of condition (i) and (ii) validates the weak or nonexistent positive correlation found in other studies. Given the clustering information flow during an event window, this also suggests an explanation for the empirical work to date that indicates that the empirical correlation of volume and price change is stronger over fixed time intervals than over a fixed number of transactions.

(2) The sensitivity of the magnitude of price change to volume is negatively correlated with total shares outstanding and positively correlated with the divergence in belief of Camp B and Camp S. Assuming a positive correlation between divergence in investors' beliefs and the number of investors, this statement is consistent with the results of Tauchen and Pitts (1983) that predict that the relationship between volume and the squared price change increases with the number of investors.

IV. Conclusion

Motivated by the development of the ECN system and information leakage observed in the stock market, this paper develops an equilibrium model of price-volume relationship where trades occur by directly matching order flows submitted by heterogeneous agents who are impinged upon by sequential information arrivals. Gradual releases of information may arise from insider information or asymmetric information access by institutional investors and individual investors. The model shows that the information content and dissemination speed are incorporated in price change and volume accumulation simultaneously. Intuitively, price change can be viewed as the market evaluation of new information, while the corresponding volume can be interpreted to reflect disagreements about the meaning of the information by individual investors. Therefore, the price and quantity available in the ECN system provides valuable information to investors.

I conclude with three major implications of my model. First, price reversion after an information shock will occur due to a need to strike a balance between two confronting investment camps. The magnitude of reversion depends on the difference in opinions of two camps, total supply of stock, and volume accumulation along the adjustment path. Based on the model setup, a simple joint statistic of price changes and volume can be used to design a convergence trading strategy to test whether it helps to time market entry and exit. Second, the results suggest that event study should take into account the abnormal changes in both price and volume. The price weighted by volume is a potential variable to combine both effects.

Finally, the widely-cited positive empirical correlation between the magnitude of price change and volume is traced to common information ties in this model. The model also accommodates negative or weak correlation as found in other research. This helps to explain the mixed empirical evidence on the relationship between price and volume. I also provide several testable empirical hypotheses. In particular, the model predicts that the relationship between price change and volume depends on the total shares outstanding and divergence in investors' beliefs, which is consistent with conjectures of Karpoff (1987) based on preliminary empirical evidence.

Appendix A (Proof of Proposition 1):

There exists an equilibrium price P^* for any set of { P_S , P_B }, such that Camp S and Camp B reach a state of equilibrium:

$$(P^* - P_S)V_S^* = (P_B - P^*)V_B^* \qquad \text{(A.1)}$$

where P_S and P_B are the expected stock price of Camp S and Camp B, V_S^* and V_B^* are the aggregate equilibrium stock holdings by Camp S and B, respectively, and $V_S^* + V_B^* = V$.

Proof

Case 1:

Assume that the aggregate beliefs of Camp S and Camp B, P_S , P_B , do not change when stock owning is transferred between two groups. Suppose there is an initial imbalance. WLOG, let

$$\begin{aligned} &(P^{'} - P_S)V_S^{'} > (P_B - P^{'})V_B^{'} \\ &V_S^{'} + V_B^{'} = V \end{aligned}$$

I can find ΔE >0 s.t.

$$(P^{'} - P_S)V_S^{'} = (P_B - P^{'})V_B^{'} + \Delta E \qquad \text{(A.2)}$$

A fraction of investors in Camp S is capable of driving down the stock price by liquidating stocks, accompanied by a transfer of shares previously owned by Camp S to Camp B. Note that the buyers who purchase these shares must belong to Camp B, whose individual valuation of the asset is higher than the purchasing price. Consequently, shares owned by Camp B increased. Following a series of price declines, ΔE gradually decreases to zero. More formally, let ΔP , ΔV be the price change and the transfer of shares between two groups, respectively. (A.2) is equivalent to the following series of equations:

$$((P' - \Delta P_1) - P_S)(V_S' - \Delta V_1) = (P_B - (P' - \Delta P_1))(V_B' + \Delta V_1) + \Delta E - [(P_B - P_S)\Delta V_1 + \Delta P_1 \cdot V]$$

$$((P' - \Delta P_1 - \Delta P_2) - P_S)(V_S' - \Delta V_1 - \Delta V_2)$$
$$= (P_B - (P' - \Delta P_1 - \Delta P_2))(V_B' + \Delta V_1 + \Delta V_2) + \Delta E - [(P_B - P_S)\Delta V_2 + (\Delta P_1 + \Delta P_2) \cdot V]$$

...

$$((P' - \sum_{i=1}^{N} \Delta P_i) - P_S)(V_S' - \sum_{i=1}^{N} \Delta V_i)$$
$$= (P_B - (P' - \sum_{i=1}^{N} \Delta P_i))(V_B' + \sum_{i=1}^{N} \Delta V_i) + \Delta E - [(P_B - P_S)\sum_{i=1}^{N} \Delta V_i + \sum_{i=1}^{N} \Delta P_i \cdot V]$$

Note that ΔE is a fixed number, and it is greater than zero. $(P_B - P_S)\sum_{i=1}^{N} \Delta V_i + \sum_{i=1}^{N} \Delta P_i \cdot V$ >0, $\sum_{i=1}^{N} \Delta V_i$ and $\sum_{i=1}^{N} \Delta P_i$ is an increasing function of N. Hence there must exist N such that

$$\Delta E - [(P_B - P_S)\sum_{i=1}^{N} \Delta V_i + \sum_{i=1}^{N} \Delta P_i \cdot V] = 0 \qquad \text{(A.3)}$$

Thus, I have

$$((P' - \sum_{i=1}^{N} \Delta P_i) - P_S)(V_S' - \sum_{i=1}^{N} \Delta V_i) = (P_B - (P' - \sum_{i=1}^{N} \Delta P_i))(V_B' + \sum_{i=1}^{N} \Delta V_i) \qquad \text{(A.4)}$$

(A.1) obtains by the following substitution:

$$P' - \sum_{i=1}^{N} \Delta P_i = \text{P*},\ V_S' - \sum_{i=1}^{N} \Delta V_i = V_S^*,\ V_B' + \sum_{i=1}^{N} \Delta V_i = V_B^* \qquad \text{(A.5)}$$

Case 2:

More generally, consider that the aggregate beliefs of Camp S and Camp B, P_S and P_B, change to new levels when stock owning is transferred between two groups.

Suppose

$$(P' - P_S)V_S' > (P_B - P')V_B'$$
$$V_S' + V_B' = V$$

Start with the first round. As price drops, the number of shares owned by Camp S decreases, and P_S^{Δ}, the weighted average of belief of the investors who sell shares, is lower than the new price $P^{'} - \Delta P_1$. Otherwise, they will not sell. On the other hand, the number of shares owned by Camp B increases, and P_B^{Δ}, the average belief of the investors who purchase the shares, is higher than the new price. Otherwise, they will not make the purchase. Let $P_S^{'}$, $P_B^{'}$ denote the new average beliefs of adjusted Camp S and Camp B after the first round.

Next I prove that LHS of the inequality is monotonically decreasing as $P^{'}$ drops and RHS is monotonically increasing as $P^{'}$ drops. If that is the case, there must exist 0<P*<$P^{'}$, such that LHS=RHS. This is because initially, LHS> RHS, while at $P^{'}$=0, LHS < RHS.

First prove that LHS of the inequality is monotonically decreases as $P^{'}$ drops.

Need to show:

$$((P^{'} - \Delta P_1) - P_S^{'})(V_S^{'} - \Delta V_1) < (P - P_S)V_S^{'}$$

$$\Leftrightarrow P'(V_s^{'} - \Delta V) - \Delta P(V_s^{'} - \Delta V) - P_S^{'}(V_s^{'} - \Delta V) - P' \cdot V_s^{'} + P_S V_s^{'} < 0$$

$$\Leftrightarrow P_S V_s^{'} - P_S^{'}(V_s^{'} - \Delta V) - P'\Delta V - \Delta P(V_s^{'} - \Delta V) < 0$$

$$\Leftrightarrow P_s^{\Delta}\Delta V - (P' - \Delta P)\Delta V - \Delta P \cdot V_S^{'} < 0$$

$$\Leftrightarrow (P_s^{\Delta} - (P' - \Delta P))\Delta V - \Delta P \cdot V_S^{'} < 0$$

The fourth inequality is equivalent to the third one due to the following relationship:

$$P_S V_s^{'} - P_S^{'}(V_s^{'} - \Delta V) = P_s^{\Delta}\Delta V$$

by definitions of P_S, $P_S^{'}$ and P_s^{Δ}. The last inequality holds since $P_s^{\Delta} < P' - \Delta P$, ΔV, ΔP, $V_S^{'}$ are greater than zero.

Last I prove that RHS monotonically increases as $P^{'}$ drops.

Need to show:

$$(P_B^{'}-(P'-\Delta P_1))(V_B^{'}+\Delta V_1)>(P_B-P')V_B^{'}$$

$$\Leftrightarrow P_B^{'}(V_{_B}^{'}+\Delta V)+\Delta P(V_{_B}^{'}+\Delta V)-P'(V_{_B}^{'}+\Delta V)+P'\cdot V_B^{'}-P_B V_B^{'}>0$$

$$\Leftrightarrow (P_B^{'}(V_{_B}^{'}+\Delta V)-P_B V_{_B}^{'})-P'\Delta V+\Delta P(V_{_B}^{'}+\Delta V)>0$$

$$\Leftrightarrow (P_{_B}^{\Delta}\Delta V-(P'-\Delta P)\Delta V)+\Delta P\cdot V_B^{'}>0$$

$$\Leftrightarrow (P_{_B}^{\Delta}-(P'-\Delta P))\Delta V+\Delta P\cdot V_B^{'}>0$$

The fourth inequality is equivalent to the third one due to the following relationship:

$$P_B^{'}(V_{_B}^{'}+\Delta V)-P_B V_{_B}^{'}= P_{_B}^{\Delta}\Delta V$$

by definitions of $P_B, P_B^{'}$ and $P_{_B}^{\Delta}$. The last inequality holds since $P_{_B}^{\Delta}>P'-\Delta P$, ΔV, ΔP, $V_B^{'}$ are greater than zero.

In sum, for the first round the LHS is decreasing and RHS is increasing in price. The same logic applies to subsequent rounds when price keeps going down due to negative news.

Note that initially, LHS> RHS, while at $P^{'}$=0, LHS < RHS. There must exist 0<P<$P^{'}$, such that LHS=RHS. Q.E.D.

Appendix B (Proposition 2):

Suppose the initial price P_0 satisfies $(P_0-P_S)V_S=(P_B-P_0)V_B$. Price after a negative information shock $\Delta\delta$<0 is:

$$P_R=P_0+\Delta\delta+\frac{\sum_{i=1}^{R}\Delta V_i}{V}(P_B-P_S) \tag{A.6}$$

Price after a positive information shock $\Delta\delta$>0 is:

$$P_R = P_0 + \Delta\delta - \frac{\sum_{i=1}^{R} \Delta V_i}{V}(P_B - P_S) \qquad \text{(A.7)}$$

Proof:

(1) Negative information shock

Let us first assume there is a negative information shock $\Delta\delta<0$ for the trading window. Both Camp S and Camp B will revise their beliefs $P_{S,}P_B$ downward to $P_S^{'}, P_B^{'}$. Assuming V_S, V_B remain unchanged, we have $(P_0 - P_S^{'})V_S > (P_B^{'} - P_0)V_B$. The initial equilibrium is destroyed. According to Proposition 1, a balance between Camp S and Camp B exists at a new equilibrium price. Since the imbalance results from the introduction of $\Delta\delta$, I can explicitly solve for the new equilibrium price in terms of $\Delta\delta$.

Intuitively, following the negative information shock, a fraction of investors in Camp S will first sell their shares, driving down the stock price, accompanied by a decrease in the total shares owned by Camp S. Those who bought these shares belong to Camp B, since their expected price must be higher than current price. Otherwise they will not buy. The total shares owned by Camp B increase. The shock will be sequentially absorbed by investors depending on their adjusted beliefs. From t=0 to 1, a total of R sequential equilibriums happens, incorporating the sequentially revealed shock. At Rth round, all information shock is absorbed.

Case 1:

Assume that the aggregate beliefs of Camp S and Camp B, P_S, P_B, do not change when stock owning is transferred between two groups.

The changes in the prices and volumes for Auctions 1 to R are described as followed:

$$((P_0 - \Delta P_1) - P_S)(V_S - \Delta V_1) - (P_B - (P_0 - \Delta P_1))(V_B + \Delta V_1) = \alpha_1 \Delta\delta \cdot V \qquad \text{(A.8-1)}$$

$$((P_0 - \Delta P_1 - \Delta P_2) - P_S)(V_S - \Delta V_1 - \Delta V_2) - (P_B - (P_0 - \Delta P_1 - \Delta P_2))(V_B + \Delta V_1 + \Delta V_2) = \\ = (\alpha_1 \Delta\delta + \alpha_2 \Delta\delta) \cdot V \quad \text{(A.8-2)}$$

…

$$((P_0 - \sum_{i=1}^{R} \Delta P_i) - P_S)(V_S - \sum_{i=1}^{R} \Delta V_i) - (P_B - (P_0 - \sum_{i=1}^{R} \Delta P_i))(V_B + \sum_{i=1}^{R} \Delta V_i) = (\sum_{i=1}^{R} \alpha_i \Delta\delta) \cdot V \quad \text{(A.8-R)}$$

Note $(P_0 - \sum_{i=1}^{R} \Delta P_i) = P_R$, $\sum_{i=1}^{R} \alpha_i \Delta\delta = \Delta\delta$, (A.8-R) becomes

$$(P_R - P_S)(V_S - \sum_{i=1}^{R} \Delta V_i) - (P_B - P_R)(V_B + \sum_{i=1}^{R} \Delta V_i) = \Delta\delta \cdot V \quad \text{(A.9)}$$

Given $(P_0 - P_S)V_S = (P_B - P_0)V_B$, and $V_S + V_B = V$, it is straightforward to obtain

$$P_R = P_0 + \Delta\delta + \frac{\sum_{i=1}^{R} \Delta V_i}{V}(P_B - P_S) \quad \text{(A.10)}$$

Now check whether P_R is a new equilibrium price.

Let $P_0 - \sum_{i=1}^{R} \Delta P_i = P_R$, $V_S - \sum_{i=1}^{R} \Delta V_i = V_{S,R}$, $V_B + \sum_{i=1}^{R} \Delta V_i = V_{B,R}$ Guess that the price drop induced by the negative shock is the sum of the drop in the expected value by new Camp S and new Camp B, weighted by their respective fractions of shares. Formally, the new expectations of final payoffs by Camp B and Camp S after Rth round, $P_{S,R}$, $P_{B,R}$ satisfy

$$(P_{S,R} - P_S)\frac{V_{S,R}}{V} + (P_{B,R} - P_B)\frac{V_{B,R}}{V} = \sum_{i=1}^{R} \alpha_i \Delta\delta \quad \text{(A.11)}$$

Plugging in (A.8-R), I have

$$(P_R - P_{S,R})V_{S,R} = (P_{B,R} - P_R)V_{B,R} \\ V_{S,R} + V_{B,R} = V \quad \text{(A.12)}$$

Hence, P_R is the new equilibrium price that balances Camp B and Camp S after the information shock $\Delta\delta$.

Similarly, let $P_0 - \sum_{i=1}^{I} \Delta P_i = P_I$, $V_S - \sum_{i=1}^{I} \Delta V_i = V_{S,I}$, $V_B + \sum_{i=1}^{I} \Delta V_i = V_{B,I}$, and the new expectation of final payoffs by Camp B and Camp S after Ith round, $P_{S,I}$, $P_{B,I}$ satisfy $(P_{S,I} - P_S)\frac{V_{S,I}}{V} + (P_{B,I} - P_B)\frac{V_{B,I}}{V} = \sum_{i=1}^{I} \alpha_i \Delta\delta$, it's easy to show that

$$P_I = P_0 + \delta_I + \frac{\sum_{i=1}^{I} \Delta V_i}{V}(P_B - P_S) \qquad \text{(A.13)}$$

is the equilibrium price after the information shock $\sum_{i=1}^{I} \alpha_i \Delta\delta = \delta_I$ for i=1,2,…,R.

Case 2:

More generally, consider that the aggregate beliefs of Camp S and Camp B, P_S and P_B, change to new levels when stock owning is transferred between two groups.

The changes in the prices and volumes for Auctions 1 to R are described as followed:

$$((P_0 - \Delta P_1) - P_s^1)(V_S - \Delta V_1) - (P_B^1 - (P_0 - \Delta P_1))(V_B + \Delta V_1) = \alpha_1 \Delta\delta \cdot V$$

$$P_1(V_S - \Delta V_1) + P_1(V_B + \Delta V_1) - P_s^1(V_S - \Delta V_1) - P_B^1(V_B + \Delta V_1) = \alpha_1 \Delta\delta \cdot V$$

Note

$$P_B^1(V_B + \Delta V_1) = P_B V_B + P_B^{\Delta_1} \Delta V_1$$

$$P_s^1(V_S - \Delta V_1) = P_S V_S - P_S^{\Delta_1} \Delta V_1$$

where $P_S^{\Delta_1}$ is the weighted average of belief of the investors who sell shares, and $P_B^{\Delta_1}$ is the weighted average of belief of the investors who buy shares, and

$$P_S V_S + P_B V_B = P_0 V$$

$$(V_S - \Delta V_1) + (V_B + \Delta V_1) = \mathrm{V}$$

I obtain

$$P_1 = P_0 + \alpha_1 \Delta\delta + (P_B^{\Delta_1} - P_S^{\Delta_1})\frac{\Delta V_1}{V}$$

Similarly, for i=R, I have

$$P_R = P_0 + \sum_{i=1}^{R} \alpha_i \Delta\delta + \sum_{i=1}^{R} (P_B^{\Delta_i} - P_S^{\Delta_i})\frac{\Delta V_i}{V}$$

$$= P_0 + \Delta\delta + \sum_{i=1}^{R} (P_B^{\Delta_i} - P_S^{\Delta_i})\frac{\Delta V_i}{V}$$

Let $P_0 - \sum_{i=1}^{R} \Delta P_i = P_R$, $V_S - \sum_{i=1}^{R} \Delta V_i = V_{S,R}$, $V_B + \sum_{i=1}^{R} \Delta V_i = V_{B,R}$ Guess the price drop induced by the negative shock is the sum of the drop in the expected value by new Camp S and new Camp B, weighted by their respective fractions of shares. Formally, the new expectations of final payoffs by Camp B and Camp S after Rth round, $P_{S,R}$, $P_{B,R}$, satisfy

$$(P_{S,R} - P_S^R)\frac{V_{S,R}}{V} + (P_{B,R} - P_B^R)\frac{V_{B,R}}{V} = \sum_{i=1}^{R} \alpha_i \Delta\delta \qquad \text{(A.14)}$$

Plug in $((P_0 - \sum_{i=1}^{R} \Delta P_i) - P_S^R)(V_S - \sum_{i=1}^{R} \Delta V_i) - (P_B^R - (P_0 - \sum_{i=1}^{R} \Delta P_i))(V_B + \sum_{i=1}^{R} \Delta V_i) = (\sum_{i=1}^{R} \alpha_i \Delta\delta) \cdot V$.

Thus,

$$\begin{aligned} &(P_R - P_{S,R})V_{S,R} = (P_{B,R} - P_R)V_{B,R} \\ &V_{S,R} + V_{B,R} = V \end{aligned} \qquad \text{(A.15)}$$

Hence, P_R is the new equilibrium price that balances Camp B and Camp S after the information shock $\Delta\delta$. It should be noted that the changes in average belief of Camp S are induced by two parts. One is the transfer of a fraction of shares out of Camp S, which results in

P_S^R, and the other is the revision of each individual's belief following the information shock, which results in $P_{S,R}$ - P_S^R . The sum of these two parts reflects the revision of aggregate belief of the whole group. The same logic applies to Camp B.

(2) Positive information shock

If good news comes, the case is mathematically parallel to the case when bad news comes. The proof for (A.7) resembles that for (A.6). Q.E.D.

Appendix C (proof of Proposition 3):

Denote $\Delta P_I \equiv P_I - P_{I-1}$. Price change at the ith round after a negative information shock is:

$$\Delta P_I = \alpha_I \Delta\delta + \frac{\Delta V_I}{V}(P_B - P_S) \quad \text{(A.16)}$$

Price change at the ith round after a positive information shock is:

$$\Delta P_I = \alpha_I \Delta\delta - \frac{\Delta V_I}{V}(P_B - P_S) \quad \text{(A.17)}$$

for I=1,2,…R.

Both the magnitude of the price change $|\Delta P_i|$ and the trading volume ΔV_i are positively correlated with the magnitude of information content, characterized by filtration parameter α_I. The sensitivity of the magnitude of price change to volume is negatively correlated with total shares outstanding and positively correlated with the divergence in belief of Camp B and Camp S.

Proof:

From (A.6), for negative information shock, I have

$$P_I = P_0 + \delta_I + \frac{\sum_{i=1}^{I} \Delta V_i}{V} (P_B - P_S)$$

and

$$P_{I-1} = P_0 + \delta_{I-1} + \frac{\sum_{i=1}^{I-1} \Delta V_i}{V} (P_B - P_S)$$

Take the difference between the above two equations, and (A.16) obtains. (A.17) can be similarly derived.

Write (A.16) as $\Delta P_I - \frac{\Delta V_I}{V}(P_B - P_S) = \alpha_I \Delta\delta$. For $\Delta\delta$<0, ΔP_I<0,

$$\frac{\partial(|\Delta P_I|)}{\partial(\alpha_I |\Delta\delta|)} = 1 > 0 \quad \text{(A.18)}$$

and

$$\frac{\partial(\Delta V_I)}{\partial(\alpha_I |\Delta\delta|)} = \frac{V}{(P_B - P_S)} > 0 . \quad \text{(A.19)}$$

Hence, for negative information innovation, both the magnitude of price change $|\Delta P_i|$ and the trading volume ΔV_i are positively correlated with the information content.

For positive information shock, write (A.17) as $\Delta P_I + \frac{\Delta V_I}{V}(P_B - P_S) = \alpha_I \Delta\delta$.

For $\Delta\delta$>0

$$\frac{\partial(\Delta P_I)}{\partial(\alpha_I \Delta\delta)} = 1 > 0 \quad \text{(A.20)}$$

and

$$\frac{\partial(\Delta V_I)}{\partial(\alpha_I \Delta\delta)} = \frac{V}{(P_B - P_S)} > 0 . \quad \text{(A.21)}$$

Hence, for positive information innovation, both the price change ΔP_i *per se* and the trading volume ΔV_i are positively correlated with the information content.

In sum, for both negative and positive information innovation, the magnitude of the price change $|\Delta P_i|$ and the trading volume ΔV_i are both positively correlated with the magnitude of information content.

$$\frac{\partial(|\Delta P_I|)}{\partial(|\Delta V_I|)} = \frac{P_B - P_S}{V} > 0 \qquad \text{(A.22)}$$

The sensitivity of the magnitude of price change to volume is negatively correlated with total shares outstanding and positively correlated with the divergence in investors' beliefs.

Q.E.D.

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