The Spirit of Capitalism, Asset Pricing and Growth in a Small Open Economy

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Abstract

Conventional models of economic behavior have failed to account for a number of observed empirical regularities in macroeconomics and international economics. This may be due to preference specifications in conventional models. In this paper, we consider preferences with the "spirit of capitalism" (the desire to accumulate wealth as a way of acquiring status). We analyze a number of potential effects of international catching-up and the spirit of capitalism on savings, growth, portfolio allocation and asset pricing. Moreover, we obtain a multi-factor Capital Asset Pricing Model (CAPM). Our results show that status concerns have non-trivial effects on savings, growth, portfolio allocation, asset prices and the foreign exchange risk premium.

Keywords: International Catching-Up; Asset Pricing; Exchange Rates; Spirit of Capitalism; Stochastic General Equilibrium Models.

1 Introduction

Conventional models of economic behavior assume that a person's well-being depends on the absolute quantities of various goods and services he consumes and not on how these quantities compare with those consumed by others. Yet, from Max Weber's pursuit of wealth as an end in itself (spirit of capitalism) to Duesenberry's "demonstration effect" in consumption, there is a long tradition in sociology and economics which acknowledges that people are concerned with their relative standing in society and individuals' consumption decisions are influenced by others (Duesenberry (1949)).

There is a growing recognition that modeling preferences to accommodate such elements enhances our understanding of saving-consumption, asset pricing, and economic growth. For example, several authors have recently shown that the spirit of capitalism has nontrivial consumption-saving, portfolio allocation, asset pricing and growth effects. Bakshi and Chen (1996b) develop a model where relative wealth status leads to a two factor capital asset pricing model (CAPM). Smith (2001) extends the model to recursive preferences and obtains a three factor CAPM. In another paper Smith (1999) examines growth effects. In a series of papers, Gong and Zou examine long run growth, saving, policy effectiveness, and monetary issues in a cash-advance setting under the spirit of capitalism (Zou (1994), Zou (1995), Gong and Zou (2001), Gong and Zou (2002)). Following Bakshi and Chen (1996b) and others, Carroll (2002) presents a model where wealth enters consumers' utility functions directly and shows that the model yields results consistent with the available data on the saving behavior of the wealthy.

Frank (1985) and others have shown that an individual's well-being depends significantly on his relative standing in a society whereas Greenfeld (2001) makes a compelling case that a country's global standing and international catching up has played a dominant role in the history of modern economic growth. According to Greenfeld, while the spirit of capitalism defined the historically exceptional inclination for the pursuit of material gain, it was competitive nationalism and not Protestanism that provided the collective consciousness and a new set of moral values for modern economic growth. Being a member of a nation confers prestige upon an individual; as such, each individual invests in the dignity and prestige of the nation which is "necessarily assessed in relation to the status of other nations." Therefore nationalism implies international competition, and "makes competitiveness a measure of success in every sphere which a nation defines as significant for its self-image, and commits societies which define themselves as nations to a race with a relative and therefore forever receding finishing line" [Greenfeld (2001), p. 23]. The sense of being a nation started in Britain by the beginning of the 17th century and stimulated nationalism elsewhere.

In this paper, we incorporate elements of 'spirit of capitalism' à la Greenfeld whereby the representative individual cares about his standing in the global wealth hierarchy and we use "international catching-up" to describe concerns about fluctuations in individual wealth relative to international wealth standards. When individuals care about relative status and the associated risks of falling out of status, they will hedge against these risks. This can have potentially important portfolio allocation, consumption-saving and growth effects. Using the "international catching-up" feature, we analyze consumption, growth, international portfolio allocation, and asset pricing in a dynamic stochastic general equilibrium setting. Our model builds on the rich equilibrium framework of Grinols and Turnovsky (1994) and well known to be capable of addressing a number of interesting issues (such as how means and variances of domestic government policy impact on the economy and isolating determinants of the real foreign exchange rate premium). Even though the capitalist spirit model of Bakshi and Chen (1996b) captures the main features of the catching-up with the Jonesses features of Abel Abel (1990), the idea has not been modeled in a stochastic general equilibrium setup in an *open* economy.

There are several distinctive features of the model developed here (the first two are familiar from Grinols and Turnovsky (1994)): First, the equilibrium involves the joint determination of the means and variances of the relevant economic variables (in terms of the first two moments of the exogenous stochastic processes impinging on the economy). Second, the model incorporates portfolio choice - thereby giving rise to an integrated analysis of exchange rate determination with a risk adjusted PPP and portfolio equilibrium. Third, our use of a recursive utility function which disentangles the two preference parameters for risk aversion and intertemporal substitution ensures that the model is fully capable of assessing the importance of the distinct and separate roles played by agents' attitudes towards risk and intertemporal substitution. Finally, the model has a utility function that includes the level of wealth and a world wealth index - thereby capturing features of 'catching up with the Joneses' models such as Abel (1990), and exogenous habit formation models such as Campbell and Cochrane (1999).[1]

This latter feature is of particular interest because it captures elements of a popular approach to explaining a number of puzzles associated with the equity premium and the foreign exchange market. For example, Campbell and Cochrane (1999) proposed a preference specification in which there is both an aggregate consumption externality and utility is time-inseparable because of habit persistence; this helped explain the US stylized facts of the equity premium puzzle. Because the habit persistence takes the form of an aggregate consumption externality, this preference specification embodies

[1]Strictly speaking, the literature distinguishes the different external references in the utility function: agents care about past aggregate consumption in the economy ("catching up with the Joneses" as in Abel (1990) and "external habit formation" as in Campbell and Cochrane (1999)); agents care about current per capita consumption levels in the economy ("keeping with the Joneses"); agents care about absolute or relative wealth in the economy ("spirit of capitalism" as in Bakshi and Chen (1996b)).

keeping up with the Joneses' effects in the spirit of Duesenberry (1949) and Abel (1990). However, the particular way in which the utility function is specified here also draws on another recent and fast-developing literature which incorporates the spirit of capitalism (the desire to accumulate wealth as a way of acquiring status). Our approach runs along the lines of Bakshi and Chen (1996b) who find that the spirit of capitalism seems to be a driving force behind stock market volatility (and economic growth).

Our paper is organized as follows. Section 2 outlines our development of existing continuous time stochastic endogenous growth models and presents the solution. Section 3 presents the the effects of status concerns on international portfolio diversification, asset prices, and exchange rates. Section 4 explores the foreign exchange risk premium and its determinants within the international catching-up framework while section 5 concludes the paper.

2 The Model

We consider a continuous-time, infinite-horizon small open economy with complete financial markets and a single production good.[2] In our model the utility maximizing and portfolio optimizing behavior of a representative household has a somewhat nonstandard element. We assume that the representative agent's preferences exhibit the "the spirit of capitalism" feature of Bakshi and Chen (1996b), Gong and Zou (2002) and Smith (2001). Specifically, we assume that household utility is a stochas-

[2]Preferences incorporating 'spirit of capitalism' have non-trivial consumption-saving, portfolio allocation, capital accumulation, and growth effects; see Smith (1999), and Gong and Zou (2002). These effects have not been studied in an open economy context; hence, we maintain a production economy setup in the spirit of Cox, Ingersoll, and Ross (1985) to study such effects. For asset pricing considerations, Bakshi and Chen (1997) demonstrate the equivalence between endowment and production economies.

tic differential function of the multiplicative of individual consumption and relative wealth. The latter is defined as the ratio of the individual's absolute wealth to the international standard of living. As in the standard constant relative risk aversion (CRRA) utility function, in this form of preferences individual risk aversion does not change over time. The model is a monetary one where money enters into the utility function. The production technology assumes a Rebelo (1991) 'AK' production function which leads to endogenous growth. The stochastic nature of the model is characterized by four exogenous stochastic shocks: (i) productivity shocks; (ii) monetary growth shocks; (iii) foreign price shocks and (iv) foreign reference wealth index shocks. Other shocks could be included but this set is characteristic of the most important exogenous stochastic influences in a small open economy. In financial markets there exist four assets: (i) equity; (ii) money; (iii) domestic bonds with zero net supply; and (iv) foreign bonds. We allow for a full range of interactions across shocks. The paper deals with only the steady-state stochastic equilibrium which is separated into deterministic and stochastic components. The remainder of this section sets out the key features of the model and its solution; more details can be found in Kenc (2004).

2.1 Prices and Asset Returns

There are three commodity prices in the model: the domestic price of the traded good (P); the foreign price level of the traded good (P^*); and the exchange rate (E). P^* is assumed to be exogenous and other two prices P and E are endogenously determined. The exchange rate (E) is measured in units of domestic currency per unit of foreign currency. Prices and returns are both generated by geometric Brownian motion (GBM) processes. Each of the prices P, P^* and E evolves according to

$$\frac{dx}{x} = (\text{drift term})\; dt + (\text{diffusion term})\; dZ \tag{1}$$

where x is either P, P^* or E; and π (σ_P), π^* (σ_{P^*}) and ϵ (σ_E) are respective drift(diffusion) terms of these price processes.[3] Thus, for example, $\pi\, dt$ is the expected mean rate of change of P and $\sigma_P\, dt$ is the volatility of this rate of change. Z_j is a Wiener process for $j = P, P^*, E$. We use ρ to denote the "instantaneous" correlation coefficient between any two Wiener processes: $\rho_{ij} = dZ_i dZ_j$.

This small open economy is linked with the rest of the world through the law of one price. Formally, it means that the exchange rate E relates foreign prices P^* to domestic prices of traded goods P through the purchasing power parity (PPP) relationship. The domestic price of the imported good is then given by

$$P = EP^* \tag{2}$$

which yields the following price process

$$\frac{dP}{P} = (\pi^* + \epsilon + \sigma_{P^*E})\, dt + \sigma_{P^*} dZ_{P^*} + \sigma_E dZ_E \tag{3}$$

There are four assets: money M, capital (equity) K, domestic bonds B and foreign bonds B^*. Of these assets only capital and foreign bonds are internationally traded. As for asset returns, the real rate of return to equity holders is calculated from the flow of new output dY per capital K as follows

$$dR_K = r_K dt + du_K \qquad r_K = Adt \qquad du_K = A\sigma_Y dZ_Y. \tag{4}$$

where A is the marginal physical product of capital and $\sigma_Y dZ_Y$ a productivity shock with σ_Y being the volatility of the shock. dY follows an AK type of aggregate

[3] We realize that in this type of monetary general equilibrium models a mean reverting process as in Stulz (1986) and Bakshi and Chen (1996a) is more plausible and realistic as compared to a GBM. However, for analytical tractability we maintain the GBM assumption.

production function with endogenous growth:[4]

$$dY = [Adt + \sigma_Y dZ_Y]K. \tag{5}$$

Returns to other assets can be described in terms of the interest rates they pay. Domestic and foreign bonds pay nominal rates of interest, i and i^*, respectively. Applying stochastic calculus and separating real returns from nominal returns, we obtain the real rates of return to domestic holders of money, domestic bonds, and foreign bonds as follows:

$$dR_j = r_j dt - \sigma_j dZ_j, \qquad j = M, B, B^*$$

where $r_M = -\pi + \sigma_P^2$, $r_B = i - \pi + \sigma_P^2$ and $r_B^* = i^* - \pi^* + \sigma_{P^*}^2$.

2.2 Monetary ad Fiscal Features of the Model

In order to address exchange rate determination and how status concerns impact upon the exchange rate, we assume a monetary economy. Since status concerns affect the optimum saving-consumption behavior [e.g. Smith (1999), and Gong and Zou (2002)], one would like to address how this behavior affects the equilibrium interest rate and the exchange rate. In our model, the monetary policy rule takes the following simple form:

$$dM/M = \phi dt + \sigma_X dZ_X \tag{6}$$

where ϕ is the mean monetary growth rate. The stochastic term $\sigma_X dZ_X$ may reflect exogenous stochastic failures to meet the monetary growth target set by the monetary authority. The correlation between the monetary growth shock and other shocks is

[4]We assume that output is produced from capital by means of the stochastic constant returns to scale technology; and the economy-wide capital stock is assumed to have a positive external effect on the individual factor capital.

important. For example, the correlation between dZ_X and dZ_B^* may reflect stochastic adjustments in the money supply as the authorities respond to exogenous stochastic movements in an intermediate target such as the exchange rate.

The proceeds from printing money are distributed as transfers to households. These are non-marketable assets. The real value of transfers is random because of the stochastic monetary rule described above. We assume that the household expects that its real transfer income dT will remain proportional to wealth, as follows:

$$dT = \tau W\, dt + \sigma_T W\, dZ_T, \qquad 0 < \tau < 1, \tag{7}$$

where τ is the transfer rate and $\sigma_T dZ_T$ a shock with σ_T being the volatility of the shock.

2.3 Preferences

At each point in time the representative consumer chooses its consumption C and allocates its portfolio of wealth, W, across four assets: money M, capital K, domestic bonds B and foreign bonds B^*. The model assumes that the representative household "comes into" period t with real wealth invested in assets so that

$$W = \frac{M}{P} + \frac{B}{P} + \frac{EB^*}{P} + K, \tag{8}$$

where E is the exchange rate and P equals the price level. Of these assets only capital and foreign bonds are internationally traded. The only source of income for the representative household is the capital income received from holding these assets. The amount of consumption $C(t)dt$ for the period $(t, t + dt)$ and the new portfolio are simultaneously chosen, and if it is assumed that all trades are made at (known) current prices, then we have that

$$\frac{dW}{W} = \psi dt + \sigma_W dZ_W, \tag{9a}$$

where for notational convenience

$$\psi = n_M r_M + n_B r_B + n_K r_K + n_B^* r_B^* - \tau - C_t/W_t \tag{9b}$$

$$\sigma_W dZ_W = - n_M \sigma_P dZ_P - n_B \sigma_P dZ_P + n_K A \sigma_Y dZ_Y - n_B^* \sigma_P^* dZ_{P^*} - \sigma_T dZ_T \tag{9c}$$

In the above notation, n_i is the share of portfolio held in asset i (More specifically, $n_M = (M/P)/W$, $n_B = (B/P)/W$, $n_K = K/W$ and $n_B^* = (E\,B^*/P)/W$).

The representative agent's intertemporal utility is defined as in Duffie and Epstein (1992b):

$$U_0 = \mathcal{E}_0 \left\{ \int_0^\infty f(C_t, M_t/P_t, S_t, U_t) dt \right\} \tag{10}$$

where

$$f(C_t, M_t/P_t, S_t, U_t) = \delta \frac{\left\{ \left([C_t^\theta (M_t/P_t)^{1-\theta}]^\alpha S_t^\lambda] \right)^{1-\frac{1}{\zeta}} - [(1-\gamma)U_t]^{\frac{\zeta-1}{\zeta(1-\gamma)}} \right\}}{\left\{ (1-\frac{1}{\zeta})[(1-\gamma)U_t]^{\frac{\zeta-1}{\zeta(1-\gamma)}-1} \right\}}. \tag{11}$$

$\mathcal{E}_0$ is the conditional expectation operator. Current utility U_t depends upon current consumption C_t, real money balances m_t, "status" S_t and expected values of *future utility* U_s, $s > t$. $f(c, U)$ is known as the normalized "aggregator" function generating ordinally equivalent utility functions. The parameter $\gamma > 0$ is the coefficient of relative risk aversion with respect to timeless gambles over x_t, while $\zeta > 0$ is the elasticity of intertemporal substitution defined over riskless paths of x_t, and $\delta > 0$ is the rate of time preference. λ measures the extent to which the investor cares about status - the degree of spirit of capitalism. Relative wealth is assumed to be status: the consumer's absolute wealth, W_t, and a *world wealth index*, V_t. Status is thus described by a function $S_t = f[W_t, V_t]$, where $f_W > 0$ and $f_V \leq 0$. More precisely, it is defined as

$$S_t = \frac{W_t}{V_t}.$$

We assume that the world-wealth index follows a diffusion process:

$$dV = [\mu_V dt + \sigma_V dZ_V]V. \tag{12}$$

The relative importance of money in composite consumption is measured by θ.

2.4 Household Optimization

The agent's objective is to select the rates of consumption, together with her portfolio of assets, to maximize the expected value of discounted utility (10) subject to $W(0) = W_0$, the wealth constraint (8), the stochastic wealth accumulation equation (9a) and the adding up condition for portfolio shares:

$$1 = n_M + n_B + n_K + n_B^*. \tag{13}$$

In maximizing utility, the representative household takes the rates of return on assets, and the relevant variances and covariances as given. However, the general equilibrium conditions, i.e. market-clearing conditions, of the model will determine these rates of return, variances and covariances.

This maximization problem represents a continuous-time stochastic dynamic optimization problem of the type pioneered by Merton (1969) and its solution strategy is based on the dynamic programming approach of Duffie and Epstein (1992b). To solve the agent's optimization problem, we introduce the value function

$$X(W_t, V_t, t) = \max_{\{C, \vec{n} \in \mathcal{C} \times R^N\}} \mathcal{E}_t \left\{ \int_t^\infty f[C_s, M_s/P_s, S_s, X(W_s, V_s)] ds \right\}, \tag{14}$$

subject to (9a) and (13).

Moreover, we specify

$$X(W_t, V_t) = e^{-\delta t} J(W_t, V_t).$$

2.4.1 Optimum Consumption and Portfolio Choice

Proposition 1. *In a small open economy with the spirit of capitalism, the optimum solution to the consumption-portfolio choice in* (14) *is*

$$\begin{aligned}\frac{\hat{C}}{W} = \kappa = \Phi\Delta + (1-\Phi)[\hat{r}_Q - \tau - \Gamma_1 \frac{\hat{\sigma}_W^2}{2}] \\ - (1-\Phi)\frac{\lambda}{\alpha+\lambda}[\mu_V - \Gamma_2 \frac{\sigma_V^2}{2}] - (1-\Phi)\lambda(1-\gamma)\hat{\sigma}_{WV},\end{aligned} \tag{15}$$

$$[r_K - \hat{r}_B] = \Gamma_1(\hat{\sigma}_{YW} - \hat{\sigma}_{PW}) + \lambda(1-\gamma)(\hat{\sigma}_{YV} - \hat{\sigma}_{PV}), \tag{16}$$

$$[r_B^* - \hat{r}_B] = \Gamma_1(\hat{\sigma}_{P^*W} - \hat{\sigma}_{PW}) + \lambda(1-\gamma)(\hat{\sigma}_{P^*V} - \hat{\sigma}_{PV}), \tag{17}$$

$$\hat{n}_M = \frac{(1-\theta)}{\theta}\frac{\hat{C}}{W}\frac{1}{\hat{i}}. \tag{18}$$

where a hat ˆ over a variable denotes optimized value, $\Delta = \frac{\alpha\theta}{\alpha+\lambda}\delta$*;* $\Phi = \frac{\zeta}{\zeta-(\zeta-1)\theta\alpha}$*;* $\hat{r}_Q = \hat{n}_M r_M + \hat{n}_B \hat{r}_B + \hat{n}_K r_K + \hat{n}_{B^*} r_{B^*}$*. Furthermore, using Duffie and Epstein (1992b) [cf. their equations (44) and (45)], it can be shown that the Bellman equation for this optimal control problem is*

$$\sup_{\{C,\vec{n}\in\mathcal{C}\times R^N\}} \mathcal{D}J(W_t, V_t) + f[C_t, M_t/P_t, S_t, J(W_t, V_t)] = 0 \tag{19}$$

where

$$\mathcal{D}J(W_t, V_t) = J_W \psi W + J_V \mu_V V + \tfrac{1}{2}\mathbf{tr}(\Sigma)$$

with

$$\Sigma = \begin{pmatrix} W\vec{n}^T\vec{\sigma} - \sigma_T W \\ \sigma_V V \end{pmatrix}^T \begin{pmatrix} J_{WW} & J_{WV} \\ J_{VW} & J_{VV} \end{pmatrix} \begin{pmatrix} W\vec{n}^T\vec{\sigma} - \sigma_T W \\ \sigma_V V \end{pmatrix}$$

and

$$\vec{n}^T\vec{\sigma} = -n_M\sigma_P - n_B\sigma_P + n_K\sigma_Y - n_{P^*}\sigma_{P^*}.$$

The spirit of capitalism impacts equilibrium consumption and portfolio choice in several ways. First, status concerns alter behavioral parameters such as the effective coefficient of relative risk aversion, $\Gamma_1 = 1 - (\alpha+\lambda)(1-\gamma)$, the effective rate of time preference, $\Delta = \frac{\alpha\theta}{\alpha+\lambda}\delta$, and $\Gamma_2 = \lambda(1-\gamma) - 1$. Assuming $\gamma > 1$, an increase in the intensity of status concerns, λ, raises Γ_1, and reduces Γ_2 and the effective rate of time preference, Δ. Second, the presence of the external wealth reference standard (V_t) introduces catching-up and hedging effects. For example, the last two terms in consumption-wealth ratio in (15) depend on the expected growth rate of the reference wealth index and its volatility (catching-up effect), and the covariance of reference wealth with the representative agent's own wealth (hedging effect). Given that both α and θ are smaller than one, the catching-up and hedging effects in consumption depend on the intertemporal elasticity of substitution. If the latter is smaller than one, then $\Phi < 1$ so that the catching-up effect is negative. That is, an increase in the international wealth standard makes the representative individual more frugal in her consumption and hence has a positive effect on saving. This is not surprising in itself as it has been demonstrated by Bakshi and Chen (1996b), Smith (2001), and Gong and Zou (2002), among others; what we show here is that depending on the attitude towards intertemporal substitution, the idea extends to international catching-up in an open economy. Similarly when $\gamma > 1$, $\zeta < 1$ and the domestic wealth and foreign reference wealth are negatively correlated so that $\sigma_{WV} < 0$, the representative agent will consume less due to the hedging effect.

2.4.2 Goods Market Equilibrium and Balance of Payments

In our small open economy net exports in real terms are given by

$$\text{Net Exports} = dY - dC - dK.$$

The balance-of-payments equilibrium condition in real terms is

$$d(EB^*/P) = (EB^*/P)dR_B^* + \text{Net Exports}. \tag{20}$$

Substituting and simplifying we derive the following expression for the rate of growth of the capital stock:

$$\frac{dK}{K} = \left[\omega\left(A - \frac{1}{n_K}\frac{C}{W}\right) + (1-\omega)r_B^*\right]dt + \omega\sigma_Y dZ_Y - (1-\omega)\sigma_B^* dZ_B^*. \tag{21}$$

where for notational convenience we define $\omega \equiv \frac{n_K}{n_K + n_B^*}$ to be the share of capital in the traded portion of the consumer's portfolio.

2.4.3 Determination of transfers

Real transfers dT depend on the money supply rule and the price level. That is,

$$dT = (1/P)dM = \phi(M/P)dt + \sigma_X(M/P)dZ_X. \tag{22}$$

Using the definition of n_M (portfolio share of money) and substituting (7) for dT we obtain

$$\tau = \phi n_M, \tag{23}$$

$$\sigma_T dZ_T = \sigma_X n_M dZ_X. \tag{24}$$

2.5 Equilibrium

A rational expectations equilibrium (REE) is defined as a set of stochastic processes $[(P, E, i, r_i; T, C, n_i); \text{with } i = M, B, K, B^*]$ satisfying the first-order conditions (A.1), the PPP relationship (2), the government budget constraint (22), the goods market clearing condition (20), the domestic bonds market clearing condition $B = 0$, and all the exogenous stochastic processes. An equilibrium exists and is unique provided that there exists a unique function J and a set of control variables $(C/W, n_M, n_B, n_K, n_{B^*})$ satisfying the Bellman equation and the stated regularity conditions.

The assumption of constant drift (mean) and diffusion (variance) parameters in the geometric Brownian motion, which describes the four stochastic shocks of the model leads to a constant investment opportunity set. This feature of the model, together with the state separable preferences (the constant elasticity utility function), generates a recurring equilibrium, implying that the consumer chooses the same portfolio shares n_M, n_B^*, n_K and consumption-wealth ratio, C/W, at each instant of time. Note that even though preferences are not time-separable in this case, they still take a generalized isoelastic (GIE) form. Since domestic bonds are in zero net supply they will not be held in equilibrium. Moreover, the multiplicity of all shocks (meaning that stochastic disturbances are proportional to the current state variables such as the capital stock and wealth) leads to an equilibrium in which means and variances of the relevant endogenous variables are jointly and consistently determined – a *mean-variance equilibrium*.

The exogenous factors apart from the four stochastic shocks explained above include (i) the preference and technology parameters $\gamma, \lambda, \delta, \theta, A$ (ii) the monetary policy parameter ϕ, and (iii) the mean foreign inflation rate π^*. The endogenous variables include (i) the stochastic adjustments in the economy including $\sigma_P dZ_P$ (the stochastic adjustment in the domestic price level), $\sigma_E dZ_E$ (the stochastic component of the PPP relationship), $\sigma_T dZ_T$ (the stochastic adjustments in transfers), $\sigma_W dZ_W$ (the stochastic component of wealth), (ii) the transfer rate τ, (iii) the optimal consumption-wealth ratio and the optimal portfolio shares, (iv) the expected equilibrium inflation rate π, the domestic nominal interest rate i, the expected exchange rate depreciation ϵ, and (v) the equilibrium growth rate ψ.

The determination of endogenous variables involves several stages. By using the constancy assumption of portfolio shares, we first solve the model for the price level and thereby π and $\sigma_P dZ_P$. The next stage is to determine stochastic adjustments. Having obtained the stochastic adjustments, one can then calculate the endogenous variances and covariances that appear in the optimality conditions for the consumption-wealth ratio, portfolio shares etc. The final stage is to substitute these variances and covariances into the deterministic components of the equilibrium.

3 International Catching-Up and the Spirit of Capitalism

In this section, we discuss how the spirit of capitalism or the concern for social status affects asset pricing, international portfolio diversification, exchange rate determination and economic growth. To facilitate this analysis we will simplify the model described in Section 2.

3.1 International Portfolio Diversification

First, we examine the equilibrium asset-pricing relationships. To make our model comparable with a baseline model without the spirit of capitalism [e.g., Giuliano and Turnovsky (2003)] we consider a real economy with only two assets, a domestic equity and foreign bonds.

Proposition 2. *In a small open economy, the spirit of capitalism generates a home bias in the optimum share of equity as follows:*

$$n_K = \frac{\textit{Sharpe ratio}}{\Gamma_1} \quad + \quad \textit{home bias} \tag{25}$$

where

$$\textit{Sharpe ratio} = \frac{r_K - [r_{B^*} - \Gamma_1(\sigma_{P^*}^2 - \sigma_{P^*Y})]}{\Lambda}, \tag{26}$$

$$\textit{home bias} = \mathcal{H}\left(\frac{\sigma_{YV} + \sigma_{B^*V}}{\Lambda}\right), \tag{27}$$

$$\Lambda = \sigma_Y^2 + \sigma_{P^*}^2 - 2\sigma_{P^*Y},$$

$$\mathcal{H} = -\frac{J_{WV}}{J_{WW}}\frac{V}{W} = -\frac{\lambda(1-\gamma)}{\Gamma_1}.$$

In equation (27), $\mathcal{H}$ denotes the 'propensity to hedge' against unanticipated fluctuations in the reference wealth index. A closer inspection of Eq (25) together with equations (26) and (27) indicates that the spirit of capitalism impacts the portfolio allocation through the effective coefficient of relative risk aversion (and therefore the *Sharpe ratio*) and the *home bias effect*. Overall, this impact seems to be ambiguous for reasonable parameter values. First, as pointed by Smith (2001), an increase in the the degree of the spirit of capitalism, λ, leads to a rise in the effective coefficient of relative risk aversion, $\Gamma_1 = 1 - (\alpha + \lambda)(1 - \gamma)$, when $\gamma > 1$, otherwise Γ_1 declines. In turn, assuming $\gamma > 1$ the rise in λ decreases the demand for equity, n_K. This confirms the point made by Carroll (2002) that the wealthy, identified as risk lovers ($\gamma < 1$) and higher λ, hold much riskier portfolios than the rest of the population. Second, the home bias depends on the correlations of asset returns and the world wealth index, V. If these correlations are positive and $\gamma > 1$, then the *home bias* effect is positive. This has been recognized by Bakshi and Chen (1996b) who contented that adding domestic equity serves to insure against a future decline in V. This is in line with the empirical findings of the home bias literature discussed in Lewis (1999). Note that the higher the degree of the spirit of capitalism, λ, the more intensive is the insurance effect. Moreover, unless $\gamma = 1$, optimal portfolio shares are convex functions of λ as shown by Smith (2001).

Proposition 3. *In a small open economy with the spirit of capitalism, the optimum portfolio shares can be decomposed into speculative demand, home bias, and a variance minimizing portfolio as follows:*[5]

$$n_K = \frac{r_K - r_{B^*}}{\Gamma_1 \Lambda} + home\ bias\ +\ \tilde{n}_K, \tag{28}$$

$$n_{B^*} = \frac{r_{B^*} - r_K}{\Gamma_1 \Lambda} + home\ bias\ +\ \tilde{n}_{B^*}, \tag{29}$$

[5]The variance minimizing portfolio was recognized by Branson and Henderson (1995), who dubbed it "hedging demand" as it is independent of preferences. They called $\frac{(r_k - r_b^*)}{\Gamma_1 \Lambda}$ "speculative demand" so the portfolio shares can be expressed as speculative demand, home bias effects and hedging demand, respectively.

where the first terms represent speculative demand and the last terms give variance minimizing portfolio shares:

$$\tilde{n}_K = \frac{\sigma_{P^*}^2 - \sigma_{P^*Y}}{\Lambda} \qquad \tilde{n}_{B^*} = \frac{\sigma_Y^2 - \sigma_{P^*Y}}{\Lambda}.$$

Corollary 1. *When $\gamma > 1$, an increase in the degree of the spirit of capitalism reduces speculative demand for equity.*

It is evident from the definition of speculative demand that it is decreasing in Γ_1. Intuitively, the intensity of status concerns raises the effective coefficient of relative risk aversion, Γ_1, and reduces the demand for the risky asset.

Giuliano and Turnovsky (2003) point out that the size of the equilibrium portfolio shares, n_K, n_{B^*}, relative to the growth *variance-minimizing* portfolio shares, $\tilde{n}_K$, $\tilde{n}_{B^*}$ is a crucial determinant of the effects of structural changes on the equilibrium. The expressions in (28) and (29) clearly show the additional channels, namely effective risk aversion and home bias, where the spirit of capitalism affects portfolio allocation [c.f., Giuliano and Turnovsky (2003)].

Proposition 4. *In a small open economy with the spirit of capitalism the equilibrium asset-pricing relationships and assets' beta coefficients are given by:*

$$r_i - r^f = \beta_i\big(r_Q - r^f\big) \qquad \textit{for } i = K, B^*, \tag{30}$$

$$r_Q = r_K n_K + r_{B^*} n_{B^*} = \left[\frac{r_K - r_{B^*}}{\Gamma_1 \Lambda} + \textit{home bias} \ + \ \tilde{n}_K\right](r_K - r_{B^*}) + r_{B^*}, \tag{31}$$

$$r^f = \chi/[\delta(1-\Gamma_1) b^{1-\gamma} W^{1-\Gamma_1} V^{1-\Gamma_2}], \tag{32}$$

$$\beta_K = \frac{\sigma_Y^2\left[\left(\frac{r_K - r_{B^*}}{\Gamma_1}\right) + \sigma_{P^*}^2\right] + \sigma_Y^2 h}{\left(\frac{r_K - r_{B^*}}{\Gamma_1}\right)^2 + \sigma_{P^*}^2\sigma_Y^2 + h^2 + 2h\left(\frac{r_K - r_{B^*}}{\Gamma_1}\right)\left(\frac{\sigma_Y^2 - \sigma_{P^*}^2}{\sigma_Y^2 + \sigma_{P^*}^2}\right) + 4\left(\frac{\sigma_Y^2\sigma_{P^*}^2 h}{\sigma_Y^2 + \sigma_{P^*}^2}\right)}, \tag{33}$$

$$\beta_{B^*} = \frac{\sigma_{P^*}^2\left[\left(\frac{r_{B^*} - r_K}{\Gamma_1}\right) + \sigma_Y^2\right] + \sigma_{P^*}^2 h}{\left(\frac{r_K - r_{B^*}}{\Gamma_1}\right)^2 + \sigma_{P^*}^2\sigma_Y^2 + h^2 + 2h\left(\frac{r_K - r_{B^*}}{\Gamma_1}\right)\left(\frac{\sigma_Y^2 - \sigma_{P^*}^2}{\sigma_Y^2 + \sigma_{P^*}^2}\right) + 4\left(\frac{\sigma_Y^2\sigma_{P^*}^2 h}{\sigma_Y^2 + \sigma_{P^*}^2}\right)}, \tag{34}$$

where r_Q is the rate of return on the equilibrium (market) portfolio, h is home bias, and $\sigma_Q dZ_Q$ is the stochastic term of this portfolio.

Without the spirit of capitalism, Giuliano and Turnovsky (2003) obtain the following expression

$$r_Q^{GT} = \left[\frac{r_K - r_{B*}}{\Gamma_1^{GT}\Lambda^{GT}} + \tilde{n}_K^{GT}\right](r_K - r_{B*}) + r_{B*}$$

where

$$\Lambda^{GT} = \sigma_Y^2 + \sigma_{P*}^2;\ \Gamma_1^{GT} = \gamma;\ \tilde{n}_K^{GT} = \frac{\sigma_{P*}^2}{\sigma_Y^2 + \sigma_{P*}^2}.$$

Again the spirit of capitalism affects the rate of return on the market portfolio through home bias and the effective risk aversion. Comparing r_Q to r_Q^{GT}, when $\gamma > 1$ the home bias increases the rate of return on the market portfolio whereas the increase in the effective relative risk aversion decreases it. The qualitative effect of the spirit of capitalism on asset pricing seems to be ambiguous due to the conflicting effects of home bias and the amplified effective risk aversion.

A simplified version of our model along the lines of Gong and Zou (2002), i.e., power utility and wealth-is-status shows that the spirit of capitalism affects risk premium through second order risk aversion so that the risk premium is influenced by variance and covariance terms. In this setting, the equity premium effect of the spirit of capitalism can be calculated as $\lambda\alpha^2(\sigma_y^2 + \sigma_z^2)$. Since the spirit effect is scaled down by the variance term, it is likely to be small for a reasonable parametrization of the model. While our model qualitatively improves on the equity premium puzzle, there remains a substantial portion of the equity premium that needs to be explained as discussed in Mehra (2003).

3.2 Growth and Consumption Effects

The effect of the spirit of capitalism on consumption, saving, and growth can be summarized in the following proposition:

Proposition 5. *In a small open economy, the spirit of capitalism affects consumption and growth through catching up, home bias, and hedging effects as follows:*

$$\frac{C}{W} = \left(\frac{C}{W}\right)^b + \text{ home bias effect } + \text{ direct catching-up effect } + \text{ hedging effect} \quad (35)$$

where

$$\left(\frac{C}{W}\right)^b = \Phi\Delta + (1-\Phi)\Big\{\Big[\frac{SR}{\Gamma_1}\Big]\Big[r_K - r_{B^*} + \Gamma_1\sigma_{P^*}^2 - 0.5\Gamma_1\Big(\frac{SR}{\Gamma_1}\Big)(\sigma_Y^2 + \sigma_{P^*}^2)\Big] + r_{B^*} - 0.5\Gamma_1\sigma_{P^*}^2\Big\}$$

$$\begin{aligned}\text{home bias effect} &= (1-\Phi)h\Big[r_K - r_{B^*} + \Gamma_1\sigma_{P^*}^2 - 0.5\Gamma_1\Big(\frac{SR}{\Gamma_1} + h\Big)(\sigma_Y^2 + \sigma_{P^*}^2)\Big] \\ &- (1-\Phi)\lambda(1-\gamma)\Big[h(\sigma_{YV} - \sigma_{P^*V}) + \sigma_{P^*V}\Big] - 0.5(1-\Phi)SR(\sigma_Y^2 + \sigma_{P^*}^2)h \quad (36)\end{aligned}$$

$$\text{direct catching-up effect} = -(1-\Phi)\frac{\lambda}{\alpha+\lambda}[\mu_V - \Gamma_2\frac{\sigma_V^2}{2}]$$

$$\text{hedging effect} = -(1-\Phi)\lambda(1-\gamma)\Big[\Big(\frac{SR}{\Gamma_1}\Big)(\sigma_{YV} - \sigma_{P^*V}) + \sigma_{P^*V}\Big]$$

$$SR = \frac{r_K - [r_{B^*} - \Gamma_1(\sigma_{P^*}^2 - \sigma_{P^*Y})]}{\Lambda}$$

$$\begin{aligned}\Psi = r_Q - \frac{C}{W} = r_Q^b + h(r_K - r_{B^*}) - \left(\frac{C}{W}\right)^b &- \text{ home bias effect} \\ - \text{ direct catching-up effect } &- \text{ hedging effect} \quad (37)\end{aligned}$$

Similarly, the variance of the equilibrium growth rate is given by

$$\sigma_\psi^2 = (\sigma_\psi^2)^b + \frac{h^2}{\sigma_{P^*}^2 + \sigma_Y^2} + \frac{2h}{(\sigma_{P^*}^2 + \sigma_Y^2)^2}\Big\{\Big(\frac{r_K - r_{B^*}}{\Gamma_1}\Big)(\sigma_Y^2 - \sigma_{P^*}^2) + 2\sigma_{P^*}^2\sigma_Y^2 h\Big\}, \quad (38)$$

$$(\sigma_\psi^2)^b = \frac{\left(\frac{r_K - r_{B^*}}{\gamma}\right) + \sigma_{P^*}^2\sigma_Y^2}{\sigma_{P^*}^2 + \sigma_Y^2},$$

where the 'b' superscript denotes base values. These base values are free from the extensions considered in this paper. They are similar to those in Smith (1999) who defines the spirit of capitalism as "preference for capital" as opposed to our definition of "relative wealth". Smith (1999) demonstrated that the growth effects of the spirit of capitalism are ambiguous: an increase in the degree of the spirit of capitalism may accelerate or decelerate growth by affecting consumption through the "precautionary savings premium" (the variance terms in the expression), depending upon the magnitudes of the risk aversion and intertemporal elasticity of substitution parameters. On the other hand, the spirit of capitalism reduces consumption thereby increasing growth through a reduction in the effective rate of time preference [see Zou (1994)]. However, the extension of this single asset (capital) model of Smith (1999) to a portfolio assets, as in our case, removes the role of the intertemporal elasticity of substitution parameter, leaving the magnitude of γ determining the overall outcome. This reflects the fact that the savings-growth effects of the spirit of capitalism are dominated by the direct portfolio effects on the expected rate of return.

It is evident that our extended version of the spirit of capitalism introduces additional terms into the consumption wealth ratio and the growth rate. From these expressions, it is not possible to ascertain the relative magnitude of the additional channels whereby the home bias international catching-up, and hedging effects influence the equilibrium growth rate and the consumption-wealth ratio. However, under certain conditions we can make several observations. First, as we noted above, when the intertemporal elasticity of substitution is smaller than one ($\zeta < 1$), a faster increase in the international wealth standard (μ_V) induces lower consumption/ higher domestic saving, and higher growth due to the direct catching-up effect. This implies that in the long run, growth in the reference country can stimulate growth elsewhere as in Greenfeld (2001). Second, when domestic income and foreign wealth are negatively correlated (assuming $\gamma > 1$, $\zeta < 1$), the consumer will consume less in response to the increase in status concerns due to the hedging effect.

A calibration analysis of a small open economy based on plausible model parameters suggests that an increase in mean productivity induces a modest increase in the consumption wealth ratio while it increases the equilibrium growth rate substantially. This is consistent with the habit formation literature where consumption takes time to catch up with higher income levels; see Deaton (1999). On the other hand an increase in status concerns induces a sizable reduction in the consumption wealth ratio and increases the equilibrium growth rate modestly when $\gamma > 1$. This is due to the increase in the effective risk aversion cited above. However, this result seems to be sensitive to key parameters such as the inter-temporal elasticity of substitution and the sign and magnitude of the covariance terms.

3.3 A Multi-Factor CAPM

Proposition 6. *In a small open economy with the spirit of capitalism*

(i) there exists a well-defined **stochastic discount factor** *(state-pricing process)*

$$\xi_t = \exp\Big[\int_0^t f_u ds\Big]\big[f_c + f_{m/p} + f_s\big], \tag{39}$$

with the differential equation,

$$\frac{d\xi_t}{\xi_t} \quad = \mu_\xi dt \quad + \quad \sigma_\xi dZ, \tag{40}$$

where

$$\mu_\xi = f_u + \frac{\mathcal{D}J_W}{J_W}$$

$$\sigma_\xi = \frac{(\eta - 1)}{c}\sigma_c - (\frac{\eta}{\varrho} - 1)\frac{1}{W}\sum_i [\sigma_i] + \lambda\sigma_V; \qquad i = P, P, Y, P^* \tag{41}$$

(ii) there exists a multifactor Capital Asset Pricing Model (CAPM) that characterizes asset returns

$$\text{(APE)} \qquad \mathcal{E}_t\Big(\frac{dP_t}{P_t}\Big) + \frac{D}{P_t}dt = r_t^f - \mathcal{E}_t\Big[\frac{d\xi_t}{\xi_t}\frac{dP_t}{P_t}\Big]$$

where f_i stands for the partial differentiation of the function $f(C, M/P, S, U)$ with respect to $i = C, M/P, S, U$; D_t is the dividend at time t; and P_t is the price of common stock at time t.

Corollary 2. *Assume an economy without money to render the results comparable to the existing literature e.g., Epstein and Zin (1991), Bakshi and Chen (1996b), Smith (2001). Applying (APE) to the expected rate of return on asset X_j yields*

$$\mathcal{E}_t\Big(\frac{dX}{X}\Big) - r_t^f = -\mathcal{E}_t\Big[\frac{d\xi_t}{\xi_t}\frac{dX_j}{X_j}\Big]$$

Simplifying

$$\mu_x - r_t^f = -\Big[\frac{(\eta-1)}{c}\rho_{cx}\sigma_c\sigma_x + (\frac{\eta}{\varrho} - 1)\frac{1}{W}\Big(\rho_{wx}\sigma_w\sigma_x + \rho_{px}\sigma_p\sigma_x + \rho_{p^*x}\sigma_{p^*}\sigma_x\Big) + \lambda\rho_{vx}\sigma_v\sigma_x\Big] \quad (42)$$

where $\eta = \frac{1}{\zeta} - 1$ and $\varrho = 1 - \gamma$.

This is a multi (five) factor CAPM model—an extension of equation (22) in Duffie and Epstein (1992b) to include domestic and foreign inflation uncertainties and the relative reference wealth uncertainty. The first factor (first term on the right hand side) is a "premium" due to the variation in consumption, as the representative agent must be compensated for bearing consumption risk. The second term represents compensation due to variation in wealth: this matters for the risk premium because in recursive utility, wealth is a proxy for future utility. Moreover, in the spirit of capitalism framework, the investor cares about wealth induced status; as such the investor must be compensated for risks associated with the variation in wealth. Similarly, agents need to be compensated for bearing domestic and foreign inflation risks in equilibrium, which is evident in the third and fourth terms in equation (42). Finally, as in Bakshi and Chen (1996b), and Smith (2001), the risk premium depends on the covariance of an asset's return with reference wealth and the investor must be compensated for risks associated with the variations in the reference wealth index.

3.4 Exchange Rate Determination

In this section, we derive the stochastic process generating the equilibrium exchange rate. To this end we use a full version of the model except we restrict preferences to the following logarithmic form for explicit expressions:

$$U_t = \int_t^{\infty} e^{-\delta s} \Big[\alpha[\theta \ln C_s + (1-\theta)\ln(M_s/P_s)] + \lambda \ln S_s \Big] ds.$$

In particular, we obtain the equilibrium drift and diffusion terms of the following stochastic process:

$$\frac{dE}{E} = \tilde{\epsilon}\, dt + \tilde{\sigma}_{\epsilon}\, dZ \;=\; \tilde{\epsilon}\, dt \;+\; \sigma_X dZ_X \;-\; \tilde{\omega}\, [\sigma_Y dZ_Y + \sigma_{P*} dZ_{P*}]$$

where $\tilde{\epsilon}$ is the equilibrium depreciation of the exchange rate. This will be determined by two equilibrium loci governing domestic and foreign real rates of return (RR) and portfolio balance (PP) as is done by Grinols and Turnovsky (1994). With logarithmic preferences the optimum share of capital in the traded component of the portfolio, ω, (derived from the portfolio optimality conditions (16) and (17)) is given by

$$\omega = \frac{r_K - (i^* - \pi^*) + \sigma_{P^*Y}}{\sigma_Y^2 + 2\sigma_{P^*Y} + \sigma_{P^*}^2} \tag{43}$$

This expression is the same as that obtained in Grinols and Turnovsky (1994) apart from covariances [cf eq. (25)]. This is not surprising because logarithmic preferences eliminate tangency portfolio effects, hence no international portfolio diversification effects.

Proposition 7. *In a small open economy with logarithmic preferences, status concerns reduce the equilibrium inflation rate and exchange rate depreciation.*

Proof. The relationship between ϵ and π that maintains equilibrium between the real rates of return is

$$(RR) \qquad \pi = i^* + \epsilon - r_K + \sigma_Y^2 \omega$$

Similarly, the relationship between ϵ and π that maintains the portfolio balance is given by

$$(PP) \qquad \pi = \phi - (i^* - \pi^*) + (i^* - \pi^* - \sigma_{P*}^2 - r_K)\omega + (\sigma_Y^2 + \sigma_{P*}^2)\omega^2 + \frac{1}{\Upsilon}\frac{\alpha\theta\delta}{\alpha+\lambda},$$

where

$$\Upsilon = 1 - \frac{(1-\theta)\alpha\delta}{(\alpha+\lambda)(i^* + \epsilon - \sigma_X^2)}.$$

The equilibrium values of π and ϵ are determined by the intersection of (RR), which is a 45 degree line in the $\pi - \epsilon$ space, and (PP), which is a rectangular hyperbola. The effects of various exogenous shocks on the equilibrium values of the inflation rate and the exchange rate depreciation can be analyzed as in Grinols and Turnovsky (1994). Here we restrict our focus to the effects of the spirit of capitalism. As seen from (RR) none of the parameters representing the spirit of capitalism such as λ, μ_V or σ_V appear in the equation. However, the degree of the spirit of capitalism λ affects the (PP) schedule causing it to shift down; this results in lower equilibrium values for inflation and exchange rate depreciation. □

As is well known from Smith (1999) and Gong and Zou (2002) the presence of the spirit of capitalism reduces consumption and hence increases savings. This is evident from equation (15) where imposing logarithmic preferences yields the following optimal consumption-wealth ratio:

$$\frac{C}{W} = \Delta = \frac{\alpha\theta}{\alpha+\lambda}\delta$$

The resulting increase in saving reduces the equilibrium interest rate. Given perfect capital mobility *uncovered interest parity* necessitates a domestic currency depreciation. Similarly, the decrease in i leads to a decline in π due a *Fisher effect*.

A numerical analysis based on a full version of the model suggests a depreciation in the region of a 30% in the equilibrium exchange rate reflecting the similar drop in the domestic inflation rate. We also observe a decline of 5%-7% in the equilibrium interest rate.

4 Foreign Exchange Risk Premium

The model developed here, with its mean-variance equilibrium and general equilibrium features, offers a convenient vehicle for examining the determinants of the foreign exchange risk premium. In addition, our unconventional preferences assumption gives scope for a quantitative analysis of the foreign exchange risk premium. In general, the literature distinguishes between real and nominal measures and across several definitions. We shall focus on the real measure as being of greater relevance and use the proper definition of Engel (1992) as used by Grinols and Turnovsky (1994). Our expression for the real foreign exchange risk premium is therefore:

Definition 1. *The real foreign exchange risk premium over the period* (t, T) *is the difference between the risk-neutral forward rate* $\mathcal{F}^{RN}(t, T)$ *and the forward rate* $\mathcal{F}(t, T)$*, divided by the risk-neutral forward rate*

$$\Theta(t, T) \equiv 1 - \frac{\mathcal{F}(t, T)}{\mathcal{F}^{RN}(t, T)} \tag{44}$$

where

$$\mathcal{F}^{RN}(t, T) = \frac{\mathcal{E}_t[E_T/P_T]}{\mathcal{E}_t[1/P_T]} \tag{45}$$

$\mathcal{F}(t, T)$ *is the forward exchange rate at time* t *of the spot exchange rate* E_T *at time* T.

Proposition 8. *In a small open economy with spirit of capitalism, the foreign exchange risk premium is given by:*

$$\Theta(t, T) = 1 - \exp[(i - i^* - \epsilon + \sigma_{EP})(T - t)] \tag{46}$$

where

$$
\begin{aligned}
i - i^* - \epsilon + \sigma_{EP} &= r_B - r_{B^*} \\
&= -\Gamma_1[\omega\sigma_{XY} - \omega^2\sigma_Y^2 + (1 - 3\omega + \omega^2)\sigma_{P^*Y} + (1-\omega)\sigma_{XP^*} - \omega(1-\omega)\sigma_{P^*}^2] \\
&\quad -\lambda(1-\gamma)(\sigma_{XV} - \omega\sigma_{YV} - \omega\sigma_{P^*V}) \qquad (47)
\end{aligned}
$$

and

$$
\omega = \frac{A - (i^* - \pi^* + \sigma_{P^*}^2) + \Gamma_1[A\sigma_{P^*Y} + \sigma_{P^*}^2]}{\Gamma_1\Lambda} + h.
$$

It is evident that the magnitude and the sign of the risk premium depends on the market price of risk (which itself is a function of the attitude toward risk and the intensity of status concerns) and variance/covariance of domestic and global shocks. Most of these variance covariance terms are scaled by terms related to the share of equity in the traded portion of the investor portfolio ω, which among other things, is a function of home bias. Thus our model sheds new light on the determinants of the foreign exchange risk premium. Note that our expression (46) is the exact version of the approximated "unconditional pure currency risk premium" expression in Brandt and Santa-Clara (2002).

One can make several observations based on eqn. (47) regarding the risk premium within the spirit of capitalism and international catching up framework:

- The presence of the spirit of capitalism raises effective risk aversion for $\gamma > 1$, and the home bias raises the share of equity in the tradable portfolio. These and the covariance terms involving the international reference wealth index are additional determinants of foreign exchange risk premium in our model.

- The effect of foreign inflation risk on the risk premium depend on the sign of $\Gamma_1\omega(1-\omega)$ whereas the effect of domestic output risk depend on $\Gamma_1\omega^2$. These expressions are ultimately determined by the interplay between the intensity of status concerns and the attitude toward risk. Everything else equal if the investor is highly status conscious and moderately risk averse (wealthy investors as in Carroll (2002)), both of these terms will be negative raising the foreign exchange risk premium.

- In addition to status concerns and attitude toward risk, the effects of other factors on the risk premium depend on the sign of the covariance terms. For example if domestic income and foreign reference wealth co-vary positively so that $\sigma_{YV} > 0$ and the investor is highly risk averse $\gamma > 1$, then the risk premium rises. However when $\sigma_{YV} < 0$, and the investor is moderately risk averse $\gamma < 1$, a positive shock to the global risk factor increases the foreign exchange risk premium, "suggesting that investors demand a risk premium when their wealth declines relative to the global portfolio". [See Bakshi, Carr, and Wu (2005)].

- When domestic money and foreign reference wealth are negatively correlated and the investor is highly risk averse, the investor will demand a higher risk premium. The intensity of status concerns acts to magnify the effect on the risk premium.

5 Conclusions

We considered preferences with the "spirit of capitalism" in a continuous-time, infinite-horizon, small open economy model with complete financial markets and a single production good. We analyzed the effects of international catching-up and the spirit of capitalism on savings, growth, portfolio allocation and asset pricing.

Our results show that incorporating the spirit of capitalism alters behavioral parameters such as the effective risk aversion and intertemporal substitution and introduces potentially important home bias, catching-up, and hedging effects into consumption, growth and asset pricing. The "home bias" effect arises due to hedging against future fluctuations in the external wealth reference index. When assets held in the portfolio are positively correlated with the world reference wealth index, the home bias effect is positive and individuals hold more of the domestic equity. In most cases though, the additional effects cannot be ascertained qualitatively as they depend on key behavioral parameters such as risk aversion and intertemporal substitution. For example, when the intertemporal elasticity of substitution is smaller than one ($\zeta < 1$), a faster increase in the international wealth standard (μ_V) induces lower consumption/ higher domestic saving, and higher growth in the representative economy due to a catching-up effect. Similarly status concerns lead to non-trivial asset pricing effects.

We also obtain an expression for the foreign exchange risk premium and a five-factor Capital Asset Pricing Model. The spirit of capitalism alters the market price of risk for foreign exchange and introduces additional domestic and global factors that can have potentially important effects on the foreign exchange risk premium. As for the CAPM, the representative investor needs to be compensated for bearing consumption and wealth risks, and those associated with domestic and foreign inflation and the relative reference wealth uncertainty.

Even though "the capitalist spirit" matters, proper care should be taken to account for government expenditure and finance, and market imperfections such as those in credit, insurance, input and output markets. Moreover, assuming more plausible forcing processes for the exogenous variables such as mean reverting drifts and square-root diffusions as in Bakshi and Chen (1997) may yield richer asset price dynamics. These are avenues for future research.

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A Appendix: Proofs

Proof of Proposition 1. In the usual fashion of maximization under constraints, we define the Lagrangian

$$L = \mathcal{D}J(W_t, V_t) + f[C_t, M_t/P_t, S_t, J(W_t, V_t)] + \chi[1 - (n_M + n_B + n_K + n_B^*)]$$

where

$$f[C_t, M_t/P_t, S_t, J(W_t, V_t)] = \delta \frac{\left\{\left([C_t^\theta (M_t/P_t)^{1-\theta}]^\alpha S_t^\lambda]\right)^{1-\frac{1}{\zeta}} - [(1-\gamma)U_t]^{\frac{\zeta-1}{\zeta(1-\gamma)}}\right\}}{\left\{(1-\frac{1}{\zeta})[(1-\gamma)J_t]^{\frac{\zeta-1}{\zeta(1-\gamma)}-1}\right\}},$$

$$\mathcal{D}J(W_t, V_t) = J_W \psi W + J_V \mu_V V + \tfrac{1}{2}\mathbf{tr}(\Sigma),$$

$$\Sigma = \begin{pmatrix} W\vec{n}^T\vec{\sigma} - \sigma_T W \\ \sigma_V V \end{pmatrix}^T \begin{pmatrix} J_{WW} & J_{WV} \\ J_{VW} & J_{VV} \end{pmatrix} \begin{pmatrix} W\vec{n}^T\vec{\sigma} - \sigma_T W \\ \sigma_V V \end{pmatrix},$$

$$\vec{n}^T\vec{\sigma} = -n_M\sigma_P - n_B\sigma_P + n_K\sigma_Y - n_{P^*}\sigma_{P^*},$$

χ is the multiplier and find the extreme points from extreme points from the first-order conditions

$$0 = L_\kappa(\hat{\kappa}, \hat{\vec{n}}) = \delta\alpha\theta \frac{\left([C_t^\theta (M_t/P_t)^{1-\theta}]^\alpha S_t^\lambda]\right)^{1-\frac{1}{\zeta}} C^{-1}}{[(1-\gamma)J]^{\frac{(1-\zeta)}{\zeta(1-\gamma)}-1}} - J_W, \tag{A.1a}$$

$$0 = L_{n_M}(\hat{\kappa}, \hat{\vec{n}}) = \delta\alpha(1-\theta) \frac{\left([C_t^\theta (M_t/P_t)^{1-\theta}]^\alpha S_t^\lambda]\right)^{1-\frac{1}{\zeta}} n_M^{-1}}{[(1-\gamma)J]^{\frac{(1-\zeta)}{\zeta(1-\gamma)}-1}} + r_M J_W W + \frac{1}{2}\sigma_{PW} J_{WW} WW + \sigma_{PV} J_{WV} WV - \chi, \tag{A.1b}$$

$$0 = L_{n_K}(\hat{\kappa}, \hat{\vec{n}}) = r_K J_W W + \sigma_{YW} J_{WW} WW + \sigma_{YV} J_{WV} WV - \chi, \tag{A.1c}$$

$$0 = L_{n_B}(\hat{\kappa}, \hat{\vec{n}}) = r_B J_W W + \sigma_{PW} J_{WW} WW + \sigma_{PV} J_{WV} WV - \chi, \tag{A.1d}$$

$$0 = L_{n_B^*}(\hat{\kappa}, \hat{\vec{n}}) = r_B^* J_W W + \sigma_{P^*W} J_{WW} WW + \sigma_{P^*V} J_{WV} WV - \chi, \tag{A.1e}$$

$$0 = L_\chi(\hat{\kappa}, \hat{\vec{n}}) = 1 - (n_M + n_B + n_K + n_B^*). \tag{A.1f}$$

If every asset return is generated by a strong diffusion process, f is strictly concave in its elements, the there exists a set of optimal rules (controls), $\hat{\kappa}$ and $\hat{\vec{n}}$, satisfying the portfolio adding-up condition and the transversality condition, we have that

$$\sup_{\{C,\vec{n}\in\mathcal{C}\times R^N\}} \mathcal{D}J(W_t,V_t)+f[C_t,M_t/P_t,S_t,J(W_t,V_t)]=0 \tag{A.2}$$

where

$$\mathcal{D}J(W_t,V_t)=J_W\psi W+J_V\mu_V V+\tfrac{1}{2}\mathbf{tr}(\Sigma).$$

We conjecture that the value function and consumption functions are

$$J(W_t,V_t)=\frac{b^{1-\gamma}}{1-\gamma}W_t^{(\alpha+\lambda)(1-\gamma)}V_t^{-\lambda(1-\gamma)} \tag{A.3}$$

$$C_t=\kappa W_t \tag{A.4}$$

and use the following definition

$$M/P=n_M W$$

where b and κ are constants to be determined. Using these conjectures, and setting $\eta=1-1/\zeta$ and $\varrho=1-\gamma$, the expression in (A.2) can be expressed as

$$\sup_{\kappa,\vec{n}}\Big\{(1-\Gamma_1)b^{\varrho}W^{1-\Gamma_1}V^{1-\Gamma_2}\psi+(1-\Gamma_2)b^{\varrho}W^{1-\Gamma_1}V^{1-\Gamma_2}\mu_V+\tfrac{1}{2}\mathbf{tr}(\Sigma)+$$
$$\delta\frac{\big([\kappa^{\theta}n_M^{1-\theta}W]^{\alpha}(W/V)^{\lambda}]\big)^{\eta}-\big[\varrho b^{\varrho}W^{1-\Gamma_1}V^{1-\Gamma_2}\big]^{\frac{\eta}{\varrho}}}{\eta\big[\varrho b^{\varrho}W^{1-\Gamma_1}V^{1-\Gamma_2}\big]^{\frac{\eta}{\varrho}-1}}\Big\}=0 \tag{A.5}$$

with

$$\Sigma=\begin{pmatrix}W\vec{n}^T\vec{\sigma}-\sigma_T W\\ \sigma_V V\end{pmatrix}^T\begin{pmatrix}-(1-\Gamma_1)\Gamma_1 b^{\varrho}W^{-\Gamma_1-1}V^{1-\Gamma_2} & (1-\Gamma_1)(1-\Gamma_2)b^{\varrho}W^{-\Gamma_1}V^{-\Gamma_2}\\ (1-\Gamma_2)(1-\Gamma_1)b^{\varrho}W^{-\Gamma_1}V^{-\Gamma_2} & -\Gamma_2(1-\Gamma_2)b^{\varrho}W^{1-\Gamma_1}V^{-\Gamma_2-1}\end{pmatrix}\begin{pmatrix}W\vec{n}^T\vec{\sigma}-\sigma_T W\\ \sigma_V V\end{pmatrix}$$

where $\Gamma_1=1-(\alpha+\lambda)(1-\gamma)$ is the effective coefficient of relative risk aversion and $\Gamma_2=\lambda(1-\gamma)-1$. Simplifying (A.5) yields:

$$\sup_{\kappa,\vec{n}}\Big\{\eta(\alpha+\lambda)[\psi-\Gamma_1\frac{\sigma_W^2}{2}]-\eta\lambda[\mu_V-\Gamma_2\frac{\sigma_V^2}{2}]$$
$$-\eta(\alpha+\lambda)\lambda(1-\gamma)\sigma_{WV}+\delta\Big(\Big[\frac{(\kappa^{\theta}n_M^{1-\theta})^{\alpha}}{b}\Big]^{\eta}-1\Big)\Big\}=0. \tag{A.6}$$

First, substitute the first-order conditions into Eq. (A.6) to obtain (15) in the text. Then (i) subtract (A.1d) from (A.1c), (ii) subtract (A.1e) from (A.1d), (iii) subtract (A.1d) from (A.1b), and make use of (A.1a) together with the wealth constraint to obtain equations (16)-(18) in the text. □

Proof of Proposition 2. With no money and domestic bonds, the first-order optimality condition for n_K is given by:

$$\begin{aligned} r_K - r_B^* &= \Gamma_1(\sigma_{YW} - \sigma_{P^*W}) + \lambda(1-\gamma)(\sigma_{YV} - \sigma_{P^*V}) \\ &= \Gamma_1[(\sigma_Y^2 + \sigma_{P^*}^2 - 2\sigma_{YP^*})n_K + (\sigma_{P^*}^2 - \sigma_{YP^*})] + \lambda(1-\gamma)(\sigma_{YV} + \sigma_{P^*V}) \end{aligned}$$

$$r_K - r_B^* - \lambda(1-\gamma)(\sigma_{YV} - \sigma_{P^*V}) - \Gamma_1[\sigma_{P^*}^2 - \sigma_{YP^*}] = \Gamma_1[\sigma_Y^2 + \sigma_{P^*}^2 + 2\sigma_{YP^*}]n_K$$

Solving this equation for n_K gives Eq. (25) in the text. □

Proof of Proposition 3. As in Giuliano and Turnovsky (2003), define $\tilde{n}_K$, $\tilde{n}_{B^*}$ to be the portfolio shares that minimize the growth volatility. $\tilde{n}_K$ and $\tilde{n}_{B^*}$ can be obtained by minimizing (9c) with respect to n_K, n_{B^*}. □

Proof of Proposition 4. Equation (30) is the standard Capital Asset Pricing relationship. The risk free rate r^f follows Gong and Zou (2002). Given the definition of the market portfolio $Q = n_K W + n_{B^*} W$, r_Q is given by $r_Q = r_K n_K + r_{B^*} n_{B^*}$. Using equations (28)-(29) yields equation (31) in the text. In a representative agent setting, equilibrium requires that $\sigma_Q dZ_Q = \sigma_W dZ_W$. Using this condition, we obtain the beta coefficients as

$$\beta_K = \frac{n_K \sigma_Y^2}{n_K^2 \sigma_Y^2 + n_{B^*}^2 \sigma_{P^*}^2 + 2 n_K n_{B^*} \sigma_{YP^*}} \tag{A.7}$$

$$\beta_{B^*} = \frac{n_{B^*} \sigma_{P^*}^2}{n_K^2 \sigma_Y^2 + n_{B^*}^2 \sigma_{P^*}^2 + 2 n_K n_{B^*} \sigma_{YP^*}} \tag{A.8}$$

Substituting (28) and (29) into (A.7) and (A.8) respectively and ignoring the cross correlations appearing in Λ yield equations (33)-(34) in the text. □

Proof of Proposition 5. The expression for the consumption wealth ratio can be obtained by substituting the equilibrium portfolio shares implied by equation (25) into the simplified (two assets) version of (15) for variance and covariance terms, and by substituting equation (31) for r_Q. The expression for the equilibrium growth rate is obtained using the fact that in equilibrium $dK/K = dW/W$ holds. The equilibrium growth rate ψ is given by the drift term of dW/W, i.e., $\psi dt = E[dW/W]$, which is equal to $\psi = r_Q - C/W$. Once the expressions for r_Q and C/W are obtained, finding an expression for ψ involves substitution of them into the expression for ψ. $\square$

Proof of Proposition 6. **(i)** As in Duffie and Epstein (1992a) [cf. eq. (35)], in terms of the normalized aggregator f, the **stochastic discount factor** (state-pricing process) is given by:

$$\xi_t = \exp\Big[\int_0^t f_u ds\Big]\big[f_c + f_{m/p} + f_s\big] \tag{A.9}$$

where f_i stands for the partial differentiation of the function $f(C, M/P, S, U)$ with respect to $i = C, M/P, S, U$. Applying Ito's formula to ξ yields

$$\frac{d\xi_t}{\xi_t} = \mu_\xi dt \quad + \quad \sigma_\xi dZ \tag{A.10}$$

where

$$\mu_\xi = f_u + \frac{\mathcal{D}J_W}{J_W}$$

$$\sigma_\xi = \frac{(\eta-1)}{c}\sigma_c - (\frac{\eta}{\varrho} - 1)\frac{1}{W}\sum_i [\sigma_i] + \lambda\sigma_V \tag{A.11}$$

$i = P, P, Y, P^*$

The first order condition for the Bellman equation for optimal interior c is $f_c = J_W$. Assuming that the optimal consumption policy is given by a smooth function $\mathcal{C}$ of states [i.e., $c_t = \mathcal{C}(W_t, S_t, t)$], we can differentiate $J_W = f_c$ with respect to W and obtain $J_{WW} = f_{cc}\mathcal{C}_W + f_{cu}J_W$. Substituting these definitions together with the definitions of σ_c, σ_i for $i = M, K, B, B^*$ and σ_V in expression for σ_ξ we obtain a new expression for σ_ξ in term of key volatility terms such as σ_c as follows:

$$\sigma_\xi = \frac{(\eta-1)}{c}\sigma_c - (\frac{\eta}{\varrho} - 1)\frac{1}{W}\sum_i [\sigma_i] + \lambda\sigma_V \tag{A.12}$$

$i = P, P, Y, P^*$.

In order to derive Eq. (41), first reproduce the expression for σ_ξ:

$$\sigma_\xi = \sum_i \left[\frac{WJ_{WW}}{J_W} n_i \sigma_i\right] + \frac{VJ_{VW}}{J_W}\sigma_V.$$

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$$\begin{aligned}
\sigma_\xi &= \textstyle\sum_i \left[\frac{W[f_{cc}C_w + f_{cu}J_W]}{J_W} n_i\sigma_i\right] + \lambda\sigma_V \\
&= \textstyle\sum_i \left[\frac{W[f_{cc}C_w n_i\sigma_i + f_{cu}J_W n_i\sigma_i]}{J_W}\right] + \lambda\sigma_V \\
&= \textstyle\sum_i \left[\frac{f_{cc}}{f_c}C_w n_i W\sigma_i + \frac{f_{cu}WJ_W n_i\sigma_i}{J_W}\right] + \lambda\sigma_V \\
&= \textstyle\frac{f_{cc}}{f_c}\sigma_c + \sum_i [f_{cu}Wn_i\sigma_i] + \lambda\sigma_V
\end{aligned} \tag{A.13}$$

Taking first and second differentiation of the normalized "aggregator" function f with with respect to c and u we obtain

$$f_c = \beta\frac{c^{\eta-1}}{(\varrho u)^{\eta/\varrho-1}} \qquad f_{cc} = (\eta-1)\beta\frac{c^{\eta-2}}{(\varrho u)^{\eta/\varrho-1}} \qquad \frac{f_{cc}}{f_c} = \frac{(\eta-1)}{c}$$

$$\begin{aligned}
f_{cu} &= -\beta\varrho(\tfrac{\eta}{\varrho}-1)\tfrac{c^{\eta-1}(\varrho u)^{\eta/\varrho-2}}{(\varrho u)^{2(\eta/\varrho-1)}} = -\beta\varrho(\tfrac{\eta}{\varrho}-1)\tfrac{cc^{\eta-2}(\varrho u)^{\eta/\varrho-2}}{(\varrho u)^{(\eta/\varrho-1)}(\varrho u)^{(\eta/\varrho-1)}} = -(\tfrac{\eta}{\varrho}-1)\tfrac{cf_{cc}}{(\eta-1)u} \\
&= -(\tfrac{\eta}{\varrho}-1)\tfrac{cf_{cc}}{(\eta-1)\frac{AW\varrho}{\varrho}} = -(\tfrac{\eta}{\varrho}-1)\tfrac{cf_{cc}}{(\eta-1)f_c W} \\
&= -(\tfrac{\eta}{\varrho}-1)\tfrac{1}{W},
\end{aligned}$$

where $\eta = \frac{1}{\zeta} - 1$ and $\varrho = 1 - \gamma$. Substituting these partial differentials in expression (A.13) yields

$$\sigma_\xi = \frac{(\eta-1)}{c}\sigma_c - (\frac{\eta}{\varrho} - 1)\frac{1}{W}\sum_i [\sigma_i] + \lambda\sigma_V \qquad \text{for } \ i = P, P, Y, P^*$$

which is equation (41) above.

(ii) Versions of the fundamental asset pricing expression (APE) can be found in Rubinstein (1976), Lucas (1978), and Breeden (1979). $\square$

Proof of Proposition 8. To derive an expression for $\Theta(t,T)$ we exploit *covered interest parity* (**CIP**), which stems from the equilibrium (no-arbitrage) condition of the forward foreign currency market:

$$\mathcal{F}(t,T)B(t,T) = E_t B^*(t,T)$$

where $B(t,T) = \exp[-i(T-t)]$ and $B^*(t,T) = \exp[-i^*(T-t)]$ are the time t prices of a domestic and foreign zero–coupon bond paying a certain dollar at date T respectively. Substituting and simplifying this into the above equation we obtain

$$\frac{\mathcal{F}(t,T)}{E_t}\exp[i^*(T-t)] = \exp[i(T-t)].$$

The discrete-time version of this expression is the usual covered interest parity condition, $1+i = (1+i^*)\frac{\mathcal{F}(t,t+1)}{E_t}$. In order to determine $\mathcal{E}_t[E_T/P_T]$, we apply stochastic calculus to

$$\frac{d[E/P]}{E/P} = \frac{dE}{E} - \frac{dP}{P} - \frac{dE}{E}\frac{dP}{P} + \left(\frac{dP}{P}\right)^2.$$

Taking expectations gives

$$\mathcal{E}_t\{[E_T/P_T]\} = [E_t/P_t]\exp\left[\left(\epsilon - \pi - \pi^* + \epsilon + \rho_{EP}\sigma_E\sigma_P + \sigma_P^2\right)(T-t)\right].$$

Similarly calculating $\mathcal{E}_t\{[1/P_T]\}$ we obtain

$$\mathcal{F}^{RN}(t,T) = \frac{\mathcal{E}_t[E_T/P_T]}{\mathcal{E}_t[1/P_T]} = E_t\exp[(\epsilon - \sigma_{EP})(T-t)].$$

Combining these terms and substituting into Eq. (44) in the text yields the foreign exchange risk premium [Eq. (46) in the text]. To obtain the term $i - i^* - \epsilon + \sigma_{EP}$ in Eq. (47) we use the general equilibrium solutions for i, ϵ, σ_{EP} and the co-variance terms. $\square$